Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/202-7.6.1-u-a+b-arccsch-c-x-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [178]. This is test number [202].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (178)	0.00 (0)
Mathematica	100.00 (178)	0.00 (0)
Fricas	69.10 (123)	30.90 (55)
Maple	57.87 (103)	42.13 (75)
Maxima	34.27 (61)	65.73 (117)
Mupad	27.53 (49)	72.47 (129)
Giac	25.84 (46)	74.16 (132)
Sympy	21.35 (38)	78.65 (140)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

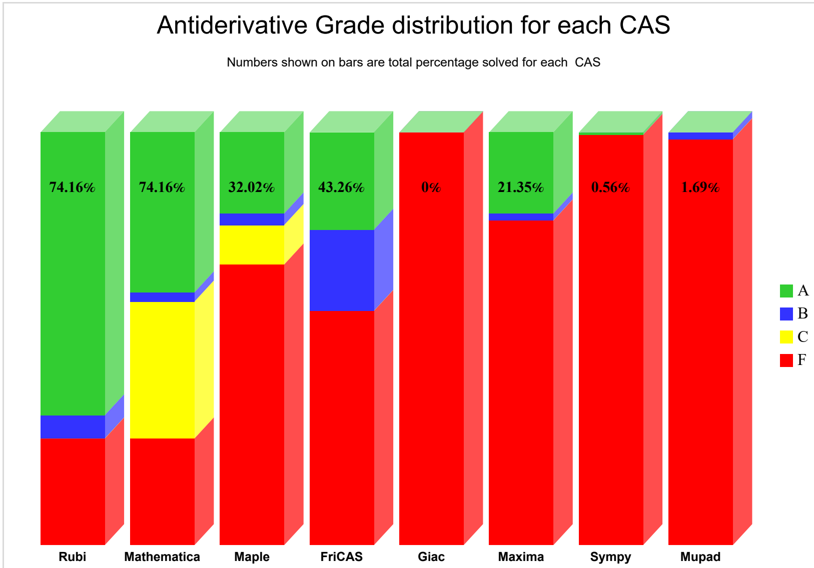
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

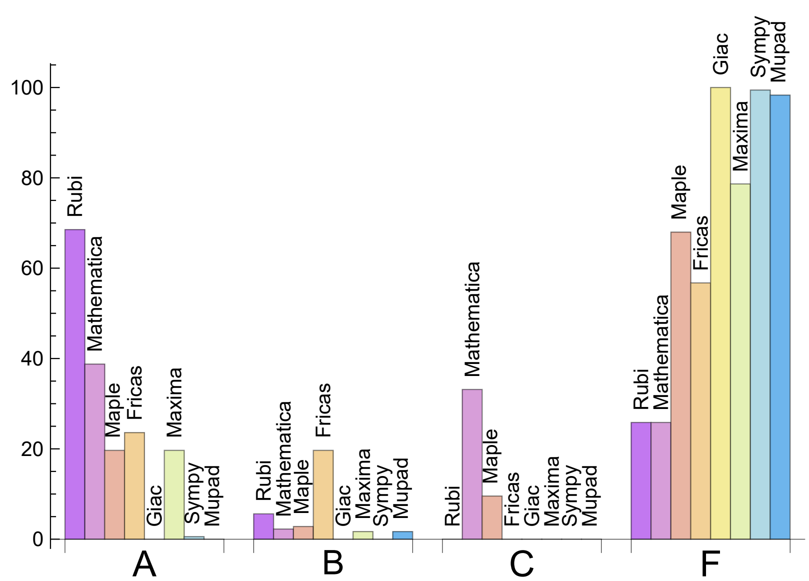
System	% A grade	% B grade	% C grade	% F grade
Rubi	57.303	8.989	7.865	25.843
Mathematica	38.764	2.247	33.146	25.843
Fricas	23.596	19.663	0.000	56.742
Maple	19.663	2.809	9.551	67.978
Maxima	19.663	1.685	0.000	78.652
Sympy	0.562	0.000	0.000	99.438
Giac	0.000	0.000	0.000	100.000
Mupad	0.000	1.685	0.000	98.315

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	55	89.09	10.91	0.00
Maple	75	100.00	0.00	0.00
Maxima	117	58.97	0.00	41.03
Mupad	129	0.00	100.00	0.00
Sympy	140	78.57	21.43	0.00
Giac	132	98.48	0.00	1.52

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Giac	0.28
Fricas	0.30
Maxima	0.70
Rubi	0.84
Maple	1.88
Mupad	5.44
Mathematica	6.03
Sympy	36.26

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	20.58	0.96	20.00	0.95
Giac	21.83	1.03	23.00	1.00
Mupad	26.80	1.21	27.00	1.17
Maxima	164.43	4.92	132.00	1.29
Rubi	315.33	1.20	131.00	1.00
Fricas	315.71	2.30	143.00	1.74
Mathematica	317.74	1.23	129.00	1.09
Maple	373.24	1.49	71.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

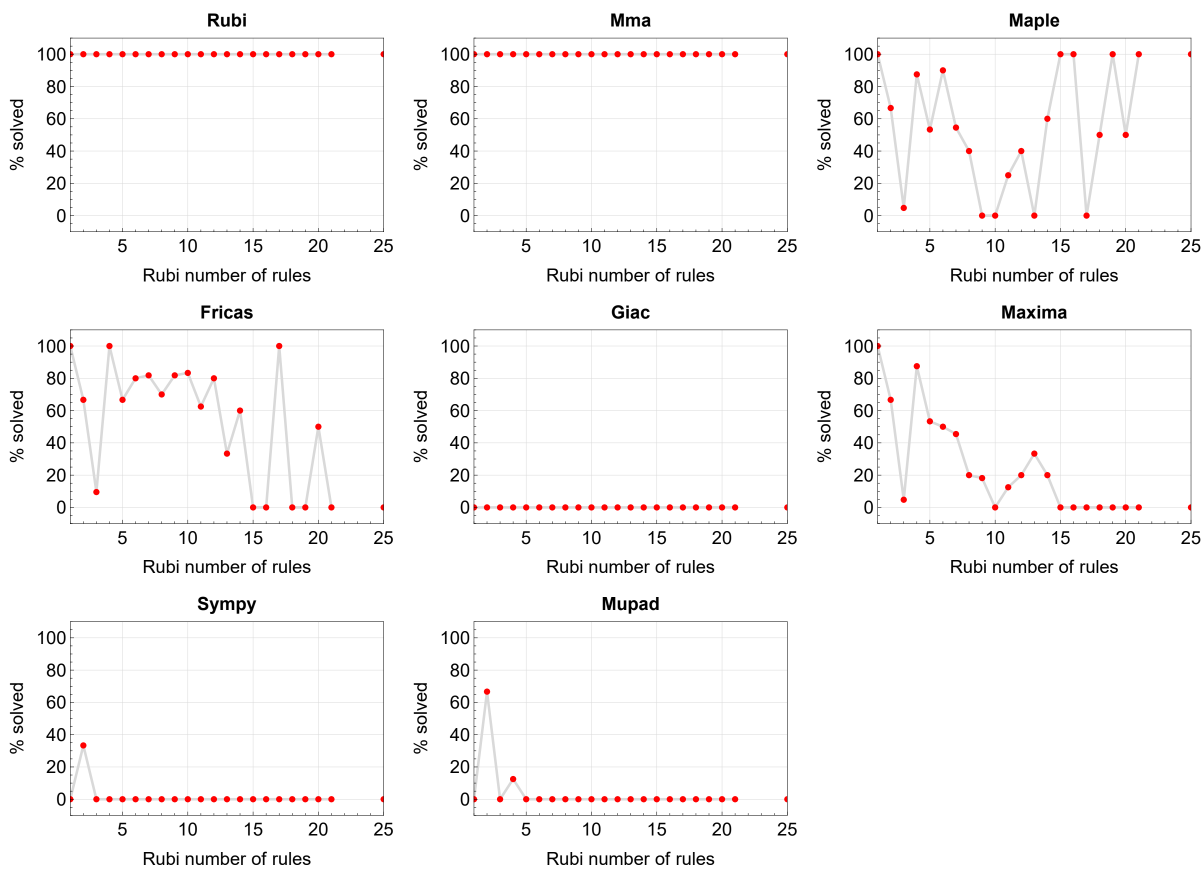


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

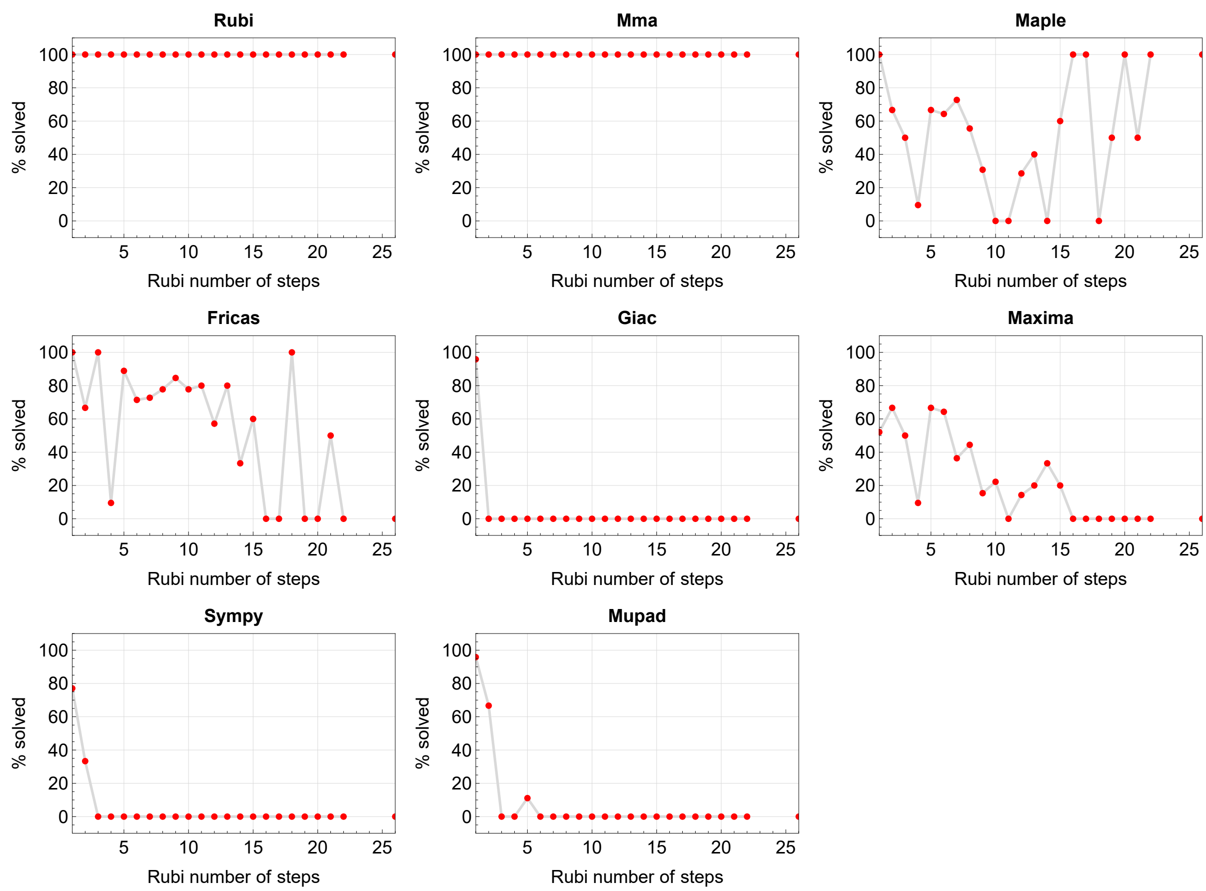


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

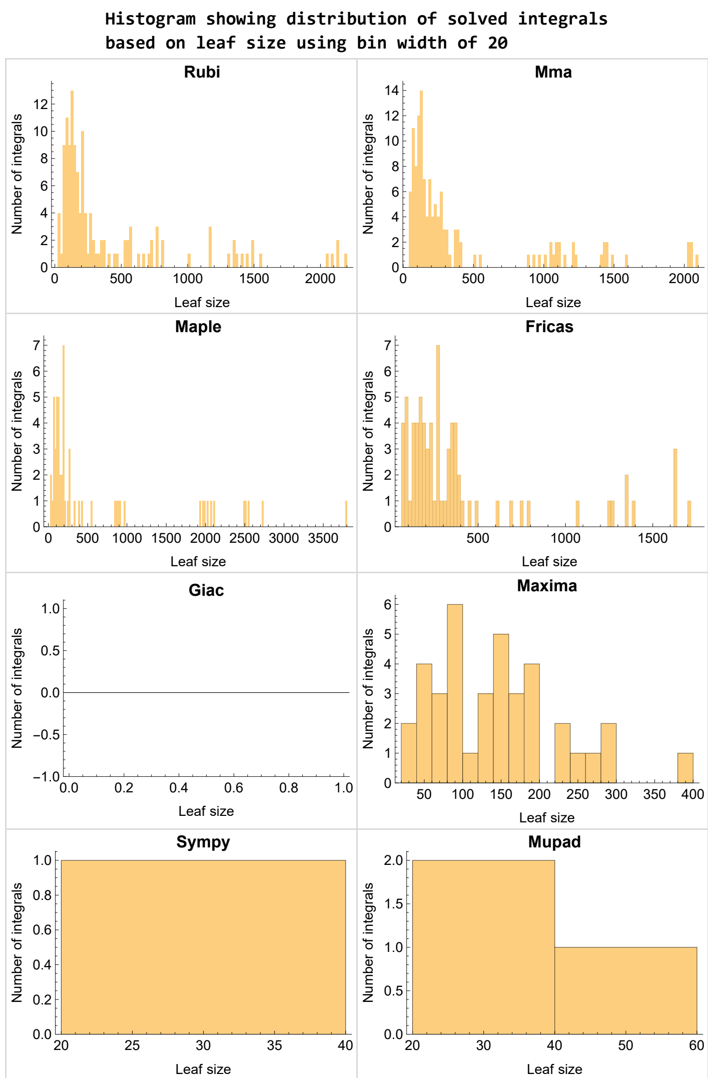


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

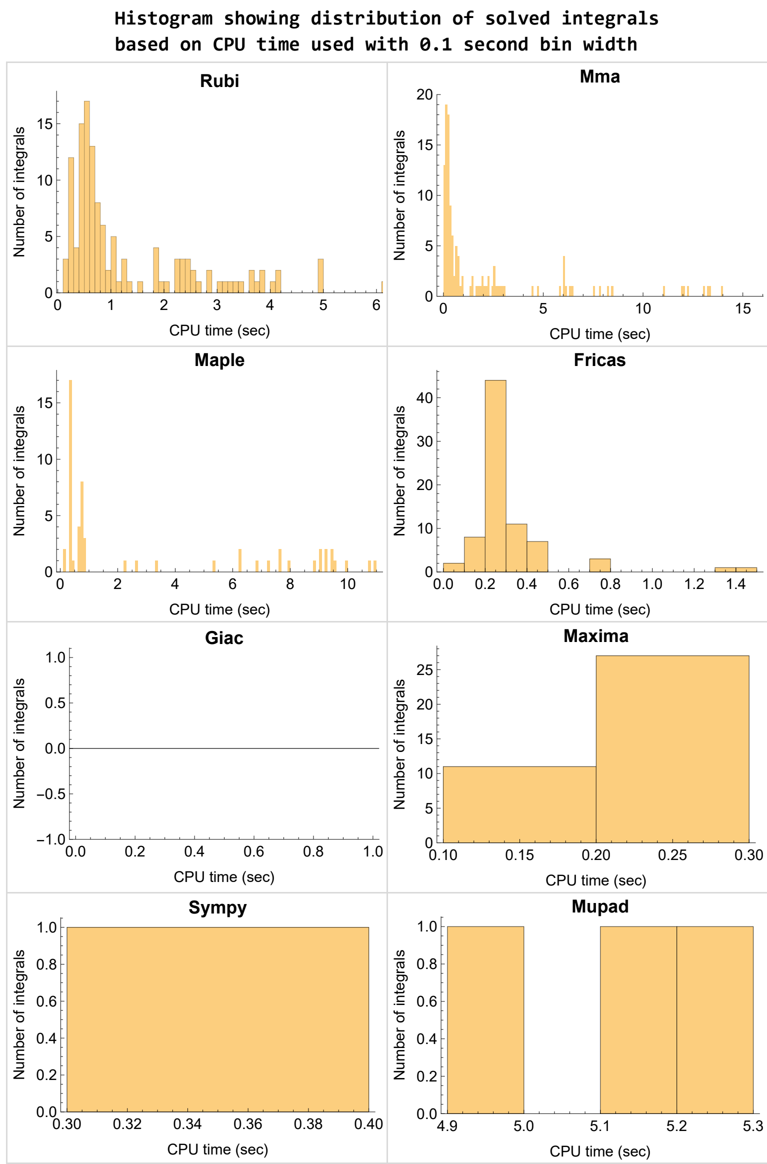


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

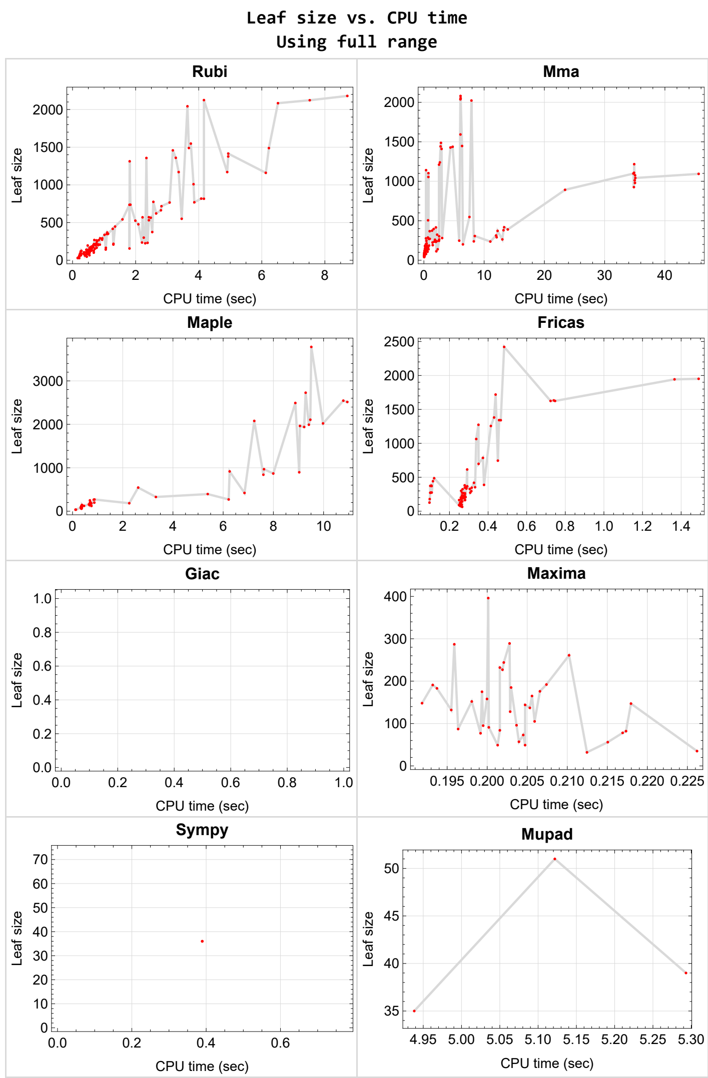


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {8, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 175}

Mathematica {51, 53, 57, 58, 63, 69, 70, 75, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

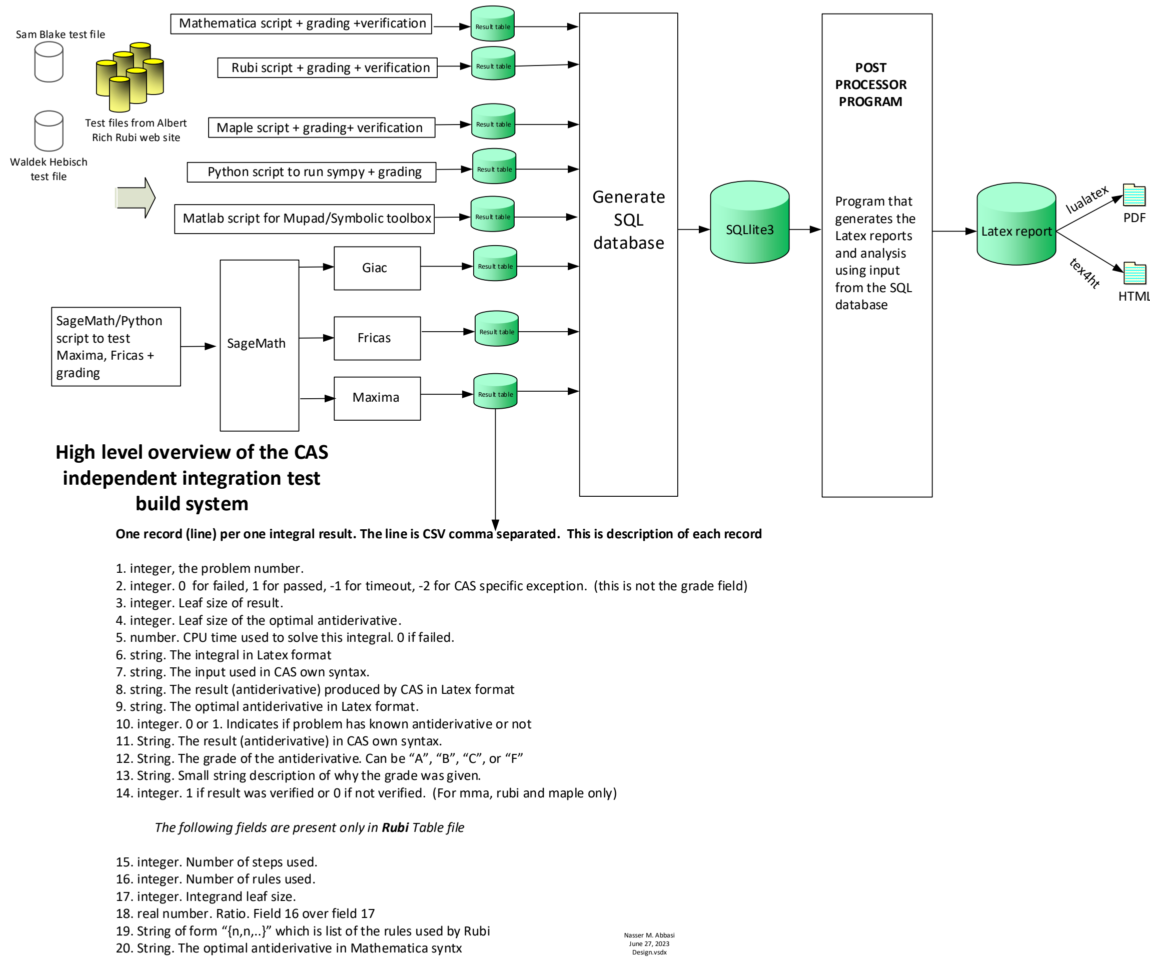
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 21, 23, 30, 32, 36, 38, 41, 44, 45, 46, 47, 48, 49, 50, 57, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

B grade { 51, 52, 53, 56, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75 }

C grade { 8, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 31, 37 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 49, 50, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 145, 154, 163, 165, 166, 167, 174, 175, 176 }

B grade { 7, 25, 27, 47 }

C grade { 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 164 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 12, 13, 14, 44, 45, 46, 47, 49, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { 10, 50, 105, 112, 113 }

C grade { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75 }

F normal fail { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 2, 4, 10, 11, 12, 13, 14, 23, 32, 76, 77, 80, 81, 82, 83, 84, 85, 88, 92, 93, 94, 95, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 145, 146, 147, 154, 155, 163, 164, 174, 175 }

B grade { 1, 3, 5, 6, 7, 9, 15, 17, 20, 21, 22, 29, 30, 31, 44, 45, 46, 47, 49, 50, 78, 79, 89, 90, 91, 105, 112, 113, 140, 148, 149, 156, 157, 158, 176 }

C grade { }

F normal fail { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 56, 57, 58, 59, 60, 63, 66, 70, 71, 72, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 165, 166, 167 }

F(-1) timedout fail { 52, 53, 64, 65, 69, 75 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 17, 20, 29, 44, 45, 46, 47, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95 }

B grade { 10, 12, 14 }

C grade { }

F normal fail { 8, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 99, 101, 103, 104, 105, 106, 111, 112, 113, 114, 120, 129, 140, 149, 154, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 98, 100, 102, 107, 108, 109, 110, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 141, 142, 143, 144, 145, 146, 147, 148, 150, 151, 152, 153, 155, 156, 159, 160, 161, 162 }

2.1.6 Giac

A grade { }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 176 }

F(-1) timedout fail { }

F(-2) exception fail { 174, 175 }

2.1.7 Mupad

A grade { }

B grade { 6, 9, 10 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 9 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 118, 119, 120, 126, 127, 129, 138, 139, 140, 145, 146, 148, 149, 154, 155, 166, 167, 176 }

F(-1) timedout fail { 74, 75, 110, 111, 112, 113, 114, 115, 116, 117, 128, 132, 136, 137, 147, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 169, 170, 174, 175 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	107	123	158	208	0	0	0
N.S.	1	1.06	0.97	1.12	1.44	1.89	0.00	0.00	0.00
time (sec)	N/A	0.257	0.163	0.458	0.200	0.283	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	94	72	79	77	97	0	0	0
N.S.	1	1.09	0.84	0.92	0.90	1.13	0.00	0.00	0.00
time (sec)	N/A	0.252	0.144	0.336	0.199	0.264	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	89	97	104	128	199	0	0	0
N.S.	1	1.03	1.13	1.21	1.49	2.31	0.00	0.00	0.00
time (sec)	N/A	0.243	0.062	0.347	0.203	0.278	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	66	62	70	57	87	0	0	0
N.S.	1	1.06	1.00	1.13	0.92	1.40	0.00	0.00	0.00
time (sec)	N/A	0.220	0.115	0.342	0.204	0.253	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	61	85	83	96	186	0	0	0
N.S.	1	0.98	1.37	1.34	1.55	3.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.069	0.326	0.204	0.265	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	50	61	35	70	0	0	39
N.S.	1	1.00	1.32	1.61	0.92	1.84	0.00	0.00	1.03
time (sec)	N/A	0.186	0.038	0.345	0.226	0.262	0.000	0.000	5.293

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	36	49	143	0	0	0
N.S.	1	1.00	2.13	1.20	1.63	4.77	0.00	0.00	0.00
time (sec)	N/A	0.167	0.128	0.135	0.201	0.263	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	97	51	0	0	0	0	0	0
N.S.	1	1.73	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.544	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	40	58	32	64	36	0	35
N.S.	1	1.00	1.33	1.93	1.07	2.13	1.20	0.00	1.17
time (sec)	N/A	0.210	0.044	0.359	0.212	0.266	0.389	0.000	4.938

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	63	66	96	105	76	0	0	51
N.S.	1	1.26	1.32	1.92	2.10	1.52	0.00	0.00	1.02
time (sec)	N/A	0.232	0.052	0.357	0.206	0.261	0.000	0.000	5.122

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	63	59	71	56	77	0	0	0
N.S.	1	1.09	1.02	1.22	0.97	1.33	0.00	0.00	0.00
time (sec)	N/A	0.253	0.067	0.344	0.215	0.257	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	94	78	116	147	89	0	0	0
N.S.	1	1.27	1.05	1.57	1.99	1.20	0.00	0.00	0.00
time (sec)	N/A	0.257	0.065	0.358	0.218	0.250	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	83	69	79	73	87	0	0	0
N.S.	1	1.05	0.87	1.00	0.92	1.10	0.00	0.00	0.00
time (sec)	N/A	0.259	0.083	0.371	0.204	0.259	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	125	88	135	185	99	0	0	0
N.S.	1	1.28	0.90	1.38	1.89	1.01	0.00	0.00	0.00
time (sec)	N/A	0.280	0.092	0.386	0.203	0.252	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	114	122	0	0	272	0	0	0
N.S.	1	1.09	1.16	0.00	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.539	0.223	0.000	0.000	0.266	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	128	225	0	0	0	0	0	0
N.S.	1	1.05	1.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.537	1.433	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	65	87	0	82	234	0	0	0
N.S.	1	1.20	1.61	0.00	1.52	4.33	0.00	0.00	0.00
time (sec)	N/A	0.411	0.236	0.000	0.217	0.265	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	73	135	0	0	0	0	0	0
N.S.	1	1.07	1.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.392	0.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	96	114	0	0	0	0	0	0
N.S.	1	1.19	1.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.538	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	49	63	70	0	78	139	0	0	0
N.S.	1	1.29	1.43	0.00	1.59	2.84	0.00	0.00	0.00
time (sec)	N/A	0.383	0.163	0.000	0.217	0.261	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	88	100	0	0	163	0	0	0
N.S.	1	1.01	1.15	0.00	0.00	1.87	0.00	0.00	0.00
time (sec)	N/A	0.353	0.145	0.000	0.000	0.280	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	121	106	0	0	178	0	0	0
N.S.	1	1.21	1.06	0.00	0.00	1.78	0.00	0.00	0.00
time (sec)	N/A	0.506	0.203	0.000	0.000	0.260	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	138	147	0	0	202	0	0	0
N.S.	1	1.05	1.11	0.00	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.440	0.178	0.000	0.000	0.259	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	207	271	0	0	0	0	0	0
N.S.	1	1.06	1.39	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.867	0.984	0.000	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	201	548	0	0	0	0	0	0
N.S.	1	1.04	2.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.813	7.563	0.000	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	128	171	0	0	0	0	0	0
N.S.	1	1.09	1.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.602	0.659	0.000	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	119	246	0	0	0	0	0	0
N.S.	1	0.99	2.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.544	0.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	126	181	0	0	0	0	0	0
N.S.	1	1.15	1.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.656	0.180	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	91	132	0	144	222	0	0	0
N.S.	1	1.17	1.69	0.00	1.85	2.85	0.00	0.00	0.00
time (sec)	N/A	0.472	0.234	0.000	0.205	0.265	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	140	182	0	0	267	0	0	0
N.S.	1	1.14	1.48	0.00	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.443	0.315	0.000	0.000	0.272	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	166	197	200	0	0	301	0	0	0
N.S.	1	1.19	1.20	0.00	0.00	1.81	0.00	0.00	0.00
time (sec)	N/A	0.740	0.320	0.000	0.000	0.260	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	204	264	277	0	0	346	0	0	0
N.S.	1	1.29	1.36	0.00	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.772	0.342	0.000	0.000	0.280	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	18
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.50
time (sec)	N/A	0.184	2.904	0.026	0.255	0.251	0.461	0.264	5.263

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	16
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.60
time (sec)	N/A	0.174	3.334	0.026	0.256	0.244	0.403	0.249	4.748

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	16	20
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	1.14	1.43
time (sec)	N/A	0.192	0.320	0.028	0.261	0.241	0.910	0.256	5.207

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	47	44	0	0	0	0	0	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.443	0.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	66	56	0	0	0	0	0	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.534	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	110	91	0	0	0	0	0	0
N.S.	1	0.94	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.501	0.181	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	1351	44	15	18	22
N.S.	1	1.00	1.12	1.00	84.44	2.75	0.94	1.12	1.38
time (sec)	N/A	0.197	5.763	0.028	7.733	0.249	22.104	0.266	5.470

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	644	30	15	18	22
N.S.	1	1.00	1.12	1.00	40.25	1.88	0.94	1.12	1.38
time (sec)	N/A	0.197	3.864	0.023	3.606	0.267	10.082	0.263	5.080

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	81	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.274	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	22
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.38
time (sec)	N/A	0.203	0.976	0.027	0.291	0.253	1.129	0.264	4.722

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	566	32	15	18	22
N.S.	1	1.00	1.12	1.00	35.38	2.00	0.94	1.12	1.38
time (sec)	N/A	0.202	1.991	0.027	0.741	0.261	5.299	0.275	4.771

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	166	165	250	261	419	0	0	0
N.S.	1	0.99	0.99	1.50	1.56	2.51	0.00	0.00	0.00
time (sec)	N/A	0.747	0.260	0.692	0.210	0.327	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	128	122	186	192	328	0	0	0
N.S.	1	1.05	1.00	1.52	1.57	2.69	0.00	0.00	0.00
time (sec)	N/A	0.532	0.170	0.691	0.207	0.305	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	85	119	98	87	207	0	0	0
N.S.	1	1.05	1.47	1.21	1.07	2.56	0.00	0.00	0.00
time (sec)	N/A	0.354	0.222	0.335	0.196	0.277	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	36	49	143	0	0	0
N.S.	1	1.00	2.13	1.20	1.63	4.77	0.00	0.00	0.00
time (sec)	N/A	0.175	0.046	0.118	0.205	0.262	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	506	0	0	0	0	0	0
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.683	0.698	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	106	134	184	0	354	0	0	0
N.S.	1	1.08	1.37	1.88	0.00	3.61	0.00	0.00	0.00
time (sec)	N/A	0.368	0.219	2.260	0.000	0.287	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	196	204	544	0	745	0	0	0
N.S.	1	1.20	1.25	3.34	0.00	4.57	0.00	0.00	0.00
time (sec)	N/A	0.478	0.430	2.615	0.000	0.451	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	918	2125	1094	2515	0	0	0	0	0
N.S.	1	2.31	1.19	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.216	45.597	10.945	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	2043	418	1963	0	0	0	0	0
N.S.	1	3.01	0.62	2.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.818	13.327	9.058	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	429	1313	926	840	0	0	0	0	0
N.S.	1	3.06	2.16	1.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.888	34.852	7.602	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	138	21	19	21	25
N.S.	1	1.00	1.10	0.90	6.57	1.00	0.90	1.00	1.19
time (sec)	N/A	0.250	43.245	0.161	2.170	0.266	13.244	0.265	5.008

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	149	21	20	21	25
N.S.	1	1.00	1.10	0.90	7.10	1.00	0.95	1.00	1.19
time (sec)	N/A	0.255	6.806	0.174	2.340	0.279	14.315	0.266	4.845

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	1357	380	1939	0	0	0	0	0
N.S.	1	2.79	0.78	3.99	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.421	13.215	9.227	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	939	1548	1098	2545	0	0	0	0	0
N.S.	1	1.65	1.17	2.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.962	34.763	10.786	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	707	1458	1012	1991	0	0	0	0	0
N.S.	1	2.06	1.43	2.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.109	34.981	9.414	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	1377	416	868	0	0	0	0	0
N.S.	1	2.91	0.88	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.933	1.999	7.996	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	737	307	395	0	0	0	0	0
N.S.	1	2.60	1.08	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.101	8.452	5.385	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	116	29	19	21	25
N.S.	1	1.00	1.10	0.90	5.52	1.38	0.90	1.00	1.19
time (sec)	N/A	0.237	4.861	0.172	1.111	0.251	8.543	0.288	4.867

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	175	31	20	21	25
N.S.	1	1.00	1.10	0.90	8.33	1.48	0.95	1.00	1.19
time (sec)	N/A	0.249	8.453	0.168	2.383	0.245	17.808	0.285	5.467

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	731	1488	1042	2021	0	0	0	0	0
N.S.	1	2.04	1.43	2.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.771	35.093	9.978	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	1415	979	896	0	0	0	0	0
N.S.	1	2.84	1.96	1.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.007	35.032	9.030	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	1009	264	421	0	0	0	0	0
N.S.	1	3.17	0.83	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.369	13.028	6.850	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	736	166	328	0	0	0	0	0
N.S.	1	4.94	1.11	2.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.088	0.698	3.320	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	179	40	19	21	25
N.S.	1	1.00	1.10	0.90	8.52	1.90	0.90	1.00	1.19
time (sec)	N/A	0.247	14.894	0.179	2.184	0.269	32.577	0.265	5.037

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	252	42	20	21	25
N.S.	1	1.00	1.10	0.90	12.00	2.00	0.95	1.00	1.19
time (sec)	N/A	0.263	19.339	0.165	2.669	0.257	59.648	0.273	4.992

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	777	2180	1108	2728	0	0	0	0	0
N.S.	1	2.81	1.43	3.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.248	34.883	9.286	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	569	2123	1076	2492	0	0	0	0	0
N.S.	1	3.73	1.89	4.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.478	35.046	8.875	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	2084	390	2106	0	0	0	0	0
N.S.	1	5.30	0.99	5.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.832	13.927	9.472	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	1359	892	2079	0	0	0	0	0
N.S.	1	3.68	2.42	5.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.883	23.432	7.239	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	239	51	19	21	25
N.S.	1	1.00	1.10	0.90	11.38	2.43	0.90	1.00	1.19
time (sec)	N/A	0.266	38.786	0.164	2.651	0.253	162.422	0.256	4.995

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	316	53	0	21	25
N.S.	1	1.00	1.10	0.90	15.05	2.52	0.00	1.00	1.19
time (sec)	N/A	0.279	34.940	0.168	3.027	0.257	0.000	0.261	4.986

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD
size	648	1489	1217	3782	0	0	0	0	0
N.S.	1	2.30	1.88	5.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.190	34.906	9.511	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	172	138	198	289	295	0	0	0
N.S.	1	0.80	0.64	0.93	1.35	1.38	0.00	0.00	0.00
time (sec)	N/A	0.345	0.266	0.701	0.203	0.313	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	140	119	158	227	273	0	0	0
N.S.	1	0.84	0.71	0.95	1.36	1.63	0.00	0.00	0.00
time (sec)	N/A	0.315	0.204	0.652	0.202	0.307	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	106	155	109	148	245	0	0	0
N.S.	1	0.92	1.35	0.95	1.29	2.13	0.00	0.00	0.00
time (sec)	N/A	0.268	0.270	0.356	0.192	0.280	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	86	109	102	84	222	0	0	0
N.S.	1	0.95	1.20	1.12	0.92	2.44	0.00	0.00	0.00
time (sec)	N/A	0.277	0.159	0.360	0.202	0.279	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	104	68	109	91	105	0	0	0
N.S.	1	0.95	0.62	1.00	0.83	0.96	0.00	0.00	0.00
time (sec)	N/A	0.288	0.120	0.346	0.200	0.257	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	135	93	127	132	127	0	0	0
N.S.	1	0.85	0.59	0.80	0.84	0.80	0.00	0.00	0.00
time (sec)	N/A	0.296	0.151	0.355	0.196	0.256	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	164	109	145	165	146	0	0	0
N.S.	1	0.80	0.53	0.71	0.80	0.71	0.00	0.00	0.00
time (sec)	N/A	0.324	0.173	0.374	0.206	0.266	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	166	114	139	176	165	0	0	0
N.S.	1	0.81	0.56	0.68	0.86	0.81	0.00	0.00	0.00
time (sec)	N/A	0.368	0.273	0.674	0.207	0.281	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	135	97	121	137	144	0	0	0
N.S.	1	0.85	0.61	0.76	0.86	0.91	0.00	0.00	0.00
time (sec)	N/A	0.343	0.233	0.753	0.205	0.266	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	122	77	181	95	123	0	0	0
N.S.	1	0.84	0.53	1.24	0.65	0.84	0.00	0.00	0.00
time (sec)	N/A	0.334	0.132	0.749	0.199	0.258	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	115	142	107	0	0	0	0	0	0
N.S.	1	1.23	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.658	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	165	124	0	0	0	0	0	0
N.S.	1	1.29	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.632	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	220	182	270	396	388	0	0	0
N.S.	1	0.85	0.70	1.04	1.52	1.49	0.00	0.00	0.00
time (sec)	N/A	0.453	0.350	0.875	0.200	0.379	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	183	149	195	287	353	0	0	0
N.S.	1	0.93	0.76	0.99	1.46	1.79	0.00	0.00	0.00
time (sec)	N/A	0.374	0.219	0.714	0.196	0.333	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	152	134	173	191	347	0	0	0
N.S.	1	0.89	0.79	1.02	1.12	2.04	0.00	0.00	0.00
time (sec)	N/A	0.378	0.195	0.705	0.193	0.315	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	151	123	187	152	334	0	0	0
N.S.	1	0.92	0.75	1.14	0.93	2.04	0.00	0.00	0.00
time (sec)	N/A	0.379	0.244	0.717	0.198	0.288	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	175	126	175	175	165	0	0	0
N.S.	1	0.93	0.67	0.93	0.93	0.87	0.00	0.00	0.00
time (sec)	N/A	0.399	0.210	0.721	0.199	0.250	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	206	152	207	232	197	0	0	0
N.S.	1	0.83	0.61	0.83	0.93	0.79	0.00	0.00	0.00
time (sec)	N/A	0.424	0.265	0.714	0.202	0.272	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	212	159	198	244	224	0	0	0
N.S.	1	0.85	0.64	0.79	0.98	0.90	0.00	0.00	0.00
time (sec)	N/A	0.482	0.397	0.868	0.202	0.272	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	165	123	266	183	189	0	0	0
N.S.	1	0.81	0.61	1.31	0.90	0.93	0.00	0.00	0.00
time (sec)	N/A	0.370	0.293	0.845	0.194	0.267	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	208	150	0	0	0	0	0	0
N.S.	1	1.17	0.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.784	0.342	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	178	218	186	0	0	0	0	0	0
N.S.	1	1.22	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.774	0.866	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	512	564	1239	0	0	0	0	0	0
N.S.	1	1.10	2.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.540	2.657	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	467	531	1103	0	0	0	0	0	0
N.S.	1	1.14	2.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.554	0.755	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	477	525	1055	0	0	0	0	0	0
N.S.	1	1.10	2.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.249	0.748	0.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	425	477	1141	0	0	0	0	0	0
N.S.	1	1.12	2.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.312	0.365	0.000	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	518	570	1211	0	0	0	0	0	0
N.S.	1	1.10	2.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.410	2.517	0.000	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	591	663	1447	0	0	0	0	0	0
N.S.	1	1.12	2.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.783	6.340	0.000	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	553	621	1410	0	0	0	0	0	0
N.S.	1	1.12	2.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.651	2.936	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	125	271	271	0	615	0	0	0
N.S.	1	0.90	1.95	1.95	0.00	4.42	0.00	0.00	0.00
time (sec)	N/A	0.327	0.792	6.219	0.000	0.292	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	515	571	1428	0	0	0	0	0	0
N.S.	1	1.11	2.77	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.510	4.411	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	756	816	1593	0	0	0	0	0	0
N.S.	1	1.08	2.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.646	6.071	0.000	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	719	775	1442	0	0	0	0	0	0
N.S.	1	1.08	2.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.604	2.774	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	713	769	1437	0	0	0	0	0	0
N.S.	1	1.08	2.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.426	4.777	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	758	818	1487	0	0	0	0	0	0
N.S.	1	1.08	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.557	2.833	0.000	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	694	766	2023	0	0	0	0	0	0
N.S.	1	1.10	2.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.924	7.894	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	153	375	967	0	1381	0	0	0
N.S.	1	0.92	2.25	5.79	0.00	8.27	0.00	0.00	0.00
time (sec)	N/A	0.366	1.356	7.626	0.000	0.431	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	202	368	916	0	1256	0	0	0
N.S.	1	0.99	1.80	4.47	0.00	6.13	0.00	0.00	0.00
time (sec)	N/A	0.394	0.963	6.258	0.000	0.414	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	657	717	2081	0	0	0	0	0	0
N.S.	1	1.09	3.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.728	6.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1106	1170	2045	0	0	0	0	0	0
N.S.	1	1.06	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.054	6.097	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1106	1170	2053	0	0	0	0	0	0
N.S.	1	1.06	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.074	6.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1096	1160	2038	0	0	0	0	0	0
N.S.	1	1.06	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.904	6.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	413	374	324	0	0	1951	0	0	0
N.S.	1	0.91	0.78	0.00	0.00	4.72	0.00	0.00	0.00
time (sec)	N/A	1.598	2.204	0.000	0.000	1.491	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	273	263	0	0	1625	0	0	0
N.S.	1	0.90	0.87	0.00	0.00	5.38	0.00	0.00	0.00
time (sec)	N/A	0.568	2.541	0.000	0.000	0.747	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	186	233	0	0	1342	0	0	0
N.S.	1	0.92	1.15	0.00	0.00	6.61	0.00	0.00	0.00
time (sec)	N/A	0.398	1.998	0.000	0.000	0.457	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.87	1.00	1.17
time (sec)	N/A	0.279	8.125	0.158	0.000	0.251	9.078	0.286	5.959

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.279	12.765	0.155	0.000	0.261	14.538	0.265	5.781

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.270	10.195	0.167	0.000	0.254	25.628	0.272	5.661

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.215	4.211	0.157	0.000	0.254	6.364	0.279	5.536

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	23	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.00	0.96	1.00	1.17
time (sec)	N/A	0.268	2.696	0.164	0.000	0.261	7.143	0.267	5.969

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	389	337	237	0	0	275	0	0	0
N.S.	1	0.87	0.61	0.00	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.615	11.029	0.000	0.000	0.108	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	527	447	314	0	0	377	0	0	0
N.S.	1	0.85	0.60	0.00	0.00	0.72	0.00	0.00	0.00
time (sec)	N/A	0.864	11.986	0.000	0.000	0.106	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	384	348	303	0	0	1943	0	0	0
N.S.	1	0.91	0.79	0.00	0.00	5.06	0.00	0.00	0.00
time (sec)	N/A	0.679	2.535	0.000	0.000	1.366	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	270	248	246	0	0	1625	0	0	0
N.S.	1	0.92	0.91	0.00	0.00	6.02	0.00	0.00	0.00
time (sec)	N/A	0.484	2.500	0.000	0.000	0.724	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.87	1.00	1.17
time (sec)	N/A	0.305	9.472	0.164	0.000	0.266	65.481	0.312	5.548

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.303	13.495	0.157	0.000	0.258	62.326	0.279	5.931

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	43	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.87	0.00	1.00	1.17
time (sec)	N/A	0.304	10.481	0.157	0.000	0.249	0.000	0.285	5.850

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	37	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.85	0.95	1.00	1.20
time (sec)	N/A	0.228	5.147	0.151	0.000	0.252	62.908	0.299	6.325

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.283	8.947	0.155	0.000	0.257	57.978	0.285	6.212

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	40	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.74	0.96	1.00	1.17
time (sec)	N/A	0.285	19.194	0.164	0.000	0.261	66.787	0.266	6.316

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	492	417	291	0	0	374	0	0	0
N.S.	1	0.85	0.59	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.821	12.079	0.000	0.000	0.101	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	643	542	372	0	0	486	0	0	0
N.S.	1	0.84	0.58	0.00	0.00	0.76	0.00	0.00	0.00
time (sec)	N/A	0.980	12.233	0.000	0.000	0.121	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	300	281	0	0	1633	0	0	0
N.S.	1	0.91	0.85	0.00	0.00	4.96	0.00	0.00	0.00
time (sec)	N/A	1.377	3.034	0.000	0.000	0.740	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	208	236	0	0	1341	0	0	0
N.S.	1	0.91	1.03	0.00	0.00	5.86	0.00	0.00	0.00
time (sec)	N/A	0.496	1.736	0.000	0.000	0.466	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	131	108	0	0	1064	0	0	0
N.S.	1	0.97	0.80	0.00	0.00	7.88	0.00	0.00	0.00
time (sec)	N/A	0.343	0.611	0.000	0.000	0.339	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	31	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.35	0.87	1.00	1.17
time (sec)	N/A	0.283	1.720	0.202	0.000	0.255	7.469	0.280	5.767

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	33	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.43	0.96	1.00	1.17
time (sec)	N/A	0.305	7.413	0.169	0.000	0.266	28.520	0.292	5.722

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	27	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.17	0.96	1.00	1.17
time (sec)	N/A	0.281	10.027	0.139	0.000	0.260	13.925	0.277	5.792

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	0	20	19	20	24
N.S.	1	1.00	1.10	0.90	0.00	1.00	0.95	1.00	1.20
time (sec)	N/A	0.216	1.113	0.159	0.000	0.264	3.109	0.269	5.594

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	294	267	139	0	0	179	0	0	0
N.S.	1	0.91	0.47	0.00	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.529	2.295	0.000	0.000	0.098	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	425	368	239	0	0	274	0	0	0
N.S.	1	0.87	0.56	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.677	8.290	0.000	0.000	0.103	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	256	235	260	0	0	1719	0	0	0
N.S.	1	0.92	1.02	0.00	0.00	6.71	0.00	0.00	0.00
time (sec)	N/A	1.364	1.841	0.000	0.000	0.439	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	160	155	191	0	0	1274	0	0	0
N.S.	1	0.97	1.19	0.00	0.00	7.96	0.00	0.00	0.00
time (sec)	N/A	0.450	1.473	0.000	0.000	0.350	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	82	79	0	0	368	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	4.49	0.00	0.00	0.00
time (sec)	N/A	0.302	0.353	0.000	0.000	0.292	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	42	20	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.83	0.87	1.00	1.17
time (sec)	N/A	0.287	11.181	0.164	0.000	0.263	70.168	0.285	6.047

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	44	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	1.91	0.00	1.00	1.17
time (sec)	N/A	0.309	14.578	0.160	0.000	0.252	0.000	0.270	5.783

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.282	15.248	0.156	0.000	0.260	134.918	0.293	6.086

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	47	22	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.04	0.96	1.00	1.17
time (sec)	N/A	0.277	7.676	0.179	0.000	0.263	33.944	0.286	5.461

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	127	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.283	2.068	0.000	0.000	0.096	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	321	291	201	0	0	272	0	0	0
N.S.	1	0.91	0.63	0.00	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.562	6.480	0.000	0.000	0.100	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	251	225	241	0	0	2421	0	0	0
N.S.	1	0.90	0.96	0.00	0.00	9.65	0.00	0.00	0.00
time (sec)	N/A	1.406	2.123	0.000	0.000	0.483	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	153	139	0	0	786	0	0	0
N.S.	1	0.91	0.82	0.00	0.00	4.65	0.00	0.00	0.00
time (sec)	N/A	0.441	0.770	0.000	0.000	0.373	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	131	132	0	0	698	0	0	0
N.S.	1	0.91	0.92	0.00	0.00	4.85	0.00	0.00	0.00
time (sec)	N/A	0.329	0.514	0.000	0.000	0.351	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	53	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.30	0.00	1.00	1.17
time (sec)	N/A	0.304	19.102	0.156	0.000	0.257	0.000	0.274	5.697

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	55	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.39	0.00	1.00	1.17
time (sec)	N/A	0.319	23.849	0.156	0.000	0.258	0.000	0.283	5.532

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.293	17.345	0.183	0.000	0.257	0.000	0.299	5.386

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	21	0	58	0	23	27
N.S.	1	1.00	1.09	0.91	0.00	2.52	0.00	1.00	1.17
time (sec)	N/A	0.286	13.460	0.154	0.000	0.252	0.000	0.287	5.266

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	359	291	189	0	0	371	0	0	0
N.S.	1	0.81	0.53	0.00	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.523	0.458	0.000	0.000	0.107	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	278	269	248	0	0	442	0	0	0
N.S.	1	0.97	0.89	0.00	0.00	1.59	0.00	0.00	0.00
time (sec)	N/A	0.416	5.869	0.000	0.000	0.114	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	550	396	0	0	0	0	0	0
N.S.	1	0.92	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.178	1.623	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	379	350	288	0	0	0	0	0	0
N.S.	1	0.92	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.702	0.679	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	220	209	167	0	0	0	0	0	0
N.S.	1	0.95	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.465	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	20	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.87	1.09	1.26
time (sec)	N/A	0.251	3.187	0.165	0.309	0.256	41.891	0.261	4.772

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	36	0	25	29
N.S.	1	1.00	1.09	1.00	1.09	1.57	0.00	1.09	1.26
time (sec)	N/A	0.256	6.777	0.167	0.303	0.252	0.000	0.271	4.913

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	42	0	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.68	0.00	1.00	1.16
time (sec)	N/A	0.288	1.531	0.160	0.317	0.260	0.000	0.304	5.038

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.284	0.196	0.172	0.314	0.264	53.733	0.296	5.506

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	25	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.00	0.96	1.00	1.16
time (sec)	N/A	0.284	1.548	0.159	0.347	0.260	23.026	0.276	5.542

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	27	23	25	45	24	25	29
N.S.	1	1.00	1.08	0.92	1.00	1.80	0.96	1.00	1.16
time (sec)	N/A	0.292	1.863	0.156	0.328	0.263	146.974	0.282	5.119

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	395	231	214	0	0	382	0	0	0
N.S.	1	0.58	0.54	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	1.431	0.461	0.000	0.000	0.279	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	156	180	0	0	324	0	0	0
N.S.	1	0.59	0.68	0.00	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	1.140	0.522	0.000	0.000	0.268	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	101	141	0	0	265	0	0	0
N.S.	1	0.78	1.08	0.00	0.00	2.04	0.00	0.00	0.00
time (sec)	N/A	0.408	0.451	0.000	0.000	0.280	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	88	37	34	26	30
N.S.	1	1.00	1.08	0.92	3.38	1.42	1.31	1.00	1.15
time (sec)	N/A	0.281	0.638	0.158	0.594	0.262	7.517	0.283	6.259

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	112	39	36	26	30
N.S.	1	1.00	1.08	0.92	4.31	1.50	1.38	1.00	1.15
time (sec)	N/A	0.287	8.863	0.158	0.628	0.262	85.157	0.289	5.489

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [32] had the largest ratio of [1.42856999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.06	12	0.583
2	A	4	4	1.09	12	0.333
3	A	7	6	1.03	12	0.500
4	A	3	3	1.06	12	0.250
5	A	6	5	0.98	12	0.417
6	A	2	2	1.00	10	0.200
7	A	1	1	1.00	8	0.125
8	C	10	9	1.73	12	0.750
9	A	2	2	1.00	12	0.167
10	A	5	4	1.26	12	0.333
11	A	5	4	1.09	12	0.333
12	A	6	5	1.27	12	0.417
13	A	5	4	1.05	12	0.333
14	A	7	6	1.28	12	0.500
15	A	13	12	1.09	14	0.857
16	C	12	11	1.05	14	0.786
17	A	10	9	1.20	12	0.750
18	C	8	7	1.07	10	0.700
19	C	10	9	1.19	14	0.643
20	C	10	9	1.29	14	0.643
21	A	7	6	1.01	14	0.429
22	C	12	11	1.21	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	10	9	1.05	14	0.643
24	C	19	18	1.06	14	1.286
25	C	14	13	1.04	14	0.929
26	C	14	13	1.09	12	1.083
27	C	9	8	0.99	10	0.800
28	C	11	10	1.15	14	0.714
29	C	14	13	1.17	14	0.929
30	A	12	11	1.14	14	0.786
31	C	18	17	1.19	14	1.214
32	A	21	20	1.29	14	1.429
33	N/A	1	0	1.00	12	0.000
34	N/A	1	0	1.00	10	0.000
35	N/A	1	0	1.00	14	0.000
36	A	9	8	1.02	14	0.571
37	C	12	11	1.05	14	0.786
38	A	4	3	0.94	14	0.214
39	N/A	1	0	1.00	16	0.000
40	N/A	1	0	1.00	16	0.000
41	A	4	3	1.00	14	0.214
42	N/A	1	0	1.00	16	0.000
43	N/A	1	0	1.00	16	0.000
44	A	15	14	0.99	16	0.875
45	A	13	12	1.05	16	0.750
46	A	12	11	1.05	14	0.786
47	A	1	1	1.00	8	0.125
48	A	2	2	1.00	16	0.125
49	A	8	7	1.08	16	0.438
50	A	9	8	1.20	16	0.500
51	B	21	20	2.31	21	0.952
52	B	20	19	3.01	19	1.000
53	B	13	12	3.06	18	0.667
54	N/A	1	0	1.00	21	0.000
55	N/A	1	0	1.00	21	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	B	16	15	2.79	18	0.833
57	A	19	18	1.65	21	0.857
58	B	17	16	2.06	21	0.762
59	B	15	14	2.91	19	0.737
60	B	7	6	2.60	18	0.333
61	N/A	1	0	1.00	21	0.000
62	N/A	1	0	1.00	21	0.000
63	B	17	16	2.04	21	0.762
64	B	15	14	2.84	21	0.667
65	B	12	11	3.17	19	0.579
66	B	7	6	4.94	18	0.333
67	N/A	1	0	1.00	21	0.000
68	N/A	1	0	1.00	21	0.000
69	B	26	25	2.81	21	1.190
70	B	26	25	3.73	21	1.190
71	B	22	21	5.30	19	1.105
72	B	16	15	3.68	18	0.833
73	N/A	1	0	1.00	21	0.000
74	N/A	1	0	1.00	21	0.000
75	B	17	16	2.30	18	0.889
76	A	8	7	0.80	19	0.368
77	A	7	6	0.84	19	0.316
78	A	6	5	0.92	16	0.312
79	A	6	5	0.95	19	0.263
80	A	4	4	0.95	19	0.211
81	A	5	5	0.85	19	0.263
82	A	6	6	0.80	19	0.316
83	A	6	5	0.81	19	0.263
84	A	6	5	0.85	19	0.263
85	A	5	4	0.84	17	0.235
86	A	6	5	1.23	19	0.263
87	A	6	5	1.29	19	0.263
88	A	9	8	0.85	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	8	7	0.93	18	0.389
90	A	9	8	0.89	21	0.381
91	A	8	7	0.92	21	0.333
92	A	6	6	0.93	21	0.286
93	A	7	7	0.83	21	0.333
94	A	6	5	0.85	21	0.238
95	A	5	4	0.81	19	0.211
96	A	6	5	1.17	21	0.238
97	A	6	5	1.22	21	0.238
98	A	4	3	1.10	21	0.143
99	A	4	3	1.14	19	0.158
100	A	4	3	1.10	18	0.167
101	A	4	3	1.12	21	0.143
102	A	4	3	1.10	21	0.143
103	A	4	3	1.12	21	0.143
104	A	4	3	1.12	21	0.143
105	A	7	6	0.90	19	0.316
106	A	4	3	1.11	21	0.143
107	A	4	3	1.08	21	0.143
108	A	4	3	1.08	21	0.143
109	A	4	3	1.08	18	0.167
110	A	4	3	1.08	21	0.143
111	A	4	3	1.10	21	0.143
112	A	7	6	0.92	21	0.286
113	A	9	8	0.99	19	0.421
114	A	4	3	1.09	21	0.143
115	A	4	3	1.06	21	0.143
116	A	4	3	1.06	21	0.143
117	A	4	3	1.06	18	0.167
118	A	15	14	0.91	23	0.609
119	A	13	12	0.90	23	0.522
120	A	10	9	0.92	21	0.429
121	N/A	1	0	1.00	23	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	N/A	1	0	1.00	23	0.000
123	N/A	1	0	1.00	23	0.000
124	N/A	1	0	1.00	20	0.000
125	N/A	1	0	1.00	23	0.000
126	A	9	9	0.87	23	0.391
127	A	10	10	0.85	23	0.435
128	A	15	14	0.91	23	0.609
129	A	12	11	0.92	21	0.524
130	N/A	1	0	1.00	23	0.000
131	N/A	1	0	1.00	23	0.000
132	N/A	1	0	1.00	23	0.000
133	N/A	1	0	1.00	20	0.000
134	N/A	1	0	1.00	23	0.000
135	N/A	1	0	1.00	23	0.000
136	A	10	10	0.85	23	0.435
137	A	11	11	0.84	23	0.478
138	A	13	12	0.91	23	0.522
139	A	11	10	0.91	23	0.435
140	A	9	8	0.97	21	0.381
141	N/A	1	0	1.00	23	0.000
142	N/A	1	0	1.00	23	0.000
143	N/A	1	0	1.00	23	0.000
144	N/A	1	0	1.00	20	0.000
145	A	9	9	0.91	23	0.391
146	A	9	9	0.87	23	0.391
147	A	11	10	0.92	23	0.435
148	A	9	8	0.97	23	0.348
149	A	5	4	1.00	21	0.190
150	N/A	1	0	1.00	23	0.000
151	N/A	1	0	1.00	23	0.000
152	N/A	1	0	1.00	23	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	N/A	1	0	1.00	23	0.000
154	A	3	3	1.00	20	0.150
155	A	9	9	0.91	23	0.391
156	A	11	10	0.90	23	0.435
157	A	8	7	0.91	23	0.304
158	A	6	5	0.91	21	0.238
159	N/A	1	0	1.00	23	0.000
160	N/A	1	0	1.00	23	0.000
161	N/A	1	0	1.00	23	0.000
162	N/A	1	0	1.00	23	0.000
163	A	7	7	0.81	23	0.304
164	A	5	5	0.97	20	0.250
165	A	8	8	0.92	23	0.348
166	A	7	7	0.92	23	0.304
167	A	5	5	0.95	21	0.238
168	N/A	1	0	1.00	23	0.000
169	N/A	1	0	1.00	23	0.000
170	N/A	1	0	1.00	25	0.000
171	N/A	1	0	1.00	25	0.000
172	N/A	1	0	1.00	25	0.000
173	N/A	1	0	1.00	25	0.000
174	A	8	7	0.58	26	0.269
175	A	10	9	0.59	26	0.346
176	A	9	8	0.78	26	0.308
177	N/A	1	0	1.00	26	0.000
178	N/A	1	0	1.00	26	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 110

$$\int x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^6}{42c}+\frac{1}{7}x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{5b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{112c^7}$$

output
```
1/7*x^7*(a+b*arccsch(c*x))-5/112*b*arctanh((1+1/c^2/x^2)^(1/2))/c^7+5/112*
b*x^2*(1+1/c^2/x^2)^(1/2)/c^5-5/168*b*x^4*(1+1/c^2/x^2)^(1/2)/c^3+1/42*b*x
^6*(1+1/c^2/x^2)^(1/2)/c
```

3.1.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{ax^7}{7}+b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(\frac{5x^2}{112c^5}-\frac{5x^4}{168c^3}+\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\operatorname{csch}^{-1}(cx)-\frac{5b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

input
```
Integrate[x^6*(a + b*ArcCsch[c*x]),x]
```

output
```
(a*x^7)/7 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) - (5*x^4)/(
168*c^3) + x^6/(42*c)) + (b*x^7*ArcCsch[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(1
+ c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

3.1.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6838, 798, 52, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6838}$$

$$\frac{b\int \frac{x^5}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{7c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\int \frac{x^8}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{14c}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{5\int \frac{x^6}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c^2}-\frac{1}{3}x^6\sqrt{\frac{1}{c^2x^2}+1}\right)}{14c}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{5\left(-\frac{3\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{\frac{1}{c^2x^2}+1}\right)}{14c}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{5\left(-\frac{3\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{\frac{1}{c^2x^2}+1}\right)}{14c}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{5\left(-\frac{3\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\int \frac{1}{\frac{c^2}{x^4}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{\frac{1}{c^2x^2}+1}\right)}{14c}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{5\left(-\frac{3\left(\frac{\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}-x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^2}-\frac{1}{3}x^6\sqrt{\frac{1}{c^2x^2}+1}\right)}{14c}$$

input

```
Int[x^6*(a + b*ArcCsch[c*x]),x]
```

output
```
(x^7*(a + b*ArcCsch[c*x]))/7 - (b*(-1/3*(Sqrt[1 + 1/(c^2*x^2)]*x^6) - (5*(
-1/2*(Sqrt[1 + 1/(c^2*x^2)]*x^4) - (3*(-(Sqrt[1 + 1/(c^2*x^2)]*x^2) + ArcT
anh[Sqrt[1 + 1/(c^2*x^2)]]/c^2))/(4*c^2)))/(6*c^2)))/(14*c)
```

3.1.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.12

method	result	size
parts	$\frac{a\,x^7}{7}+\frac{b\left(\frac{c^7x^7\operatorname{arccsch}(cx)}{7}-\frac{\sqrt{c^2x^2+1}\,\left(-8c^5x^5\sqrt{c^2x^2+1}+10c^3x^3\sqrt{c^2x^2+1}-15cx\sqrt{c^2x^2+1}+15\operatorname{arcsinh}(cx)\right)}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^7}$	123
derivativedivides	$\frac{\frac{a\,c^7x^7}{7}+b\left(\frac{c^7x^7\operatorname{arccsch}(cx)}{7}-\frac{\sqrt{c^2x^2+1}\,\left(-8c^5x^5\sqrt{c^2x^2+1}+10c^3x^3\sqrt{c^2x^2+1}-15cx\sqrt{c^2x^2+1}+15\operatorname{arcsinh}(cx)\right)}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^7}$	127
default	$\frac{\frac{a\,c^7x^7}{7}+b\left(\frac{c^7x^7\operatorname{arccsch}(cx)}{7}-\frac{\sqrt{c^2x^2+1}\,\left(-8c^5x^5\sqrt{c^2x^2+1}+10c^3x^3\sqrt{c^2x^2+1}-15cx\sqrt{c^2x^2+1}+15\operatorname{arcsinh}(cx)\right)}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^7}$	127

input
```
int(x^6*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/7*a*x^7+b/c^7*(1/7*c^7*x^7*arccsch(c*x)-1/336*(c^2*x^2+1)^(1/2)*(-8*c^5*
x^5*(c^2*x^2+1)^(1/2)+10*c^3*x^3*(c^2*x^2+1)^(1/2)-15*c*x*(c^2*x^2+1)^(1/2
)+15*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(92) = 184.

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.89

$$\int x^6\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{48\,ac^7x^7+48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+15\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right.\cdots}{336\,c^7}$$

input
```
integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/336*(48*a*c^7*x^7 + 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
 + 1) - 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 15*b*l
og(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 48*(b*c^7*x^7 - b*c^7)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*b*c^6*x^6 - 10*b*c^4*x^
4 + 15*b*c^2*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

3.1.6 Sympy [F]

$$\int x^6 \left(a + b\text{csch}^{-1}(cx)\right)\,dx = \int x^6 (a + b\,\text{acsch}\,(cx))\,dx$$

input
```
integrate(x**6*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**6*(a + b*acsch(c*x)), x)
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.44

$$\int x^6 \left(a + b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{7}ax^7 + \frac{1}{672}\left(96\,x^7\,\text{arcsch}\,(cx) + \frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3 - 3\,c^6\left(\frac{1}{c^2x^2}+1\right)^2 + 3\,c^6\left(\frac{1}{c^2x^2}+1\right) - c^6} - \frac{15\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6} + \frac{15\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right.$$

input
```
integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*x^7 + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) -
40*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) +
 1)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*lo
g(sqrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/
c)*b
```

3.1.8 Giac [F]

$$\int x^6\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (b\,\text{arcsch}\,(cx)+a)x^6\,dx$$

input
```
integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^6, x)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^6\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^6\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^6*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^6*(a + b*asinh(1/(c*x))), x)
```

3.2 $\int x^5 \left(a + b\text{csch}^{-1}(cx)\right) \, dx$

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^5 \left(a + b\text{csch}^{-1}(cx)\right) \, dx = \frac{4b\sqrt{1+\frac{1}{c^2x^2}}x}{45c^5} - \frac{2b\sqrt{1+\frac{1}{c^2x^2}}x^3}{45c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^5}{30c} + \frac{1}{6}x^6\left(a + b\text{csch}^{-1}(cx)\right)$$

output
```
1/6*x^6*(a+b*arccsch(c*x))+4/45*b*x*(1+1/c^2/x^2)^(1/2)/c^5-2/45*b*x^3*(1+
1/c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1+1/c^2/x^2)^(1/2)/c
```

3.2.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int x^5 \left(a + b\text{csch}^{-1}(cx)\right) \, dx = \frac{ax^6}{6} + b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(\frac{4x}{45c^5} - \frac{2x^3}{45c^3} + \frac{x^5}{30c}\right) + \frac{1}{6}bx^6\text{csch}^{-1}(cx)$$

input
```
Integrate[x^5*(a + b*ArcCsch[c*x]),x]
```

output
```
(a*x^6)/6 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) - (2*x^3)/(45*
c^3) + x^5/(30*c)) + (b*x^6*ArcCsch[c*x])/6
```

3.2.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6838, 803, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6838}$$

$$\frac{b\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{6c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{803}$$

$$\frac{b\left(\frac{1}{5}x^5\sqrt{\frac{1}{c^2x^2}+1}-\frac{4\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{5c^2}\right)}{6c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{803}$$

$$\frac{b\left(\frac{1}{5}x^5\sqrt{\frac{1}{c^2x^2}+1}-\frac{4\left(\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{3c^2}\right)}{5c^2}\right)}{6c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{746}$$

$$\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\left(\frac{1}{5}x^5\sqrt{\frac{1}{c^2x^2}+1}-\frac{4\left(\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2x\sqrt{\frac{1}{c^2x^2}+1}}{3c^2}\right)}{5c^2}\right)}{6c}$$

input
```
Int[x^5*(a + b*ArcCsch[c*x]),x]
```

output
```
(b*((Sqrt[1 + 1/(c^2*x^2)]*x^5)/5 - (4*((-2*Sqrt[1 + 1/(c^2*x^2)]*x)/(3*c^
2) + (Sqrt[1 + 1/(c^2*x^2)]*x^3)/3))/(5*c^2)))/(6*c) + (x^6*(a + b*ArcCsch
[c*x]))/6
```

3.2.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{a\,x^{6}}{6}+\frac{b\left(\frac{c^{6} x^{6} \operatorname{arccsch}\left(c x \right)}{6}+\frac{\left(c^{2} x^{2}+1\right) \left(3 c^{4} x^{4}-4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	79
derivativedivides	$\frac{\frac{a\,c^{6} x^{6}}{6}+b\left(\frac{c^{6} x^{6} \operatorname{arccsch}\left(c x \right)}{6}+\frac{\left(c^{2} x^{2}+1\right) \left(3 c^{4} x^{4}-4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	83
default	$\frac{\frac{a\,c^{6} x^{6}}{6}+b\left(\frac{c^{6} x^{6} \operatorname{arccsch}\left(c x \right)}{6}+\frac{\left(c^{2} x^{2}+1\right) \left(3 c^{4} x^{4}-4 c^{2} x^{2}+8\right)}{90 \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, c x}\right)}{c^{6}}$	83

input
```
int(x^5*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+b/c^6*(1/6*c^6*x^6*arccsch(c*x)+1/90*(c^2*x^2+1)*(3*c^4*x^4-4*c^
2*x^2+8)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{15\,bc^5x^6\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+15\,ac^5x^6+\left(3\,bc^4x^5-4\,bc^2x^3+8\,bx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{90\,c^5}$$

input
```
integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*b*c^5*x^6*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 15
*a*c^5*x^6 + (3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x
^2)))/c^5
```

3.2.6 Sympy [F]

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^5(a+b\,\text{acsch}\,(cx))\,dx$$

input
```
integrate(x**5*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**5*(a + b*acsch(c*x)), x)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{6}ax^6$$
$$+\frac{1}{90}\left(15\,x^6\,\text{arcsch}\,(cx)+\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)b$$

input
```
integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2)
 - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b
```

3.2.8 Giac [F]

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int(b\,\text{arcsch}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^5, x)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^5\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^5*(a + b*asinh(1/(c*x))), x)
```

3.3 $\int x^4 (a + b\text{csch}^{-1}(cx)) \, dx$

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^4 (a + b\text{csch}^{-1}(cx)) \, dx = -\frac{3b\sqrt{1+\frac{1}{c^2x^2}}x^2}{40c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^4}{20c} + \frac{1}{5}x^5(a + b\text{csch}^{-1}(cx)) + \frac{3b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{40c^5}$$

output
```
1/5*x^5*(a+b*arccsch(c*x))+3/40*b*arctanh((1+1/c^2/x^2)^(1/2))/c^5-3/40*b*
x^2*(1+1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1+1/c^2/x^2)^(1/2)/c
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int x^4 (a + b\text{csch}^{-1}(cx)) \, dx = \frac{ax^5}{5} + b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(-\frac{3x^2}{40c^3} + \frac{x^4}{20c}\right) + \frac{1}{5}bx^5\text{csch}^{-1}(cx) + \frac{3b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

input
```
Integrate[x^4*(a + b*ArcCsch[c*x]),x]
```

output
```
(a*x^5)/5 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) + x^4/(20*c
)) + (b*x^5*ArcCsch[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)
])])/(40*c^5)
```

3.3.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6838, 798, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\operatorname{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 6838$$

$$\frac{b\int \frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{5c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 798$$

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\int \frac{x^6}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{10c}$$

$$\Big\downarrow \; 52$$

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{3\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{10c}$$

$$\Big\downarrow \; 52$$

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{3\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{10c}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{3\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\int\frac{1}{\frac{c^2}{x^4}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{10c}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}-x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{4c^2}-\frac{1}{2}x^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{10c}$$

input
```
Int[x^4*(a + b*ArcCsch[c*x]),x]
```

output
```
(x^5*(a + b*ArcCsch[c*x]))/5 - (b*(-1/2*(Sqrt[1 + 1/(c^2*x^2)]*x^4) - (3*(
-(Sqrt[1 + 1/(c^2*x^2)]*x^2) + ArcTanh[Sqrt[1 + 1/(c^2*x^2)]]/c^2))/(4*c^2
)))/(10*c)
```

3.3.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

method	result	size
parts	$\frac{a\,x^5}{5}+\frac{b\left(\frac{c^5x^5\,\mathrm{arccsch}(cx)}{5}+\frac{\sqrt{c^2x^2+1}\,\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\,\mathrm{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	104
derivativedivides	$\frac{\frac{c^5x^5a}{5}+b\left(\frac{c^5x^5\,\mathrm{arccsch}(cx)}{5}+\frac{\sqrt{c^2x^2+1}\,\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\,\mathrm{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	108
default	$\frac{\frac{c^5x^5a}{5}+b\left(\frac{c^5x^5\,\mathrm{arccsch}(cx)}{5}+\frac{\sqrt{c^2x^2+1}\,\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\,\mathrm{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	108

input
```
int(x^4*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arccsch(c*x)+1/40*(c^2*x^2+1)^(1/2)*(2*c^3*x^
3*(c^2*x^2+1)^(1/2)-3*c*x*(c^2*x^2+1)^(1/2)+3*arcsinh(c*x))/((c^2*x^2+1)/c
^2/x^2)^(1/2)/c/x)
```

3.3.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.31

$$\int x^4\left(a+b\mathrm{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{8\,ac^5x^5+8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-3\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right.}{40\,c^5}$$

input
```
integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/40*(8*a*c^5*x^5 + 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x +
1) - 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*b*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 8*(b*c^5*x^5 - b*c^5)*log((c*x*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^4*x^4 - 3*b*c^2*x^2)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

3.3.6 Sympy [F]

$$\int x^4\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^4(a+b\,\text{acsch}\,(cx))\,dx$$

input
```
integrate(x**4*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**4*(a + b*acsch(c*x)), x)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int x^4\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{80}\left(16\,x^5\,\text{arcsch}\,(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

input
```
integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*
sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1)
+ c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1
) - 1)/c^4)/c)*b
```

3.3.8 Giac [F]

$$\int x^4 \left(a + b\text{csch}^{-1}(cx)\right) \, dx = \int (b \,\text{arcsch}\,(cx) + a)x^4 \, dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^4, x)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + b\text{csch}^{-1}(cx)\right) \, dx = \int x^4 \left(a + b \,\text{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^4*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^4*(a + b*asinh(1/(c*x))), x)
```

3.4 $\int x^3 \left(a + b\text{csch}^{-1}(cx)\right) \, dx$

3.4.1 Optimal result

Integrand size = 12, antiderivative size = 62

$$\int x^3 \left(a + b\text{csch}^{-1}(cx)\right) \, dx = -\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{1}{4}x^4\left(a + b\text{csch}^{-1}(cx)\right)$$

output
```
1/4*x^4*(a+b*arccsch(c*x))-1/6*b*x*(1+1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1+1
/c^2/x^2)^(1/2)/c
```

3.4.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int x^3 \left(a + b\text{csch}^{-1}(cx)\right) \, dx = \frac{ax^4}{4} + b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(-\frac{x}{6c^3} + \frac{x^3}{12c}\right) + \frac{1}{4}bx^4\text{csch}^{-1}(cx)$$

input
```
Integrate[x^3*(a + b*ArcCsch[c*x]),x]
```

output
```
(a*x^4)/4 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*(-1/6*x/c^3 + x^3/(12*c)) + (b
*x^4*ArcCsch[c*x])/4
```

3.4.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6838, 803, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6838}$$

$$\frac{b\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{4c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 803}$$

$$\frac{b\left(\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{3c^2}\right)}{4c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 746}$$

$$\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\left(\frac{1}{3}x^3\sqrt{\frac{1}{c^2x^2}+1}-\frac{2x\sqrt{\frac{1}{c^2x^2}+1}}{3c^2}\right)}{4c}$$

input
```
Int[x^3*(a + b*ArcCsch[c*x]),x]
```

output
```
(b*((-2*Sqrt[1 + 1/(c^2*x^2)]*x)/(3*c^2) + (Sqrt[1 + 1/(c^2*x^2)]*x^3)/3))
/(4*c) + (x^4*(a + b*ArcCsch[c*x]))/4
```

3.4.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c^{4} x^{4} \operatorname{arccsch}\left(c x \right)}{4}+\frac{\left(c^{2} x^{2}+1\right) \left(c^{2} x^{2}-2\right)}{12 \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, c x}\right)}{c^{4}}$	70
derivativedivides	$\frac{\frac{a \,c^{4} x^{4}}{4}+b \left(\frac{c^{4} x^{4} \operatorname{arccsch}\left(c x \right)}{4}+\frac{\left(c^{2} x^{2}+1\right) \left(c^{2} x^{2}-2\right)}{12 \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, c x}\right)}{c^{4}}$	74
default	$\frac{\frac{a \,c^{4} x^{4}}{4}+b \left(\frac{c^{4} x^{4} \operatorname{arccsch}\left(c x \right)}{4}+\frac{\left(c^{2} x^{2}+1\right) \left(c^{2} x^{2}-2\right)}{12 \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, c x}\right)}{c^{4}}$	74

input
```
int(x^3*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arccsch(c*x)+1/12*(c^2*x^2+1)*(c^2*x^2-2)/((c
^2*x^2+1)/c^2/x^2)^(1/2)/c/x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.40

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{3\,bc^3x^4\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+3\,ac^3x^4+\left(bc^2x^3-2\,bx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

input
```
integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*b*c^3*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 3*a
*c^3*x^4 + (b*c^2*x^3 - 2*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

3.4.6 Sympy [F]

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^3(a+b\operatorname{acsch}(cx))\,dx$$

input
```
integrate(x**3*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x)), x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)b$$

input
```
integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b
```

3.4.8 Giac [F]

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (b\,\text{arcsch}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3, x)
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^3\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^3*(a + b*asinh(1/(c*x))), x)
```

3.5 $\int x^2 \left(a + b\text{csch}^{-1}(cx)\right) \, dx$

3.5.1 Optimal result

Integrand size = 12, antiderivative size = 62

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} + \frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
1/3*x^3*(a+b*arccsch(c*x))-1/6*b*arctanh((1+1/c^2/x^2)^(1/2))/c^3+1/6*b*x^
2*(1+1/c^2/x^2)^(1/2)/c
```

3.5.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \frac{ax^3}{3} + \frac{bx^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{6c} + \frac{1}{3}bx^3\text{csch}^{-1}(cx) - \frac{b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(a*x^3)/3 + (b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsch[c
*x])/3 - (b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6838, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{csch}^{-1}(cx))\,dx$$

$$\Big\downarrow \text{6838}$$

$$\frac{b\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{3c}+\frac{1}{3}x^3(a+b\text{csch}^{-1}(cx))$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{3}x^3(a+b\text{csch}^{-1}(cx))-\frac{b\int \frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{6c}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{3}x^3(a+b\text{csch}^{-1}(cx))-\frac{b\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}}{2c^2}\right)}{6c}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}x^3(a+b\text{csch}^{-1}(cx))-\frac{b\left(x^2\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\int \frac{1}{\frac{c^2}{x^4}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{3}x^3(a+b\text{csch}^{-1}(cx))-\frac{b\left(\frac{\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^2}-x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c}$$

input
```
Int[x^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(x^3*(a + b*ArcCsch[c*x]))/3 - (b*(-(Sqrt[1 + 1/(c^2*x^2)]*x^2) + ArcTanh[
Sqrt[1 + 1/(c^2*x^2)]]/c^2))/(6*c)
```

3.5.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

method	result	size
parts	$\frac{a\,x^{3}}{3}+\frac{b\left(\frac{c^{3}x^{3}\operatorname{arccsch}\left(cx\right)}{3}-\frac{\sqrt{c^{2}x^{2}+1}\,\left(-cx\sqrt{c^{2}x^{2}+1}+\operatorname{arcsinh}\left(cx\right)\right)}{6\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,xc}\right)}{c^{3}}$	83
derivativedivides	$\frac{\frac{a\,c^{3}x^{3}}{3}+b\left(\frac{c^{3}x^{3}\operatorname{arccsch}\left(cx\right)}{3}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(cx\sqrt{c^{2}x^{2}+1}-\operatorname{arcsinh}\left(cx\right)\right)}{6\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,cx}\right)}{c^{3}}$	88
default	$\frac{\frac{a\,c^{3}x^{3}}{3}+b\left(\frac{c^{3}x^{3}\operatorname{arccsch}\left(cx\right)}{3}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(cx\sqrt{c^{2}x^{2}+1}-\operatorname{arcsinh}\left(cx\right)\right)}{6\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,cx}\right)}{c^{3}}$	88

input
```
int(x^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arccsch(c*x)-1/6*(c^2*x^2+1)^(1/2)*(-c*x*(c^2
*x^2+1)^(1/2)+arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/c)
```

3.5.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(52) = 104.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.00

$$\int x^{2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{2\,ac^{3}x^{3}+bc^{2}x^{2}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+2\,bc^{3}\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx+1\right)-2\,bc^{3}\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx-1\right)+b\log\left(c\right.}{6\,c^{3}}$$

input
```
integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*x^3 + b*c^2*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*b*c^3*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*b*c^3*log(c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) - c*x - 1) + b*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x) + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)))/c^3
```

3.5.6 Sympy [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int x^2 (a + b \,\text{acsch}\,(cx)) \, dx$$

input
```
integrate(x**2*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x)), x)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= \frac{1}{3}\,ax^3 + \frac{1}{12}\left(4\,x^3\,\text{arcsch}\,(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

input
```
integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2
*x^2) + 1) - 1)/c^2)/c)*b
```

3.5.8 Giac [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (b\,\text{arcsch}\,(cx) + a)x^2 \, dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2, x)
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^2\left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^2*(a + b*asinh(1/(c*x))), x)
```

3.6 $\int x(a + b\text{csch}^{-1}(cx))\,dx$

3.6.1 Optimal result

Integrand size = 10, antiderivative size = 38

$$\int x(a + b\text{csch}^{-1}(cx))\,dx = \frac{b\sqrt{1 + \frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}x^2(a + b\text{csch}^{-1}(cx))$$

output
```
1/2*x^2*(a+b*arccsch(c*x))+1/2*b*x*(1+1/c^2/x^2)^(1/2)/c
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int x(a + b\text{csch}^{-1}(cx))\,dx = \frac{ax^2}{2} + \frac{bx\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c} + \frac{1}{2}bx^2\text{csch}^{-1}(cx)$$

input
```
Integrate[x*(a + b*ArcCsch[c*x]),x]
```

output
```
(a*x^2)/2 + (b*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsch[c*x
])/2
```

3.6.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6838, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6838}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{746}$$

$$\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2c}$$

input
```
Int[x*(a + b*ArcCsch[c*x]),x]
```

output
```
(b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsch[c*x]))/2
```

3.6.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.6.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\left(\frac{c^2x^2\,\mathrm{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	61
derivativedivides	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\mathrm{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	65
default	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\,\mathrm{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	65

input
```
int(x*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arccsch(c*x)+1/2/((c^2*x^2+1)/c^2/x^2)^(1/2)/
c/x*(c^2*x^2+1))
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx=\frac{bcx^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+acx^2+bx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,c}$$

input
```
integrate(x*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/2*(b*c*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2
+ b*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c
```

3.6.6 Sympy [F]

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{acsch}\,(cx))\,dx$$

input
```
integrate(x*(a+b*acsch(c*x)),x)
```

output
```
Integral(x*(a + b*acsch(c*x)), x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\,\text{arcsch}\,(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)b$$

input
```
integrate(x*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b
```

3.6.8 Giac [F]

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x, x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{2}+\frac{b\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,c}$$

input
```
int(x*(a + b*asinh(1/(c*x))),x)
```

output
```
(a*x^2)/2 + (b*x^2*asinh(1/(c*x)))/2 + (b*x*(1/(c^2*x^2) + 1)^(1/2))/(2*c)
```

3.7 $\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.7.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx=ax+bx\text{csch}^{-1}(cx)+\frac{b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

output
```
a*x+b*x*arccsch(c*x)+b*arctanh((1+1/c^2/x^2)^(1/2))/c
```

3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx=ax+bx\text{csch}^{-1}(cx)+\frac{2b\sqrt{1+\frac{1}{c^2x^2}}\,x\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

input
```
Integrate[a + b*ArcCsch[c*x],x]
```

output
```
a*x + b*x*ArcCsch[c*x] + (2*b*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(-1 + Sqrt[1
 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

3.7.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \ 2009$$
$$ax+\frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+bx\text{csch}^{-1}(cx)$$

input
```
Int[a + b*ArcCsch[c*x],x]
```

output
```
a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.7.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

method	result	size
default	$ax+bx\,\text{arccsch}\left(cx\right)+\frac{b\ln\left(cx+cx\sqrt{1+\frac{1}{c^{2}x^{2}}}\right)}{c}$	36
parts	$ax+bx\,\text{arccsch}\left(cx\right)+\frac{b\ln\left(cx+cx\sqrt{1+\frac{1}{c^{2}x^{2}}}\right)}{c}$	36
derivativedivides	$\frac{acx+b\left(cx\,\text{arccsch}(cx)+\ln\left(cx+cx\sqrt{1+\frac{1}{c^{2}x^{2}}}\right)\right)}{c}$	39

input
```
int(a+b*arccsch(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.77

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{acx + bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (bcx -}{c}$$

input
```
integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

output
```
(a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)))/c
```

3.7.6 Sympy [F]

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a + b\,\text{acsch}\,(cx)\right)\,dx$$

input
```
integrate(a+b*acsch(c*x),x)
```

output
```
Integral(a + b*acsch(c*x), x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx$$
$$= ax + \frac{\left(2\,cx\,\text{arcsch}\,(cx) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(
1/(c^2*x^2) + 1) - 1))*b/c
```

3.7.8 Giac [F]

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx = \int b\,\text{arcsch}\,(cx) + a\,dx$$

input
```
integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

output
```
integrate(b*arccsch(c*x) + a, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx = \int a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\,dx$$

input
```
int(a + b*asinh(1/(c*x)),x)
```

output
```
int(a + b*asinh(1/(c*x)), x)
```

3.8 $\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx$

3.8.1 Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx = -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right) + \frac{1}{2}b\,\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)$$

output
```
-1/2*(a+b*arccsch(c*x))^2/b-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)
^(1/2))^2)+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx = \frac{1}{2}b\text{csch}^{-1}(cx)^2 - b\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + a\log(x) - \frac{1}{2}b\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x,x]
```

output
```
(b*ArcCsch[c*x]^2)/2 - b*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + a*Log[
x] - (b*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

3.8.3 Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.73, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6836, 6190, 25, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx$$

$$\Big\downarrow \text{ 6836}$$

$$-\int x\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{ 6190}$$

$$-\frac{\int -\left(\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{b}\right)\right)d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\coth\left(\frac{a}{b}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{b}\right)d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -i\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\tan\left(\frac{ia}{b}-\frac{i\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}+\frac{\pi}{2}\right)d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{i\int \left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\tan\left(\frac{1}{2}\left(\frac{2ia}{b}+\pi\right)-\frac{i\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}\right)d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}$$

$$\Big\downarrow \text{ 4201}$$

$$-\frac{i\left(2i\int \frac{e^{\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}-i\pi}\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{1+e^{\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}-i\pi}}d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)-\frac{i}{2x^2}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{i\left(2i\left(\frac{1}{2}b\int\log\left(1+e^{\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}-i\pi}\right)d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)-\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1+e^{-\frac{2\left(a+b\text{arcsi}\right.}{b}}\right.\right.\right.}{b}$$

$$\downarrow 2715$$

$$-\frac{i\left(2i\left(-\frac{1}{4}b^2\int x\log\left(1+e^{\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}-i\pi}\right)de^{\frac{2a}{b}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}-i\pi}-\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1+e^{\right.\right.\right.}{b}$$

$$\downarrow 2838$$

$$-\frac{i\left(2i\left(\frac{1}{4}b^2\text{PolyLog}\left(2,-a-b\text{arcsinh}\left(\frac{1}{cx}\right)\right)-\frac{1}{2}b\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1+e^{-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{b}+\frac{2a}{b}-i\pi}\right)\right)-\frac{i}{2x^2}\right.}{b}$$

input
```
Int[(a + b*ArcCsch[c*x])/x,x]
```

output
```
((-I)*((-1/2*I)/x^2 + (2*I)*(-1/2*(b*(a + b*ArcSinh[1/(c*x)])*Log[1 + E^((
2*a)/b - I*Pi - (2*(a + b*ArcSinh[1/(c*x)]))/b)]) + (b^2*PolyLog[2, -a - b
*ArcSinh[1/(c*x)]])/4)))/b
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

rule 6836
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

3.8.4 Maple [F]

$$\int \frac{a + b\ \text{arccsch}\,(cx)}{x} dx$$

input
```
int((a+b*arccsch(c*x))/x,x)
```

output
```
int((a+b*arccsch(c*x))/x,x)
```

3.8.5 Fricas [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x} \, dx = \int \frac{b \, \text{arcsch}\,(cx) + a}{x} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/x, x)
```

3.8.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x} \, dx = \int \frac{a + b \, \text{acsch}\,(cx)}{x} \, dx$$

input
```
integrate((a+b*acsch(c*x))/x,x)
```

output
```
Integral((a + b*acsch(c*x))/x, x)
```

3.8.7 Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x} \, dx = \int \frac{b \, \text{arcsch}\,(cx) + a}{x} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

output
```
-1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log
(x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x)
)*log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) +
 2*integrate(log(x)/(c^2*x^3 + x), x))*b + a*log(x)
```

3.8.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x, x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/x,x)
```

output
```
int((a + b*asinh(1/(c*x)))/x, x)
```

3.9 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx$

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx = bc\sqrt{1+\frac{1}{c^2x^2}} - \frac{a+b\text{csch}^{-1}(cx)}{x}$$

output
```
(-a-b*arccsch(c*x))/x+b*c*(1+1/c^2/x^2)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx = -\frac{a}{x} + bc\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{x}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x^2,x]
```

output
```
-(a/x) + b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/x
```

3.9.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6838, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx$$

$$\Big\downarrow \text{6838}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^3}dx}{c}-\frac{a+b\text{csch}^{-1}(cx)}{x}$$

$$\Big\downarrow \text{793}$$

$$bc\sqrt{\frac{1}{c^2x^2}+1}-\frac{a+b\text{csch}^{-1}(cx)}{x}$$

input
```
Int[(a + b*ArcCsch[c*x])/x^2,x]
```

output
```
b*c*Sqrt[1 + 1/(c^2*x^2)] - (a + b*ArcCsch[c*x])/x
```

3.9.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.93

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arccsch}(cx)}{cx}+\frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)$	58
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsch}(cx)}{cx}+\frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)\right)$	62
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsch}(cx)}{cx}+\frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)\right)$	62

input
```
int((a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*c*(-1/c/x*arccsch(c*x)+1/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2*x
^2+1))
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx=\frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-a}{x}$$

input
```
integrate((a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

output
```
(b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 1)/(c*x)) - a)/x
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = \begin{cases} -\frac{a}{x}+bc\sqrt{1+\frac{1}{c^2x^2}}-\frac{b\operatorname{acsch}(cx)}{x} & \text{for } c\neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*acsch(c*x))/x**2,x)
```

output
```
Piecewise((-a/x + b*c*sqrt(1 + 1/(c**2*x**2)) - b*acsch(c*x)/x, Ne(c, 0)),
 (-(a + zoo*b)/x, True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = \left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)b-\frac{a}{x}$$

input
```
integrate((a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b - a/x
```

3.9.8 Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x^2, x)
```

3.9.9 Mupad [B] (verification not implemented)

Time = 4.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2}\, dx = b\, c\, \sqrt{\frac{1}{c^2\, x^2} + 1} - \frac{a}{x} - \frac{b\, \text{asinh}\left(\frac{1}{c\, x}\right)}{x}$$

input
```
int((a + b*asinh(1/(c*x)))/x^2,x)
```

output
```
b*c*(1/(c^2*x^2) + 1)^(1/2) - a/x - (b*asinh(1/(c*x)))/x
```

3.10 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3} dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3} dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2\text{csch}^{-1}(cx) - \frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

output
```
-1/4*b*c^2*arccsch(c*x)+1/2*(-a-b*arccsch(c*x))/x^2+1/4*b*c*(1+1/c^2/x^2)^
(1/2)/x
```

3.10.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3} dx = -\frac{a}{2x^2} + \frac{bc\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{4x} - \frac{b\text{csch}^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\text{arcsinh}\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 + (b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsch[c*x])/
(2*x^2) - (b*c^2*ArcSinh[1/(c*x)])/4
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6838, 858, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 6838}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}dx}{2c}-\frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}d\frac{1}{x}}{2c}-\frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)}{2c}-\frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^3\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2c}-\frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

input
```
Int[(a + b*ArcCsch[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcCsch[c*x])/x^2 + (b*((c^2*Sqrt[1 + 1/(c^2*x^2)])/(2*x) - (c
^3*ArcSinh[1/(c*x)])/2))/(2*c)
```

3.10.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(45) = 90$.

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2}-\frac{\sqrt{c^2x^2+1}\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2-\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x^3c^3}\right)$	96
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2+\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3x^3}\right)\right)$	99
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2+\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3x^3}\right)\right)$	99

input
```
int((a+b*arccsch(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arccsch(c*x)-1/4*(c^2*x^2+1)^(1/2)*(arctanh
(1/(c^2*x^2+1)^(1/2))*c^2*x^2-(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/
2)/x^3/c^3)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3} \, dx = \frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - (bc^2x^2 + 2\,b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - 2\,a}{4\,x^2}$$

input
```
integrate((a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*(b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b*c^2*x^2 + 2*b)*log((c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

3.10.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3} \, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x^3} \, dx$$

input
```
integrate((a+b*acsch(c*x))/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))/x**3, x)
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.10

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = \frac{1}{8}\,b\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}+1\right)-1}-c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)+c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c}-\frac{4\,\operatorname{arcsch}(cx)}{x^2}\right) - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^
3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) -
 1))/c - 4*arccsch(c*x)/x^2) - 1/2*a/x^2
```

3.10.8 Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x^3, x)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 5.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^3}\,dx = \frac{b\,c\,\sqrt{\frac{1}{c^2\,x^2}+1}}{4\,x} - \frac{b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\,\left(\frac{c^2\,x}{4}+\frac{1}{2\,x}\right)}{x} - \frac{a}{2\,x^2}$$

input
```
int((a + b*asinh(1/(c*x)))/x^3,x)
```

output
```
(b*c*(1/(c^2*x^2) + 1)^(1/2))/(4*x) - (b*asinh(1/(c*x))*((c^2*x)/4 + 1/(2*
x)))/x - a/(2*x^2)
```

3.11 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^4}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4}\,dx=-\frac{1}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1+\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\text{csch}^{-1}(cx)}{3x^3}$$

output
```
1/9*b*c^3*(1+1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsch(c*x))/x^3-1/3*b*c^3*(1+1/c
^2/x^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4}\,dx=-\frac{a}{3x^3}+b\left(-\frac{2c^3}{9}+\frac{c}{9x^2}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}}-\frac{b\text{csch}^{-1}(cx)}{3x^3}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 + b*((-2*c^3)/9 + c/(9*x^2))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b
*ArcCsch[c*x])/(3*x^3)
```

3.11.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6838, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx$$

$$\Big\downarrow \text{6838}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^5}dx}{3c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}d\frac{1}{x^2}}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{53}$$

$$\frac{b\int \left(c^2\sqrt{1+\frac{1}{c^2x^2}}-\frac{c^2}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x^2}}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{2}{3}c^4\left(\frac{1}{c^2x^2}+1\right)^{3/2}-2c^4\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}$$

input
```
Int[(a + b*ArcCsch[c*x])/x^4,x]
```

output
```
(b*(-2*c^4*Sqrt[1 + 1/(c^2*x^2)] + (2*c^4*(1 + 1/(c^2*x^2))^(3/2))/3))/(6*
c) - (a + b*ArcCsch[c*x])/(3*x^3)
```

3.11.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3}-\frac{(c^2x^2+1)(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)$	71
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3}-\frac{(c^2x^2+1)(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)\right)$	75
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3}-\frac{(c^2x^2+1)(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)\right)$	75

input
```
int((a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arccsch(c*x)-1/9*(c^2*x^2+1)*(2*c^2*x^2-1)/
((c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4)
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^4} \, dx = -\frac{3\, b \log \left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + (2\, bc^3x^3 - bcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 3\, a}{9\, x^3}$$

input
```
integrate((a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^3*x^
3 - b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 3*a)/x^3
```

3.11.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^4} \, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x^4} \, dx$$

input
```
integrate((a+b*acsch(c*x))/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))/x**4, x)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^4} \, dx = \frac{1}{9}\, b \left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\, c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\, \text{arcsch}\,(cx)}{x^3}\right) - \frac{a}{3\, x^3}$$

input
```
integrate((a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/9*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*a
rccsch(c*x)/x^3) - 1/3*a/x^3
```

3.11.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{x^4}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x^4, x)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/x^4,x)
```

output
```
int((a + b*asinh(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx=\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x}+\frac{3}{32}bc^4\text{csch}^{-1}(cx)-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

output
```
3/32*b*c^4*arccsch(c*x)+1/4*(-a-b*arccsch(c*x))/x^4+1/16*b*c*(1+1/c^2/x^2)
^(1/2)/x^3-3/32*b*c^3*(1+1/c^2/x^2)^(1/2)/x
```

3.12.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx=-\frac{a}{4x^4}+b\left(\frac{c}{16x^3}-\frac{3c^3}{32x}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}}-\frac{b\text{csch}^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\text{arcsinh}\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 + b*(c/(16*x^3) - (3*c^3)/(32*x))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]
 - (b*ArcCsch[c*x])/(4*x^4) + (3*b*c^4*ArcSinh[1/(c*x)])/32
```

3.12.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6838, 858, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx$$

$$\Big\downarrow \text{6838}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^6}dx}{4c}-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}d\frac{1}{x}}{4c}-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{4x^3}-\frac{3}{4}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}d\frac{1}{x}\right)}{4c}-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{262}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{4x^3}-\frac{3}{4}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)\right)}{4c}-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

$$\Big\downarrow \text{222}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{4x^3}-\frac{3}{4}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^3\text{arcsinh}\left(\frac{1}{cx}\right)\right)\right)}{4c}-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

input
```
Int[(a + b*ArcCsch[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcCsch[c*x])/x^4 + (b*((c^2*Sqrt[1 + 1/(c^2*x^2)])/(4*x^3) -
(3*c^2*((c^2*Sqrt[1 + 1/(c^2*x^2)])/(2*x) - (c^3*ArcSinh[1/(c*x)])/2))/4))
/(4*c)
```

3.12.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.12.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.57

method	result
parts	$-\frac{a}{4x^4}+b\,c^4\left(-\frac{\operatorname{arccsch}(cx)}{4c^4x^4}-\frac{\sqrt{c^2x^2+1}\left(-3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-2\sqrt{c^2x^2+1}\right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right)$
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arccsch}(cx)}{4c^4x^4}-\frac{\sqrt{c^2x^2+1}\left(-3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-2\sqrt{c^2x^2+1}\right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right)\right)$
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arccsch}(cx)}{4c^4x^4}-\frac{\sqrt{c^2x^2+1}\left(-3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-2\sqrt{c^2x^2+1}\right)}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right)\right)$

input
```
int((a+b*arccsch(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/x^4+b*c^4*(-1/4/c^4/x^4*arccsch(c*x)-1/32*(c^2*x^2+1)^(1/2)*(-3*arc
tanh(1/(c^2*x^2+1)^(1/2))*c^4*x^4+3*(c^2*x^2+1)^(1/2)*c^2*x^2-2*(c^2*x^2+1
)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^5/x^5)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx = \frac{(3\,bc^4x^4-8\,b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(3\,bc^3x^3-2\,bcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-8\,a}{32\,x^4}$$

input
```
integrate((a+b*arccsch(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) - (3*b*c^3*x^3 - 2*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 8*a)/x^4
```

3.12.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x^5}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**5,x)
```

output
```
Integral((a + b*acsch(c*x))/x**5, x)
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(62) = 124.

Time = 0.22 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.99

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx$$
$$= \frac{1}{64}b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}+1\right)+1}}{c}-\frac{16\,\text{a}}{}\right.$$
$$-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arccsch(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/(
c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1
) + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*a/x^4
```

3.12.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x^5, x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^5}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/x^5,x)
```

output
```
int((a + b*asinh(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx$

3.13.1 Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx = \frac{1}{5}bc^5\sqrt{1+\frac{1}{c^2x^2}}-\frac{2}{15}bc^5\left(1+\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{25}bc^5\left(1+\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\text{csch}^{-1}(cx)}{5x^5}$$

output
```
-2/15*b*c^5*(1+1/c^2/x^2)^(3/2)+1/25*b*c^5*(1+1/c^2/x^2)^(5/2)+1/5*(-a-b*a
rccsch(c*x))/x^5+1/5*b*c^5*(1+1/c^2/x^2)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx = -\frac{a}{5x^5}+b\left(\frac{8c^5}{75}+\frac{c}{25x^4}-\frac{4c^3}{75x^2}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}}-\frac{b\text{csch}^{-1}(cx)}{5x^5}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 + b*((8*c^5)/75 + c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(1 + c^2*
x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(5*x^5)
```

3.13.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6838, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx$$

$$\Big\downarrow \text{6838}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^7}dx}{5c}-\frac{a+b\text{csch}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{798}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}d\frac{1}{x^2}}{10c}-\frac{a+b\text{csch}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{53}$$

$$\frac{b\int \left(\left(1+\frac{1}{c^2x^2}\right)^{3/2}c^4-2\sqrt{1+\frac{1}{c^2x^2}}c^4+\frac{c^4}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x^2}}{10c}-\frac{a+b\text{csch}^{-1}(cx)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(\frac{2}{5}c^6\left(\frac{1}{c^2x^2}+1\right)^{5/2}-\frac{4}{3}c^6\left(\frac{1}{c^2x^2}+1\right)^{3/2}+2c^6\sqrt{\frac{1}{c^2x^2}+1}\right)}{10c}-\frac{a+b\text{csch}^{-1}(cx)}{5x^5}$$

input
```
Int[(a + b*ArcCsch[c*x])/x^6,x]
```

output
```
(b*(2*c^6*Sqrt[1 + 1/(c^2*x^2)] - (4*c^6*(1 + 1/(c^2*x^2))^(3/2))/3 + (2*c
^6*(1 + 1/(c^2*x^2))^(5/2))/5))/(10*c) - (a + b*ArcCsch[c*x])/(5*x^5)
```

3.13.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.13.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arccsch}(cx)}{5c^5x^5}+\frac{(c^2x^2+1)\,(8c^4x^4-4c^2x^2+3)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)$	79
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsch}(cx)}{5c^5x^5}+\frac{(c^2x^2+1)\,(8c^4x^4-4c^2x^2+3)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arccsch}(cx)}{5c^5x^5}+\frac{(c^2x^2+1)\,(8c^4x^4-4c^2x^2+3)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

input
```
int((a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arccsch(c*x)+1/75*(c^2*x^2+1)*(8*c^4*x^4-4*
c^2*x^2+3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = -\frac{15\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (8\,bc^5x^5 - 4\,bc^3x^3 + 3\,bcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 15\,a}{75\,x^5}$$

input
```
integrate((a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*(15*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*
x^5 - 4*b*c^3*x^3 + 3*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 15*a)/x^5
```

3.13.6 Sympy [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = \int \frac{a + b\operatorname{acsch}(cx)}{x^6}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**6,x)
```

output
```
Integral((a + b*acsch(c*x))/x**6, x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^6}\,dx = \frac{1}{75}\,b\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{15\,\operatorname{arcsch}(cx)}{x^5}\right) - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) +
15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) - 1/5*a/x^5
```

3.13.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{x^6}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x^6, x)
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/x^6,x)
```

output
```
int((a + b*asinh(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^7}\,dx$

3.14.1 Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^7}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x} - \frac{5}{96}bc^6\text{csch}^{-1}(cx) - \frac{a+b\text{csch}^{-1}(cx)}{6x^6}$$

output
```
-5/96*b*c^6*arccsch(c*x)+1/6*(-a-b*arccsch(c*x))/x^6+1/36*b*c*(1+1/c^2/x^2
)^(1/2)/x^5-5/144*b*c^3*(1+1/c^2/x^2)^(1/2)/x^3+5/96*b*c^5*(1+1/c^2/x^2)^(
1/2)/x
```

3.14.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^7}\,dx = -\frac{a}{6x^6} + b\left(\frac{c}{36x^5} - \frac{5c^3}{144x^3} + \frac{5c^5}{96x}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\text{arcsinh}\left(\frac{1}{cx}\right)$$

input
```
Integrate[(a + b*ArcCsch[c*x])/x^7,x]
```

output
```
-1/6*a/x^6 + b*(c/(36*x^5) - (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(1 +
 c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(6*x^6) - (5*b*c^6*ArcSinh[1/(c*x)
])/96
```

3.14.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6838, 858, 262, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}dx$$

$$\Big\downarrow \text{ 6838}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^8}dx}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^6}d\frac{1}{x}}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{6x^5}-\frac{5}{6}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^4}d\frac{1}{x}\right)}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{6x^5}-\frac{5}{6}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{4x^3}-\frac{3}{4}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2}d\frac{1}{x}\right)\right)}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{6x^5}-\frac{5}{6}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{4x^3}-\frac{3}{4}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^2\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)\right)\right)}{6c}-\frac{a+b\operatorname{csch}^{-1}(cx)}{6x^6}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{b\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{6x^5}-\frac{5}{6}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{4x^3}-\frac{3}{4}c^2\left(\frac{c^2\sqrt{\frac{1}{c^2x^2}+1}}{2x}-\frac{1}{2}c^3\text{arcsinh}\left(\frac{1}{cx}\right)\right)\right)\right)}{6c}-\frac{a+b\text{csch}^{-1}(cx)}{6x^6}$$

input
```
Int[(a + b*ArcCsch[c*x])/x^7,x]
```

output
```
-1/6*(a + b*ArcCsch[c*x])/x^6 + (b*((c^2*Sqrt[1 + 1/(c^2*x^2)])/(6*x^5) -
(5*c^2*((c^2*Sqrt[1 + 1/(c^2*x^2)])/(4*x^3) - (3*c^2*((c^2*Sqrt[1 + 1/(c^2
*x^2)])/(2*x) - (c^3*ArcSinh[1/(c*x)])/2))/4))/6))/(6*c)
```

3.14.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.14.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.38

method	result
parts	$-\frac{a}{6x^6}+b\,c^6\left(-\frac{\operatorname{arccsch}(cx)}{6c^6x^6}-\frac{\sqrt{c^2x^2+1}\left(15\ \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6-15\sqrt{c^2x^2+1}\,c^4x^4+10\sqrt{c^2x^2+1}\,c^2x^2-8\right.}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^7x^7}\right.$
derivativedivides	$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\operatorname{arccsch}(cx)}{6c^6x^6}-\frac{\sqrt{c^2x^2+1}\left(15\ \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6-15\sqrt{c^2x^2+1}\,c^4x^4+10\sqrt{c^2x^2+1}\,c^2x^2\right.}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^7x^7}\right.\right.$
default	$c^6\left(-\frac{a}{6c^6x^6}+b\left(-\frac{\operatorname{arccsch}(cx)}{6c^6x^6}-\frac{\sqrt{c^2x^2+1}\left(15\ \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^6x^6-15\sqrt{c^2x^2+1}\,c^4x^4+10\sqrt{c^2x^2+1}\,c^2x^2\right.}{288\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^7x^7}\right.\right.$

input
```
int((a+b*arccsch(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6+b*c^6*(-1/6/c^6/x^6*arccsch(c*x)-1/288*(c^2*x^2+1)^(1/2)*(15*ar
ctanh(1/(c^2*x^2+1)^(1/2))*c^6*x^6-15*(c^2*x^2+1)^(1/2)*c^4*x^4+10*(c^2*x^
2+1)^(1/2)*c^2*x^2-8*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^7/x^
7)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^7}\,dx = -\frac{3\left(5\,bc^6x^6+16\,b\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(15\,bc^5x^5-10\,bc^3x^3+8\,bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+48\,a}{288\,x^6}$$

input
```
integrate((a+b*arccsch(c*x))/x^7,x, algorithm="fricas")
```

output
```
-1/288*(3*(5*b*c^6*x^6 + 16*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)) - (15*b*c^5*x^5 - 10*b*c^3*x^3 + 8*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + 48*a)/x^6
```

3.14.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^7}\,dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x^7}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**7,x)
```

output
```
Integral((a + b*acsch(c*x))/x**7, x)
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(82) = 164.

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.89

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^7}\,dx =$$

$$-\frac{1}{576}\,b\left(\frac{15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right) - 15\,c^7\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right) - \frac{2\left(15\,c^{12}x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 40\,c^{10}x^3\left(\frac{1}{c^2x^2}+1\right)\right)}{c^6x^6\left(\frac{1}{c^2x^2}+1\right)^3 - 3\,c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2 + 3}}{c}\right)$$

$$-\frac{a}{6\,x^6}$$

input
```
integrate((a+b*arccsch(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 15*c^7*log(c*x*sqrt
(1/(c^2*x^2) + 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) + 1)^(5/2) - 40*c^10*
x^3*(1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) + 1))/(c^6*x^6*(1/
(c^2*x^2) + 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) + 1)^2 + 3*c^2*x^2*(1/(c^2*x^2)
+ 1) - 1))/c + 96*arccsch(c*x)/x^6) - 1/6*a/x^6
```

3.14.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^7} \, dx = \int \frac{b \operatorname{arcsch}(cx) + a}{x^7} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^7,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/x^7, x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^7} \, dx = \int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^7} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/x^7,x)
```

output
```
int((a + b*asinh(1/(c*x)))/x^7, x)
```

3.15 $\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$

3.15.1 Optimal result

Integrand size = 14, antiderivative size = 105

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = \frac{b^2x^2}{12c^2} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)}{6c} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{b^2\log(x)}{3c^4}$$

output
```
1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsch(c*x))^2-1/3*b^2*ln(x)/c^4-1/3*b*x*(a
+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsch(c*x))*(1+1
/c^2/x^2)^(1/2)/c
```

3.15.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = \frac{cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1+\frac{1}{c^2x^2}}(-2+c^2x^2)\right)+2bcx\left(3ac^3x^3+b\sqrt{1+\frac{1}{c^2x^2}}(-2+c^2x^2)\right)\text{csch}^{-1}(cx)+}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcCsch[c*x])^2,x]
```

output
```
(c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2)
) + 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))*ArcCsch
[c*x] + 3*b^2*c^4*x^4*ArcCsch[c*x]^2 - 4*b^2*Log[x])/(12*c^4)
```

3.15.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6840, 5975, 3042, 4673, 25, 3042, 25, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx$$

$$\Big\downarrow \text{6840}$$

$$-\frac{\int c^5\sqrt{1+\frac{1}{c^2x^2}}x^5\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{5975}$$

$$-\frac{\frac{1}{2}b\int c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{1}{2}b\int \left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^4d\text{csch}^{-1}(cx)}{c^4}$$

$$\Big\downarrow \text{4673}$$

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\int -c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{3}c^3x^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{6}bc^2x^2\right)-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}b\left(-\frac{2}{3}\int c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{3}c^3x^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{6}bc^2x^2\right)-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{1}{2}b\left(-\frac{2}{3}\int -\left(\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^2\right)d\text{csch}^{-1}(cx)-\frac{1}{3}c^3x^3\sqrt{\frac{1}{c^2x^2}+1}}{c^4}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\int\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{1}{3}c^3x^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\right.\right.}{c^4}$$

$\Big\downarrow$ 4672

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-ib\int -ic\sqrt{1+\frac{1}{c^2x^2}}xd\text{csch}^{-1}(cx)\right)-\frac{1}{3}c^3x^3\right.}{c^4}$$

$\Big\downarrow$ 26

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-b\int c\sqrt{1+\frac{1}{c^2x^2}}xd\text{csch}^{-1}(cx)\right)-\frac{1}{3}c^3x^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{6}\right.}{c^4}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-b\int -i\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)\right.\right.}{c^4}$$

$\Big\downarrow$ 26

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)+ib\int\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)\right)\right.}{c^4}$$

$\Big\downarrow$ 3956

$$-\frac{\frac{1}{2}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-b\log\left(\frac{1}{cx}\right)\right)-\frac{1}{3}c^3x^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{6}bc^2x^2\right)-\frac{1}{4}c^4x^4\left(a\right.}{c^4}$$

input
```
Int[x^3*(a + b*ArcCsch[c*x])^2,x]
```

output
```
-((-1/4*(c^4*x^4*(a + b*ArcCsch[c*x])^2) + (b*(-1/6*(b*c^2*x^2) - (c^3*Sqr
t[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*x]))/3 + (2*(c*Sqrt[1 + 1/(c^2*x^2
)]*x*(a + b*ArcCsch[c*x]) - b*Log[1/(c*x)]))/3))/2)/c^4)
```

3.15. $\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.15.4 Maple [F]

$$\int x^3(a + b\,\text{arccsch}\,(cx))^2\,dx$$

input
```
int(x^3*(a+b*arccsch(c*x))^2,x)
```

output
```
int(x^3*(a+b*arccsch(c*x))^2,x)
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(91) = 182.

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.59

$$\int x^3\left(a + b\text{csch}^{-1}(cx)\right)^2\,dx$$
$$= \frac{3\,b^2c^4x^4\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 + 3\,a^2c^4x^4 + 6\,abc^4\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 6\,abc^4\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right.}{}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*b^2*c^4*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 +
 3*a^2*c^4*x^4 + 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + b^2*c^2*x
^2 - 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 + (b^2*c^3*x^3 - 2*b^2*c*
x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + 2*(a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^
4
```

3.15.6 Sympy [F]

$$\int x^3\left(a + b\text{csch}^{-1}(cx)\right)^2\,dx = \int x^3(a + b\,\text{acsch}\,(cx))^2\,dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))**2,x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))**2, x)
```

3.15.7 Maxima [F]

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\int (b\operatorname{arcsch}(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*x^4 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) -
 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a*b + 1/288*(72*x^4*log(sqrt(c^2*x^2 + 1)
 + 1)^2 + 1152*c^2*integrate(1/2*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 1152*c^2*integrate(1/2*x^5*lo
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) + 576*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(
x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1
152*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) +
 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 57
6*c^2*integrate(1/2*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x) - 1152*c^2*integrate(1/2*x^5*log(x)*log(sqrt(c^2*
x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1
), x) + 1152*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 1152*integrate(1/2*x^3*log(sqrt(c^2
*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x)*log(c) - 24*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2)
- 12*sqrt(c^2*x^2 + 1) + 6)*log(c)^2/c^4 - 48*(3*c^2*x^2 - 2*(c^2*x^2 + 1)
^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*log(c)^2/c^4 + 144*
(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2/c^4 + 144*(2*sqrt(c^2*x^2 + 1
) - log(c^2*x^2 + 1))*log(c)^2/c^4 - 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 ...
```

3.15.8 Giac [F]

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\int (b\operatorname{arcsch}(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2*x^3, x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx = \int x^3 \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2 \, dx$$

input
```
int(x^3*(a + b*asinh(1/(c*x)))^2,x)
```

output
```
int(x^3*(a + b*asinh(1/(c*x)))^2, x)
```

3.16 $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

3.16.1 Optimal result

Integrand size = 14, antiderivative size = 122

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2b\left(a+b\text{csch}^{-1}(cx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{3c^3}-\frac{b^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{3c^3}$$

output

```
1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsch(c*x))^2-2/3*b*(a+b*arccsch(c*x))*arctan
h(1/c/x+(1+1/c^2/x^2)^(1/2))/c^3-1/3*b^2*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1
/2))/c^3+1/3*b^2*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b*a
rccsch(c*x))*(1+1/c^2/x^2)^(1/2)/c
```

3.16.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.84

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$$

$$=\frac{b^2cx+abc^2\sqrt{1+\frac{1}{c^2x^2}}x^2+a^2c^3x^3+b^2c^2\sqrt{1+\frac{1}{c^2x^2}}x^2\text{csch}^{-1}(cx)+2abc^3x^3\text{csch}^{-1}(cx)+b^2c^3x^3\text{csch}^{-1}(cx)}{}$$

input
```
Integrate[x^2*(a + b*ArcCsch[c*x])^2,x]
```

output
```
(b^2*c*x + a*b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2 + a^2*c^3*x^3 + b^2*c^2*Sqrt[
1 + 1/(c^2*x^2)]*x^2*ArcCsch[c*x] + 2*a*b*c^3*x^3*ArcCsch[c*x] + b^2*c^3*x
^3*ArcCsch[c*x]^2 + b^2*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])] - b^2*ArcC
sch[c*x]*Log[1 + E^(-ArcCsch[c*x])] + (a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Log[-
(c*x) + Sqrt[1 + c^2*x^2]])/Sqrt[1 + c^2*x^2] + b^2*PolyLog[2, -E^(-ArcCsc
h[c*x])] - b^2*PolyLog[2, E^(-ArcCsch[c*x])])/(3*c^3)
```

3.16.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6840, 5975, 3042, 26, 4673, 26, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \ 6840$$

$$-\frac{\int c^4\sqrt{1+\frac{1}{c^2x^2}}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \ 5975$$

$$-\frac{\frac{2}{3}b\int c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{2}{3}b\int -i\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^3d\text{csch}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \ 26$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\int \left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^3d\text{csch}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \ 4673$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(\frac{1}{2}\int -icx\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{2}ic^2x^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{c^3}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(-\frac{1}{2}i\int cx\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{2}ic^2x^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(-\frac{1}{2}i\int i\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{2}ic^2x^2\sqrt{\frac{1}{c^2x^2}+1}(a}{c^3}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(\frac{1}{2}\int\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{2}ic^2x^2\sqrt{\frac{1}{c^2x^2}+1}(a+}{c^3}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(\frac{1}{2}\left(ib\int\log\left(1-e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-ib\int\log\left(1+e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(c\right.\right.}{c^3}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(\frac{1}{2}\left(ib\int e^{-\text{csch}^{-1}(cx)}\log\left(1-e^{\text{csch}^{-1}(cx)}\right)de^{\text{csch}^{-1}(cx)}-ib\int e^{-\text{csch}^{-1}(cx)}\log\left(1+\right.\right.\right.}{}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2}{3}ib\left(\frac{1}{2}\left(2i\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)+ib\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)-\right.\right.}{c^3}$$

input
```
Int[x^2*(a + b*ArcCsch[c*x])^2,x]
```

output
```
-((-1/3*(c^3*x^3*(a + b*ArcCsch[c*x])^2) - ((2*I)/3)*b*((-1/2*I)*b*c*x - (
I/2)*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcCsch[c*x]) + ((2*I)*(a + b*Ar
cCsch[c*x])*ArcTanh[E^ArcCsch[c*x]] + I*b*PolyLog[2, -E^ArcCsch[c*x]] - I*
b*PolyLog[2, E^ArcCsch[c*x]])/2))/c^3)
```

3.16. $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

3.16.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.16.4 Maple [F]

$$\int x^2(a+b\ \mathrm{arccsch}\ (cx))^2\,dx$$

input
```
int(x^2*(a+b*arccsch(c*x))^2,x)
```

output
```
int(x^2*(a+b*arccsch(c*x))^2,x)
```

3.16.5 Fricas [F]

$$\int x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)^2\,dx=\int (b\,\mathrm{arcsch}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arccsch(c*x)^2 + 2*a*b*x^2*arccsch(c*x) + a^2*x^2, x)
```

3.16.6 Sympy [F]

$$\int x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)^2\,dx=\int x^2(a+b\,\mathrm{acsch}\,(cx))^2\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))**2, x)
```

3.16.7 Maxima [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^2 \,dx = \int (b \,\text{arcsch}\,(cx) + a)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(
c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^
2*x^2) + 1) - 1)/c^2)/c)*a*b + 1/3*(x^3*log(sqrt(c^2*x^2 + 1) + 1)^2 - 3*i
ntegrate(-1/3*(3*c^2*x^4*log(c)^2 + 3*x^2*log(c)^2 + 3*(c^2*x^4 + x^2)*log
(x)^2 + 6*(c^2*x^4*log(c) + x^2*log(c))*log(x) - 2*(3*c^2*x^4*log(c) + 3*x
^2*log(c) + 3*(c^2*x^4 + x^2)*log(x) + (c^2*x^4*(3*log(c) + 1) + 3*x^2*log
(c) + 3*(c^2*x^4 + x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) +
 1) + 3*(c^2*x^4*log(c)^2 + x^2*log(c)^2 + (c^2*x^4 + x^2)*log(x)^2 + 2*(c
^2*x^4*log(c) + x^2*log(c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2
 + 1)^(3/2) + 1), x))*b^2
```

3.16.8 Giac [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^2 \,dx = \int (b \,\text{arcsch}\,(cx) + a)^2 x^2 \,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2*x^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^2 \,dx = \int x^2 \left(a + b \,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2 \,dx$$

input
```
int(x^2*(a + b*asinh(1/(c*x)))^2,x)
```

output
```
int(x^2*(a + b*asinh(1/(c*x)))^2, x)
```

3.17 $\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 54

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

output
```
1/2*x^2*(a+b*arccsch(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsch(c*x))*(1+1/c^2
/x^2)^(1/2)/c
```

3.17.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\frac{acx\left(2b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)+2bcx\left(b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)\text{csch}^{-1}(cx)+b^2c^2x^2\text{csch}^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

input
```
Integrate[x*(a + b*ArcCsch[c*x])^2,x]
```

output
```
(a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 + 1/(c^2*x^
2)] + a*c*x)*ArcCsch[c*x] + b^2*c^2*x^2*ArcCsch[c*x]^2 + 2*b^2*Log[c*x])/(
2*c^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6840, 5975, 3042, 25, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$$

$$\Big\downarrow \text{6840}$$

$$-\frac{\int c^3\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{5975}$$

$$-\frac{b\int c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2+b\int -\left(\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^2\right)d\text{csch}^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-b\int \left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c^2}$$

$$\Big\downarrow \text{4672}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-ib\int -ic\sqrt{1+\frac{1}{c^2x^2}}xd\text{csch}^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-b\int c\sqrt{1+\frac{1}{c^2x^2}}xd\text{csch}^{-1}(cx)\right)-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-b\int -i\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)+ib\int\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)\right)}{c^2}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-b\log\left(\frac{1}{cx}\right)\right)}{c^2}$$

input
```
Int[x*(a + b*ArcCsch[c*x])^2,x]
```

output
```
-((-1/2*(c^2*x^2*(a + b*ArcCsch[c*x])^2) - b*(c*Sqrt[1 + 1/(c^2*x^2)]*x*(a
 + b*ArcCsch[c*x]) - b*Log[1/(c*x)]))/c^2)
```

3.17.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.17.4 Maple [F]

$$\int x(a+b\,\mathrm{arccsch}\,(cx))^2\,dx$$

input
```
int(x*(a+b*arccsch(c*x))^2,x)
```

output
```
int(x*(a+b*arccsch(c*x))^2,x)
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.33

$$\int x\left(a+b\mathrm{csch}^{-1}(cx)\right)^2\,dx$$
$$=\frac{b^2c^2x^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2+a^2c^2x^2+2\,abc^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-2\,abc^2\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-\right.}{2\,c^2}$$

input
```
integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*c^2*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^
2*c^2*x^2 + 2*a*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2
*a*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*a*b*c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 + b^2*c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 1)/(c*x)))/c^2
```

3.17.6 Sympy [F]

$$\int x(a+b\text{csch}^{-1}(cx))^2\,dx=\int x(a+b\,\text{acsch}\,(cx))^2\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))**2,x)
```

output
```
Integral(x*(a + b*acsch(c*x))**2, x)
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int x(a+b\text{csch}^{-1}(cx))^2\,dx=\frac{1}{2}\,b^2x^2\,\text{arcsch}\,(cx)^2+\frac{1}{2}\,a^2x^2+\left(x^2\,\text{arcsch}\,(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)ab+\left(\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\text{arcsch}\,(cx)}{c}+\frac{\log(x)}{c^2}\right)b^2$$

input
```
integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arccsch(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c
^2*x^2) + 1)/c)*a*b + (x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + log(x)/c^2
)*b^2
```

3.17.8 Giac [F]

$$\int x(a+b\text{csch}^{-1}(cx))^2\,dx=\int (b\,\text{arcsch}\,(cx)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2*x, x)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\int x\,{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^2\,dx$$

input
```
int(x*(a + b*asinh(1/(c*x)))^2,x)
```

output
```
int(x*(a + b*asinh(1/(c*x)))^2, x)
```

3.18 $\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=x\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{2b^2\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c}-\frac{2b^2\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arccsch(c*x))^2+4*b*(a+b*arccsch(c*x))*arctanh(1/c/x+(1+1/c^2/x^2)^
(1/2))/c+2*b^2*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c-2*b^2*polylog(2,1/c
/x+(1+1/c^2/x^2)^(1/2))/c
```

3.18.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.99

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\frac{a^2cx+2abcx\text{csch}^{-1}(cx)+b^2cx\text{csch}^{-1}(cx)^2-2b^2\text{csch}^{-1}(cx)\log\left(1-e^{-\text{csch}^{-1}(cx)}\right)+2b^2\text{csch}^{-1}(cx)\log\left(1\right.}{}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^2,x]
```

output
```
(a^2*c*x + 2*a*b*c*x*ArcCsch[c*x] + b^2*c*x*ArcCsch[c*x]^2 - 2*b^2*ArcCsch
[c*x]*Log[1 - E^(-ArcCsch[c*x])] + 2*b^2*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[
c*x])] + 2*a*b*Log[Cosh[ArcCsch[c*x]/2]] - 2*a*b*Log[Sinh[ArcCsch[c*x]/2]]
 - 2*b^2*PolyLog[2, -E^(-ArcCsch[c*x])] + 2*b^2*PolyLog[2, E^(-ArcCsch[c*x
])])/c
```

3.18.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {6834, 5975, 3042, 26, 4670, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

$$\Big\downarrow \ 6834$$

$$-\frac{\int c^2\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c}$$

$$\Big\downarrow \ 5975$$

$$-\frac{2b\int cx\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-cx\left(a+b\text{csch}^{-1}(cx)\right)^2}{c}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^2+2b\int i\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)}{c}$$

$$\Big\downarrow \ 26$$

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\int\left(a+b\text{csch}^{-1}(cx)\right)\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)}{c}$$

$$\Big\downarrow \ 4670$$

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(ib\int\log\left(1-e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-ib\int\log\left(1+e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)+2\right.}{c}$$

$$\Big\downarrow \ 2715$$

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(ib\int e^{-\text{csch}^{-1}(cx)}\log\left(1-e^{\text{csch}^{-1}(cx)}\right)de^{\text{csch}^{-1}(cx)}-ib\int e^{-\text{csch}^{-1}(cx)}\log\left(1+e^{\text{csch}^{-1}}\right.\right.}{c}$$

$\Big\downarrow$ 2838

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(2i\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)+ib\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)-ib\,\text{Poly}\right.}{c}$$

input
```
Int[(a + b*ArcCsch[c*x])^2,x]
```

output
```
-((-(c*x*(a + b*ArcCsch[c*x])^2) + (2*I)*b*((2*I)*(a + b*ArcCsch[c*x])*Arc
Tanh[E^ArcCsch[c*x]] + I*b*PolyLog[2, -E^ArcCsch[c*x]] - I*b*PolyLog[2, E^
ArcCsch[c*x]]))/c)
```

3.18.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6834
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

3.18.4 Maple [F]

$$\int (a+b\,\mathrm{arccsch}\,(cx))^2\,dx$$

input
```
int((a+b*arccsch(c*x))^2,x)
```

output
```
int((a+b*arccsch(c*x))^2,x)
```

3.18.5 Fricas [F]

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\int \left(b\,\mathrm{arcsch}\,(cx)+a\right)^2\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2, x)
```

3.18.6 Sympy [F]

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{acsch}\,(cx)\right)^2\,dx$$

input
```
integrate((a+b*acsch(c*x))**2,x)
```

output
```
Integral((a + b*acsch(c*x))**2, x)
```

3.18.7 Maxima [F]

$$\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=\int \left(b\operatorname{arcsch}\left(cx\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

output
```
(x*log(sqrt(c^2*x^2 + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2
+ 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x) - 2*(c^2*x^2
*log(c) + (c^2*x^2 + 1)*log(x) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 + 1)*log
(x) + log(c))*sqrt(c^2*x^2 + 1) + log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^
2*x^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + l
og(c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x))
*b^2 + a^2*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(
sqrt(1/(c^2*x^2) + 1) - 1))*a*b/c
```

3.18.8 Giac [F]

$$\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=\int \left(b\operatorname{arcsch}\left(cx\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\,dx=\int \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^2,x)
```

output
```
int((a + b*asinh(1/(c*x)))^2, x)
```

3.19 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 81

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\operatorname{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right) + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right)$$

output
```
1/3*(a+b*arccsch(c*x))^3/b-(a+b*arccsch(c*x))^2*ln(1-(1/c/x+(1+1/c^2/x^2)^
(1/2))^2)-b*(a+b*arccsch(c*x))*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/
2*b^2*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.41

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}\,dx = ab\operatorname{csch}^{-1}(cx)^2 + \frac{1}{3}b^2\operatorname{csch}^{-1}(cx)^3 - 2ab\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - b^2\operatorname{csch}^{-1}(cx)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) + a^2\log(cx) - b\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right) + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right)$$

input
```
Integrate[(a + b*ArcCsch[c*x])^2/x,x]
```

output
```
a*b*ArcCsch[c*x]^2 + (b^2*ArcCsch[c*x]^3)/3 - 2*a*b*ArcCsch[c*x]*Log[1 - E
^(2*ArcCsch[c*x])] - b^2*ArcCsch[c*x]^2*Log[1 - E^(2*ArcCsch[c*x])] + a^2*
Log[c*x] - b*(a + b*ArcCsch[c*x])*PolyLog[2, E^(2*ArcCsch[c*x])] + (b^2*Po
lyLog[3, E^(2*ArcCsch[c*x])])/2
```

3.19.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6840, 3042, 26, 4199, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x} \, dx$$

$$\Big\downarrow \text{6840}$$

$$-\int c\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\int -i\left(a+b\text{csch}^{-1}(cx)\right)^2\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$i\int \left(a+b\text{csch}^{-1}(cx)\right)^2\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{4199}$$

$$i\left(2i\int -\frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)^2}{1-e^{2\text{csch}^{-1}(cx)}}d\text{csch}^{-1}(cx)-\frac{i\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-2i\int \frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)^2}{1-e^{2\text{csch}^{-1}(cx)}}d\text{csch}^{-1}(cx)-\frac{i\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(b\int\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-\frac{1}{2}\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^2\right)-\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-2i\left(b\left(\frac{1}{2}b\int\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)\right)-\frac{1}{2}\log\left(\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(-2i\left(b\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(cx)}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)de^{2\text{csch}^{-1}(cx)}-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx\right)\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$i\left(-2i\left(b\left(\frac{1}{4}b\,\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)\right)-\frac{1}{2}\log\left(1-e^{2\text{csch}^{-1}(cx)}\right.\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^2/x,x]
```

output
```
I*(((-1/3*I)*(a + b*ArcCsch[c*x])^3)/b - (2*I)*(-1/2*((a + b*ArcCsch[c*x])
^2*Log[1 - E^(2*ArcCsch[c*x])]) + b*(-1/2*((a + b*ArcCsch[c*x])*PolyLog[2,
 E^(2*ArcCsch[c*x])]) + (b*PolyLog[3, E^(2*ArcCsch[c*x])])/4)))
```

3.19.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.19. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}\,dx$

3.19.4 Maple [F]

$$\int \frac{(a + b\ \text{arccsch}\,(cx))^2}{x} dx$$

input
```
int((a+b*arccsch(c*x))^2/x,x)
```

output
```
int((a+b*arccsch(c*x))^2/x,x)
```

3.19.5 Fricas [F]

$$\int \frac{\left(a + b\text{csch}^{-1}(cx)\right)^2}{x} dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)^2}{x} dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)/x, x)
```

3.19.6 Sympy [F]

$$\int \frac{\left(a + b\text{csch}^{-1}(cx)\right)^2}{x} dx = \int \frac{(a + b\,\text{acsch}\,(cx))^2}{x} dx$$

input
```
integrate((a+b*acsch(c*x))**2/x,x)
```

output
```
Integral((a + b*acsch(c*x))**2/x, x)
```

3.19.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{arcsch}\,(cx)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x,x, algorithm="maxima")
```

output
```
b^2*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2 + a^2*log(x) - integrate(-(b^2*log
(c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2 - 2*a*b*log(c) + (b^2*c^
2*x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a
*b)*log(x) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c
^2*x^2 + b^2)*log(x) + sqrt(c^2*x^2 + 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 +
 b^2*log(c) - a*b + (2*b^2*c^2*x^2 + b^2)*log(x)))*log(sqrt(c^2*x^2 + 1) +
 1) + sqrt(c^2*x^2 + 1)*(b^2*log(c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(
c))*x^2 - 2*a*b*log(c) + (b^2*c^2*x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c)
 - a*b*c^2)*x^2 + b^2*log(c) - a*b)*log(x)))/(c^2*x^3 + (c^2*x^3 + x)*sqrt
(c^2*x^2 + 1) + x), x)
```

3.19.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{arcsch}\,(cx)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2/x, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^2/x,x)
```

output
```
int((a + b*asinh(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 49

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx=-\frac{2b^2}{x}+2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x}$$

output
```
-2*b^2/x-(a+b*arccsch(c*x))^2/x+2*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/
2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^2}\,dx$$
$$=-\frac{a^2+2b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x+2b\left(a-bc\sqrt{1+\frac{1}{c^2x^2}}x\right)\operatorname{csch}^{-1}(cx)+b^2\operatorname{csch}^{-1}(cx)^2}{x}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^2/x^2,x]
```

output
```
-((a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 2*b*(a - b*c*Sqrt[1 + 1
/(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*ArcCsch[c*x]^2)/x)
```

3.20.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6840, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{ 6840}$$

$$-c\int \sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\int \left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{ 3777}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}-2ib\int -\frac{i\left(a+b\text{csch}^{-1}(cx)\right)}{cx}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 26}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}-2b\int \frac{a+b\text{csch}^{-1}(cx)}{cx}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}-2b\int -i\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 26}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}+2ib\int \left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3777}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}+2ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-ib\int \sqrt{1+\frac{1}{c^2x^2}}d\text{csch}^{-1}(cx)\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}+2ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-ib\int\sin\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}+2ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-\frac{ib}{cx}\right)\right)$$

input
```
Int[(a + b*ArcCsch[c*x])^2/x^2,x]
```

output
```
-(c*((a + b*ArcCsch[c*x])^2/(c*x) + (2*I)*b*(((-I)*b)/(c*x) + I*Sqrt[1 + 1
/(c^2*x^2)]*(a + b*ArcCsch[c*x]))))
```

3.20.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.20.4 Maple [F]

$$\int \frac{(a+b \operatorname{arccsch}(cx))^2}{x^2} dx$$

input
```
int((a+b*arccsch(c*x))^2/x^2,x)
```

output
```
int((a+b*arccsch(c*x))^2/x^2,x)
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(47) = 94.

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^2}{x^2} dx = \frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - a^2 - 2\,b^2 + 2\left(b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - ab\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{x}$$

input
```
integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
(2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b^2*log((c*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - a*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

3.20.6 Sympy [F]

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^2}{x^2} dx = \int \frac{(a+b\operatorname{acsch}(cx))^2}{x^2} dx$$

input
```
integrate((a+b*acsch(c*x))**2/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))**2/x**2, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx = 2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\mathrm{arcsch}\,(cx)}{x}\right)ab + 2\left(c\sqrt{\frac{1}{c^2x^2}+1}\,\mathrm{arcsch}\,(cx)-\frac{1}{x}\right)b^2 - \frac{b^2\,\mathrm{arcsch}\,(cx)^2}{x} - \frac{a^2}{x}$$

input
```
integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) +
 1)*arccsch(c*x) - 1/x)*b^2 - b^2*arccsch(c*x)^2/x - a^2/x
```

3.20.8 Giac [F]

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2/x^2, x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^2/x^2,x)
```

output
```
int((a + b*asinh(1/(c*x)))^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx$

3.21.1 Optimal result

Integrand size = 14, antiderivative size = 87

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{b^2}{4x^2}-\frac{1}{2}abc^2\operatorname{csch}^{-1}(cx)-\frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2x^2}$$

output
```
-1/4*b^2/x^2-1/2*a*b*c^2*arccsch(c*x)-1/4*b^2*c^2*arccsch(c*x)^2-1/2*(a+b*
arccsch(c*x))^2/x^2+1/2*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/x
```

3.21.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx = -\frac{2a^2+b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x-2b\left(-2a+bc\sqrt{1+\frac{1}{c^2x^2}}x\right)\operatorname{csch}^{-1}(cx)+b^2(2+c^2x^2)\operatorname{csch}^{-1}(cx)^2+2abc^2}{4x^2}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^2/x^3,x]
```

output
```
-1/4*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 2*b*(-2*a + b*c*Sqrt
[1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*(2 + c^2*x^2)*ArcCsch[c*x]^2 + 2*a
*b*c^2*x^2*ArcSinh[1/(c*x)])/x^2
```

3.21.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6840, 5969, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{6840}$$

$$-c^2\int \frac{\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{5969}$$

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2x^2}-b\int \frac{a+b\text{csch}^{-1}(cx)}{c^2x^2}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2x^2}-b\int -\left(\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)^2\right)d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{25}$$

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2x^2}+b\int \left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^2\left(b\left(\frac{1}{2}\int \left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{2cx}+\frac{b}{4c^2x^2}\right)+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2x^2}\right)$$

$$\Big\downarrow \text{17}$$

$$-c^2\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^2x^2}+b\left(-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2cx}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4b}+\frac{b}{4c^2x^2}\right)\right)$$

input
```
Int[(a + b*ArcCsch[c*x])^2/x^3,x]
```

output
```
-(c^2*((a + b*ArcCsch[c*x])^2/(2*c^2*x^2) + b*(b/(4*c^2*x^2) - (Sqrt[1 + 1
/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*c*x) + (a + b*ArcCsch[c*x])^2/(4*b)))
)
```

3.21.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.21.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsch}\,(cx))^2}{x^3}dx$$

input
```
int((a+b*arccsch(c*x))^2/x^3,x)
```

output
```
int((a+b*arccsch(c*x))^2/x^3,x)
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(75) = 150.

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.87

$$\int \frac{(a+b\mathrm{csch}^{-1}(cx))^2}{x^3}\,dx = \frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - (b^2c^2x^2+2\,b^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 2\,a^2 - b^2 - 2\left(abc^2x^2 - b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2\,ab\right)\log}{4\,x^2}$$

input
```
integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/4*(2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b^2*c^2*x^2 + 2*b^2)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 - 2*(a*b*c^2
*x^2 - b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b)*log((c*x*sqrt((c^2*x
^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

3.21.6 Sympy [F]

$$\int \frac{(a+b\mathrm{csch}^{-1}(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))^2}{x^3}\,dx$$

input
```
integrate((a+b*acsch(c*x))**2/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))**2/x**3, x)
```

3.21.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(b\,\text{arcsch}\left(cx\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) -
c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
 - 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^2
 + 2*integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*
(c^2*x^2*log(c) + log(c))*log(x) - (2*c^2*x^2*log(c) + 2*(c^2*x^2 + 1)*log
(x) + (c^2*x^2*(2*log(c) - 1) + 2*(c^2*x^2 + 1)*log(x) + 2*log(c))*sqrt(c^
2*x^2 + 1) + 2*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^2*log(c)^2 + (c
^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt
(c^2*x^2 + 1))/(c^2*x^5 + x^3 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)) -
1/2*a^2/x^2
```

3.21.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(b\,\text{arcsch}\left(cx\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2/x^3, x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^2/x^3,x)
```

output
```
int((a + b*asinh(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx$

3.22.1 Optimal result

Integrand size = 14, antiderivative size = 100

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right) + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^3}$$

output
```
-2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsch(c*x))^2/x^3-4/9*b*c^3*(a+b*ar
ccsch(c*x))*(1+1/c^2/x^2)^(1/2)+2/9*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(
1/2)/x^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx$$
$$= \frac{-9a^2+6abc\sqrt{1+\frac{1}{c^2x^2}}x(1-2c^2x^2)+2b^2(-1+6c^2x^2)-6b\left(3a+bc\sqrt{1+\frac{1}{c^2x^2}}x(-1+2c^2x^2)\right)\text{csch}^{-1}(ca}{27x^3}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^2/x^4,x]
```

output
```
(-9*a^2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*
c^2*x^2) - 6*b*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2))*ArcCsc
h[c*x] - 9*b^2*ArcCsch[c*x]^2)/(27*x^3)
```

3.22.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6840, 5969, 3042, 26, 3791, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx$$

$$\Big\downarrow \text{6840}$$

$$-c^3\int \frac{\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{c^2x^2}d\operatorname{csch}^{-1}(cx)$$

$$\Big\downarrow \text{5969}$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\int \frac{a+b\operatorname{csch}^{-1}(cx)}{c^3x^3}d\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}b\int i\left(a+b\operatorname{csch}^{-1}(cx)\right)\sin\left(i\operatorname{csch}^{-1}(cx)\right)^3d\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{26}$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\int \left(a+b\operatorname{csch}^{-1}(cx)\right)\sin\left(i\operatorname{csch}^{-1}(cx)\right)^3d\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(\frac{2}{3}\int \frac{i\left(a+b\operatorname{csch}^{-1}(cx)\right)}{cx}d\operatorname{csch}^{-1}(cx)-\frac{i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3c^2x^2}+\frac{ib}{9c^3x^3}\right)\right)$$

$$\Big\downarrow \text{26}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(\frac{2}{3}i\int\frac{a+b\text{csch}^{-1}(cx)}{cx}d\text{csch}^{-1}(cx)-\frac{i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^2x^2}+\frac{ib}{9c^3x^3}\right)\right)$$

$\downarrow$ 3042

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(\frac{2}{3}i\int-i\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csc}\right.}{3c^2x^2}\right.\right.$$

$\downarrow$ 26

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(\frac{2}{3}\int\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-}\right.}{3c^2x^2}\right.\right.$$

$\downarrow$ 3777

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-ib\int\sqrt{1+\frac{1}{c^2x^2}}d\text{csch}^{-1}(cx)\right)-\frac{i\sqrt{\frac{1}{c^2x^2}+}}{}\right.\right.$$

$\downarrow$ 3042

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-ib\int\sin\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)\right)\right.\right.$$

$\downarrow$ 3117

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^3x^3}-\frac{2}{3}ib\left(-\frac{i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^2x^2}+\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)-\frac{ib}{cx}\right)+\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^2/x^4,x]
```

output
```
-(c^3*((a + b*ArcCsch[c*x])^2/(3*c^3*x^3) - ((2*I)/3)*b*(((I/9)*b)/(c^3*x^
3) - ((I/3)*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(c^2*x^2) + (2*(((
-I)*b)/(c*x) + I*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])))/3)))
```

3.22. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.22. $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^4}\,dx$

3.22.4 Maple [F]

$$\int \frac{(a+b\,\text{arccsch}\,(cx))^2}{x^4}dx$$

input
```
int((a+b*arccsch(c*x))^2/x^4,x)
```

output
```
int((a+b*arccsch(c*x))^2/x^4,x)
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\text{csch}^{-1}(cx))^2}{x^4}\,dx = \frac{12\,b^2c^2x^2 - 9\,b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 9\,a^2 - 2\,b^2 - 6\left(3\,ab + (2\,b^2c^3x^3 - b^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\cdots}{cx}\right.}{27\,x^3}$$

input
```
integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/27*(12*b^2*c^2*x^2 - 9*b^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x))^2 - 9*a^2 - 2*b^2 - 6*(3*a*b + (2*b^2*c^3*x^3 - b^2*c*x)*sqrt((c^2*x
^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 6
*(2*a*b*c^3*x^3 - a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^3
```

3.22.6 Sympy [F]

$$\int \frac{(a+b\text{csch}^{-1}(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))^2}{x^4}\,dx$$

input
```
integrate((a+b*acsch(c*x))**2/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))**2/x**4, x)
```

3.22.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
2/9*a*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3
*arccsch(c*x)/x^3) - 1/3*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^3 + 3*integra
te(-1/3*(3*c^2*x^2*log(c)^2 + 3*(c^2*x^2 + 1)*log(x)^2 + 3*log(c)^2 + 6*(c
^2*x^2*log(c) + log(c))*log(x) - 2*(3*c^2*x^2*log(c) + 3*(c^2*x^2 + 1)*log
(x) + (c^2*x^2*(3*log(c) - 1) + 3*(c^2*x^2 + 1)*log(x) + 3*log(c))*sqrt(c^
2*x^2 + 1) + 3*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 3*(c^2*x^2*log(c)^2 +
(c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sq
rt(c^2*x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1)), x))
- 1/3*a^2/x^3
```

3.22.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2/x^4, x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^2/x^4,x)
```

output
```
int((a + b*asinh(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2}{x^5}\,dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 132

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx = -\frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\text{csch}^{-1}(cx)+\frac{3}{32}b^2c^4\text{csch}^{-1}(cx)^2+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3}-\frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{16x}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4}$$

output
```
-1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsch(c*x)+3/32*b^2*c^4*arcc
sch(c*x)^2-1/4*(a+b*arccsch(c*x))^2/x^4+1/8*b*c*(a+b*arccsch(c*x))*(1+1/c^
2/x^2)^(1/2)/x^3-3/16*b*c^3*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/x
```

3.23.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \frac{-8a^2-b^2+4abc\sqrt{1+\frac{1}{c^2x^2}}x+3b^2c^2x^2-6abc^3\sqrt{1+\frac{1}{c^2x^2}}x^3-2b\left(8a+bc\sqrt{1+\frac{1}{c^2x^2}}x(-2+3c^2x^2)\right)\text{csch}}{32x^4}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^2/x^5,x]
```

output
```
(-8*a^2 - b^2 + 4*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^
3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 +
 3*c^2*x^2))*ArcCsch[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^2 + 6*a*b*c^
4*x^4*ArcSinh[1/(c*x)])/(32*x^4)
```

3.23.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6840, 5969, 3042, 3791, 25, 3042, 25, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{6840}$$

$$-c^4\int \frac{\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^3x^3}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{5969}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\int \frac{a+b\text{csch}^{-1}(cx)}{c^4x^4}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\int \left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)^4d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3791}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\int -\frac{a+b\text{csch}^{-1}(cx)}{c^2x^2}d\text{csch}^{-1}(cx)+\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{4c^3x^3}-\frac{b}{16c^4x^4}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(-\frac{3}{4}\int\frac{a+b\text{csch}^{-1}(cx)}{c^2x^2}d\text{csch}^{-1}(cx)+\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{4c^3x^3}-\frac{b}{16c^4x^4}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(-\frac{3}{4}\int-\left(\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)^2\right)d\text{csch}^{-1}(cx)+\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\right.}{4c^3x^3}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}\right.}{4c^3x^3}\right.\right.$$

$$\Big\downarrow \text{3791}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{2cx}+\frac{b}{4c^2x^2}\right.\right.\right.$$

$$\Big\downarrow \text{17}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c^4x^4}-\frac{1}{2}b\left(\frac{3}{4}\left(-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{2cx}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4b}+\frac{b}{4c^2x^2}\right)+\frac{\sqrt{\frac{1}{c^2x^2}+\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^2/x^5,x]
```

output
```
-(c^4*((a + b*ArcCsch[c*x])^2/(4*c^4*x^4) - (b*(-1/16*b/(c^4*x^4) + (Sqrt[
1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(4*c^3*x^3) + (3*(b/(4*c^2*x^2) - (
Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*c*x) + (a + b*ArcCsch[c*x])
^2/(4*b)))/4))/2))
```

3.23. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx$

3.23.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.23.4 Maple [F]

$$\int \frac{(a + b \operatorname{arccsch}(cx))^2}{x^5} dx$$

input
```
int((a+b*arccsch(c*x))^2/x^5,x)
```

output
```
int((a+b*arccsch(c*x))^2/x^5,x)
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.53

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx$$
$$= \frac{3\,b^2c^2x^2 + \left(3\,b^2c^4x^4 - 8\,b^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 8\,a^2 - b^2 + 2\left(3\,abc^4x^4 - 8\,ab - \left(3\,b^2c^3x^3 - 2\,b^2cx\right)\sqrt{\frac{c^2}{}}\right.}{32\,x^4}$$

input
```
integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 1)/(c*x))^2 - 8*a^2 - b^2 + 2*(3*a*b*c^4*x^4 - 8*a*b - (3*b^2
*c^3*x^3 - 2*b^2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/x^4
```

3.23.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(a+b\operatorname{acsch}\left(cx\right)\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*acsch(c*x))**2/x**5,x)
```

output
```
Integral((a + b*acsch(c*x))**2/x**5, x)
```

3.23.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx = \int \frac{\left(b\operatorname{arcsch}\left(cx\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1
/(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt
(1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) +
 1) + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2
/x^4 + 4*integrate(-1/2*(2*c^2*x^2*log(c)^2 + 2*(c^2*x^2 + 1)*log(x)^2 + 2
*log(c)^2 + 4*(c^2*x^2*log(c) + log(c))*log(x) - (4*c^2*x^2*log(c) + 4*(c^
2*x^2 + 1)*log(x) + (c^2*x^2*(4*log(c) - 1) + 4*(c^2*x^2 + 1)*log(x) + 4*l
og(c))*sqrt(c^2*x^2 + 1) + 4*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(c^2*x
^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(
c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c^2*x^7 + x^5)*sqrt(c^2*x
^2 + 1)), x)) - 1/4*a^2/x^4
```

3.23.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx=\int \frac{\left(b\,\text{arcsch}\left(cx\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2/x^5, x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx=\int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^2/x^5,x)
```

output
```
int((a + b*asinh(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 195

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = \frac{b^3\sqrt{1+\frac{1}{c^2x^2}}x}{4c^3}+\frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{b^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{c^4} + \frac{b^3\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2c^4}$$

output
```
1/4*b^2*x^2*(a+b*arccsch(c*x))/c^2-1/2*b*(a+b*arccsch(c*x))^2/c^4+1/4*x^4*
(a+b*arccsch(c*x))^3+b^2*(a+b*arccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2
))^2)/c^4+1/2*b^3*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(
1+1/c^2/x^2)^(1/2)/c^3-1/2*b*x*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c^
3+1/4*b*x^3*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c
```

3.24.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.39

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$
$$= \frac{-2a^2bc\sqrt{1+\frac{1}{c^2x^2}}x+b^3c\sqrt{1+\frac{1}{c^2x^2}}x+ab^2c^2x^2+a^2bc^3\sqrt{1+\frac{1}{c^2x^2}}x^3+a^3c^4x^4+b^2\left(3ac^4x^4+b\left(2-2c\sqrt{1\right.\right.}{}$$

input
```
Integrate[x^3*(a + b*ArcCsch[c*x])^3,x]
```

output
```
(-2*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x + a*b^
2*c^2*x^2 + a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c
^4*x^4 + b*(2 - 2*c*Sqrt[1 + 1/(c^2*x^2)]*x + c^3*Sqrt[1 + 1/(c^2*x^2)]*x^
3))*ArcCsch[c*x]^2 + b^3*c^4*x^4*ArcCsch[c*x]^3 + b*ArcCsch[c*x]*(c*x*(b^2
*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2)) + 4*b^2
*Log[1 - E^(-2*ArcCsch[c*x])]) + 4*a*b^2*Log[1/(c*x)] - 2*b^3*PolyLog[2, E
^(-2*ArcCsch[c*x])])/(4*c^4)
```

3.24.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {6840, 5975, 3042, 4674, 25, 3042, 25, 4254, 24, 4672, 26, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$
$$\Big\downarrow \ 6840$$
$$-\frac{\int c^5\sqrt{1+\frac{1}{c^2x^2}}x^5\left(a+b\text{csch}^{-1}(cx)\right)^3 d\text{csch}^{-1}(cx)}{c^4}$$
$$\Big\downarrow \ 5975$$
$$-\frac{\frac{3}{4}b\int c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^2 d\text{csch}^{-1}(cx)-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3}{c^4}$$

$\downarrow$ 3042

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^4d\text{csch}^{-1}(cx)}{c^4}$$

$\downarrow$ 4674

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\int-c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{1}{3}b^2\int-c^2x^2d\text{csch}^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{3}c^3x^3\sqrt{\ldots}}{c^4}$$

$\downarrow$ 25

$$-\frac{\frac{3}{4}b\left(-\frac{2}{3}\int c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{3}b^2\int c^2x^2d\text{csch}^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{3}c^3x^3\sqrt{\ldots}}{c^4}$$

$\downarrow$ 3042

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(-\frac{2}{3}\int-\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{3}b^2\int-\csc\left(i\text{csch}\ldots\right.\right.}{c^4}$$

$\downarrow$ 25

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{1}{3}b^2\int\csc\left(i\text{csch}^{-1}(cx\ldots\right.\right.}{c^4}$$

$\downarrow$ 4254

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{1}{3}ib^2\int1d\left(-ic\sqrt{1+\ldots}\right.\right.}{c^4}$$

$\downarrow$ 24

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{1}{3}bc^2x^2\left(a+b\text{csch}^{-1}(\ldots\right.\right.}{c^4}$$

$\downarrow$ 4672

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\int-ic\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)\ldots\right.\right.}{c^4}$$

$\downarrow$ 26

3.24. $\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

$$-\frac{\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2b\int c\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)\right)-\frac{1}{3}bc^2x^2\left(a+b\text{csch}\right.}{c^4}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2b\int -i\left(a+b\text{csch}^{-1}(cx)\right)\tan\left(i\text{csch}^{-1}(c\right.\right.\right.}{c^4}$$

$$\Big\downarrow \ 26$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\int\left(a+b\text{csch}^{-1}(cx)\right)\tan\left(i\text{csch}^{-1}(cx\right.\right.\right.}{c^4}$$

$$\Big\downarrow \ 4199$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(2i\int-\frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)}{1-e^{2\text{csch}^{-1}(cx)}}dcs\right.\right.\right.}{}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\int\frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)}{1-e^{2\text{csch}^{-1}(cx)}}dcs\right.\right.\right.}{}$$

$$\Big\downarrow \ 2620$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\left(\frac{1}{2}b\int\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)dcs\right.\right.\right.\right.}{}$$

$$\Big\downarrow \ 2715$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(cx)}\log\left(1-e^{2c}\right.\right.\right.\right.\right.}{}$$

$$\Big\downarrow \ 2838$$

$$-\frac{-\frac{1}{4}c^4x^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{4}b\left(\frac{2}{3}\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+\right.\right.\right.\right.\right.}{}$$

3.24. $\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

input
```
Int[x^3*(a + b*ArcCsch[c*x])^3,x]
```

output
```
-((-1/4*(c^4*x^4*(a + b*ArcCsch[c*x])^3) + (3*b*(-1/3*(b^2*c*Sqrt[1 + 1/(c
^2*x^2)]*x) - (b*c^2*x^2*(a + b*ArcCsch[c*x]))/3 - (c^3*Sqrt[1 + 1/(c^2*x^
2)]*x^3*(a + b*ArcCsch[c*x])^2)/3 + (2*(c*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*A
rcCsch[c*x])^2 + (2*I)*b*(((-1/2*I)*(a + b*ArcCsch[c*x])^2)/b - (2*I)*(-1/
2*((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])]) - (b*PolyLog[2, E^(2*
ArcCsch[c*x])])/4))))/3))/4)/c^4)
```

3.24.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.24. $\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$

3.24.4 Maple [F]

$$\int x^3 (a + b \operatorname{arccsch}(cx))^3 \, dx$$

input
```
int(x^3*(a+b*arccsch(c*x))^3,x)
```

output
```
int(x^3*(a+b*arccsch(c*x))^3,x)
```

3.24.5 Fricas [F]

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int (b \operatorname{arcsch}(cx) + a)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arccsch(c*x)^3 + 3*a*b^2*x^3*arccsch(c*x)^2 + 3*a^2*b*x^3
*arccsch(c*x) + a^3*x^3, x)
```

3.24.6 Sympy [F]

$$\int x^3 \left(a + b\operatorname{csch}^{-1}(cx)\right)^3 \, dx = \int x^3 (a + b \operatorname{acsch}(cx))^3 \, dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))**3, x)
```

3.24.7 Maxima [F]

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int (b\,\text{arcsch}\,(cx)+a)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4*log(sqrt(c^2*x^2 + 1) + 1)^3 - 12*b^3*c^2*integrate(1/4*x^5*lo
g(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log
(c)^2 + 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 1/4*a^3
*x^4 - 12*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x
^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2
*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12
*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
 12*b^3*c^2*integrate(1/4*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^
2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integrate(1/4*x^5*log(x
)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c
^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^
2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1
), x)*log(c) + 24*a*b^2*c^2*integrate(1/4*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^
2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*c^2*integra
te(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 4*b^3*c^2*integrate(1/4*sqrt(c^2*x^
2 + 1)*x^5*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2...
```

3.24.8 Giac [F]

$$\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int (b\,\text{arcsch}\,(cx)+a)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3*x^3, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int x^3 \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^3 dx$$

input
```
int(x^3*(a + b*asinh(1/(c*x)))^3,x)
```

output
```
int(x^3*(a + b*asinh(1/(c*x)))^3, x)
```

3.25 $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

3.25.1 Optimal result

Integrand size = 14, antiderivative size = 194

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2\operatorname{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c^3}+\frac{b^3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^3}-\frac{b^2\left(a+b\text{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c^3}+\frac{b^2\left(a+b\text{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c^3}+\frac{b^3\operatorname{PolyLog}\left(3,-e^{\text{csch}^{-1}(cx)}\right)}{c^3}-\frac{b^3\operatorname{PolyLog}\left(3,e^{\text{csch}^{-1}(cx)}\right)}{c^3}$$

output
```
b^2*x*(a+b*arccsch(c*x))/c^2+1/3*x^3*(a+b*arccsch(c*x))^3-b*(a+b*arccsch(c
*x))^2*arctanh(1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1+1/c^2/x^2)^(1
/2))/c^3-b^2*(a+b*arccsch(c*x))*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c^3+
b^2*(a+b*arccsch(c*x))*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+b^3*polylo
g(3,-1/c/x-(1+1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,1/c/x+(1+1/c^2/x^2)^(1/2
))/c^3+1/2*b*x^2*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c
```

3.25.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 548 vs. 2(194) = 388.

Time = 7.56 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.82

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$$

$$=\frac{a^3x^3}{3}+\frac{a^2bx^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c}+a^2bx^3\text{csch}^{-1}(cx)-\frac{a^2b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{2c^3}$$

$$+\frac{ab^2\left(8\,\text{PolyLog}\left(2,-e^{-\text{csch}^{-1}(cx)}\right)+2c^3x^3\left(-2+4\text{csch}^{-1}(cx)^2+2\cosh\left(2\text{csch}^{-1}(cx)\right)-\frac{3\text{csch}^{-1}(cx)\log(\ldots}{cx\ldots}\right.\right.}{\ldots}$$

$$+\frac{b^3\left(24\text{csch}^{-1}(cx)\coth\left(\frac{1}{2}\text{csch}^{-1}(cx)\right)-4\text{csch}^{-1}(cx)^3\coth\left(\frac{1}{2}\text{csch}^{-1}(cx)\right)+6\text{csch}^{-1}(cx)^2\text{csch}^2\left(\frac{1}{2}\text{csch}^{-1}(\ldots\right.\right.}{\ldots}$$

input
```
Integrate[x^2*(a + b*ArcCsch[c*x])^3,x]
```

output
```
(a^3*x^3)/3 + (a^2*b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + a^2*b*x^3*
ArcCsch[c*x] - (a^2*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(2*c^3)
+ (a*b^2*(8*PolyLog[2, -E^(-ArcCsch[c*x])] + 2*c^3*x^3*(-2 + 4*ArcCsch[c*x
]^2 + 2*Cosh[2*ArcCsch[c*x]] - (3*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])])
/(c*x) + (3*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*x])])/(c*x) - (4*PolyLog[2,
 E^(-ArcCsch[c*x])])/(c^3*x^3) + 2*ArcCsch[c*x]*Sinh[2*ArcCsch[c*x]] + Arc
Csch[c*x]*Log[1 - E^(-ArcCsch[c*x])]*Sinh[3*ArcCsch[c*x]] - ArcCsch[c*x]*L
og[1 + E^(-ArcCsch[c*x])]*Sinh[3*ArcCsch[c*x]])))/(8*c^3) + (b^3*(24*ArcCs
ch[c*x]*Coth[ArcCsch[c*x]/2] - 4*ArcCsch[c*x]^3*Coth[ArcCsch[c*x]/2] + 6*A
rcCsch[c*x]^2*Csch[ArcCsch[c*x]/2]^2 + (ArcCsch[c*x]^3*Csch[ArcCsch[c*x]/2
]^4)/(c*x) + 24*ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] - 24*ArcCsch[c*x
]^2*Log[1 + E^(-ArcCsch[c*x])] - 48*Log[Tanh[ArcCsch[c*x]/2]] + 48*ArcCsch
[c*x]*PolyLog[2, -E^(-ArcCsch[c*x])] - 48*ArcCsch[c*x]*PolyLog[2, E^(-ArcC
sch[c*x])] + 48*PolyLog[3, -E^(-ArcCsch[c*x])] - 48*PolyLog[3, E^(-ArcCsch
[c*x])] + 6*ArcCsch[c*x]^2*Sech[ArcCsch[c*x]/2]^2 + 16*c^3*x^3*ArcCsch[c*x
]^3*Sinh[ArcCsch[c*x]/2]^4 - 24*ArcCsch[c*x]*Tanh[ArcCsch[c*x]/2] + 4*ArcC
sch[c*x]^3*Tanh[ArcCsch[c*x]/2]))/(48*c^3)
```

3.25.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6840, 5975, 3042, 26, 4674, 26, 3042, 26, 4257, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^3 dx$$

$$\Big\downarrow \text{ 6840}$$

$$-\frac{\int c^4 \sqrt{1+\frac{1}{c^2x^2}}x^4 \left(a+b\text{csch}^{-1}(cx)\right)^3 d\text{csch}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{ 5975}$$

$$-\frac{b\int c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 d\text{csch}^{-1}(cx)-\frac{1}{3}c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3}{c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{3}c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3+b\int -i\left(a+b\text{csch}^{-1}(cx)\right)^2 \csc\left(i\text{csch}^{-1}(cx)\right)^3 d\text{csch}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-\frac{1}{3}c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3-ib\int \left(a+b\text{csch}^{-1}(cx)\right)^2 \csc\left(i\text{csch}^{-1}(cx)\right)^3 d\text{csch}^{-1}(cx)}{c^3}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{-\frac{1}{3}c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\int -icx\left(a+b\text{csch}^{-1}(cx)\right)^2 d\text{csch}^{-1}(cx)+b^2\left(-\int -icxd\text{csch}^{-1}(cx)\right)-\frac{1}{2}ic^2x^2}{c^3}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{-\frac{1}{3}c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(-\frac{1}{2}i\int cx\left(a+b\text{csch}^{-1}(cx)\right)^2 d\text{csch}^{-1}(cx)+ib^2\int cxd\text{csch}^{-1}(cx)-\frac{1}{2}ic^2x^2\sqrt{\frac{1}{c^2x^2}}}{c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{1}{3}c^3x^3 \left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(-\frac{1}{2}i\int i\left(a+b\text{csch}^{-1}(cx)\right)^2 \csc\left(i\text{csch}^{-1}(cx)\right) d\text{csch}^{-1}(cx)+ib^2\int i\csc\left(i\text{csch}^{-1}(\right.}{c^3}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)+b^2\big(-\int\csc\left(i\text{csch}^{-1}(cx\right.}{c^3}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)-\frac{1}{2}ic^2x^2\sqrt{\frac{1}{c^2x^2}+1}(a+}{c^3}$$

$$\Big\downarrow \text{4670}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\Big(2ib\int\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-2ib\int\left(a+b\text{csch}\right.}{}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\Big(-2ib\Big(b\int\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\Big(-2ib\Big(b\int e^{-\text{csch}^{-1}(cx)}\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)de^{\text{csch}^{-1}(cx)}-\text{PolyLog}\left(2,-\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{-\frac{1}{3}c^3x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-ib\Big(\frac{1}{2}\Big(2i\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\Big(b\,\text{PolyLog}\left(3,-e^{\text{csch}^{-1}(cx)}\right)}{}$$

input
```
Int[x^2*(a + b*ArcCsch[c*x])^3,x]
```

output
```
-((-1/3*(c^3*x^3*(a + b*ArcCsch[c*x])^3) - I*b*((-I)*b*c*x*(a + b*ArcCsch[
c*x]) - (I/2)*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcCsch[c*x])^2 - I*b^2
*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]] + ((2*I)*(a + b*ArcCsch[c*x])^2*ArcTanh[E^
ArcCsch[c*x]] - (2*I)*b*(-((a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]
]) + b*PolyLog[3, -E^ArcCsch[c*x]]) + (2*I)*b*(-((a + b*ArcCsch[c*x])*Poly
Log[2, E^ArcCsch[c*x]]) + b*PolyLog[3, E^ArcCsch[c*x]]))/2))/c^3)
```

3.25. $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

3.25.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.25.4 Maple [F]

$$\int x^2(a+b\,\mathrm{arccsch}\,(cx))^3\,dx$$

input
```
int(x^2*(a+b*arccsch(c*x))^3,x)
```

output
```
int(x^2*(a+b*arccsch(c*x))^3,x)
```

3.25.5 Fricas [F]

$$\int x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)^3\,dx=\int (b\,\mathrm{arcsch}\,(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arccsch(c*x)^3 + 3*a*b^2*x^2*arccsch(c*x)^2 + 3*a^2*b*x^2
*arccsch(c*x) + a^3*x^2, x)
```

3.25.6 Sympy [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = \int x^2 (a+b\,\text{acsch}\,(cx))^3 \, dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))**3, x)
```

3.25.7 Maxima [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arcsch}\,(cx)+a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3*log(sqrt(c^2*x^2 + 1) + 1)^3 + 1/3*a^3*x^3 + 1/4*(4*x^3*arccsc
h(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt
(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b
- integrate(((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b^3*c^2*x^4
+ b^3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*((b^3*c^2*
log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + (3*(b^3*c^2
*log(c) - a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b^2)*x^2 + 3*(b^3*c^2*x^4 + b
^3*x^2)*log(x) + ((b^3*c^2*(3*log(c) + 1) - 3*a*b^2*c^2)*x^4 + 3*(b^3*log(
c) - a*b^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log
(sqrt(c^2*x^2 + 1) + 1)^2 + 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4
 + (b^3*log(c)^2 - 2*a*b^2*log(c))*x^2)*log(x) - 3*((b^3*c^2*log(c)^2 - 2*
a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(c))*x^2 + (b^3*c^2*x^4
 + b^3*x^2)*log(x)^2 + 2*((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) -
 a*b^2)*x^2)*log(x) + ((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*
log(c)^2 - 2*a*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*log(x)^2 + 2*((b^
3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x))*sqrt(c^2
*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + ((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*l
og(c)^2)*x^4 + (b^3*c^2*x^4 + b^3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*
log(c)^2)*x^2 + 3*((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)
*x^2)*log(x)^2 + 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*...
```

3.25.8 Giac [F]

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int (b\,\text{arcsch}\,(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3*x^2, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3 \,dx$$

input
```
int(x^2*(a + b*asinh(1/(c*x)))^3,x)
```

output
```
int(x^2*(a + b*asinh(1/(c*x)))^3, x)
```

3.26 $\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

3.26.1 Optimal result

Integrand size = 12, antiderivative size = 117

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{c^2}-\frac{3b^3\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2c^2}$$

output
```
3/2*b*(a+b*arccsch(c*x))^2/c^2+1/2*x^2*(a+b*arccsch(c*x))^3-3*b^2*(a+b*arc
csch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/c^2-3/2*b^3*polylog(2,(1/c/
x+(1+1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(
1/2)/c
```

3.26.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$$
$$=\frac{3b^2\left(ac^2x^2+b\left(-1+c\sqrt{1+\frac{1}{c^2x^2}}x\right)\right)\text{csch}^{-1}(cx)^2+b^3c^2x^2\text{csch}^{-1}(cx)^3+3b\text{csch}^{-1}(cx)\left(acx\left(2b\sqrt{1+\frac{1}{c^2x^2}}\right.\right.}{}$$

input
```
Integrate[x*(a + b*ArcCsch[c*x])^3,x]
```

output
```
(3*b^2*(a*c^2*x^2 + b*(-1 + c*Sqrt[1 + 1/(c^2*x^2)]*x))*ArcCsch[c*x]^2 + b
^3*c^2*x^2*ArcCsch[c*x]^3 + 3*b*ArcCsch[c*x]*(a*c*x*(2*b*Sqrt[1 + 1/(c^2*x
^2)] + a*c*x) - 2*b^2*Log[1 - E^(-2*ArcCsch[c*x])]) + a*(a*c*x*(3*b*Sqrt[1
 + 1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + 3*b^3*PolyLog[2, E^(-2*Ar
cCsch[c*x])])/(2*c^2)
```

3.26.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.083, Rules used = {6840, 5975, 3042, 25, 4672, 26, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$$
$$\Big\downarrow \text{6840}$$
$$-\frac{\int c^3\sqrt{1+\frac{1}{c^2x^2}}x^3\left(a+b\text{csch}^{-1}(cx)\right)^3d\text{csch}^{-1}(cx)}{c^2}$$
$$\Big\downarrow \text{5975}$$
$$-\frac{\frac{3}{2}b\int c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3}{c^2}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3}{2}b\int -\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c^2}$$

$\downarrow$ 25

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\int \left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)}{c^2}$$

$\downarrow$ 4672

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\int -ic\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)d\text{cs}}{c^2}$$

$\downarrow$ 26

$$-\frac{-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2b\int c\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)\right)-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}\right.}{c^2}$$

$\downarrow$ 3042

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2b\int -i\left(a+b\text{csch}^{-1}(cx)\right)\tan\left(i\text{csch}^{-1}(cx)\right.\right.}{c^2}$$

$\downarrow$ 26

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\int \left(a+b\text{csch}^{-1}(cx)\right)\tan\left(i\text{csch}^{-1}(cx)+\right.\right.}{c^2}$$

$\downarrow$ 4199

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(2i\int -\frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)}{1-e^{2\text{csch}^{-1}(cx)}}d\text{csch}\right.\right.}{c^2}$$

$\downarrow$ 25

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\int \frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)}{1-e^{2\text{csch}^{-1}(cx)}}d\text{csch}\right.\right.}{c^2}$$

$\downarrow$ 2620

3.26. $\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\left(\frac{1}{2}b\int\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\left(\frac{1}{4}b\int e^{-2\text{csch}^{-1}(cx)}\log\left(1-e^{2\text{csch}^{-}}\right.\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{-\frac{1}{2}c^2x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{2}b\left(cx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2+2ib\left(-2i\left(-\frac{1}{2}\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)(a+bc\right.\right.\right.}{c^2}$$

input
```
Int[x*(a + b*ArcCsch[c*x])^3,x]
```

output
```
-((-1/2*(c^2*x^2*(a + b*ArcCsch[c*x])^3) - (3*b*(c*Sqrt[1 + 1/(c^2*x^2)]*x
*(a + b*ArcCsch[c*x])^2 + (2*I)*b*(((-1/2*I)*(a + b*ArcCsch[c*x])^2)/b - (
2*I)*(-1/2*((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])]) - (b*PolyLog
[2, E^(2*ArcCsch[c*x])])/4))))/2)/c^2)
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.26. $\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

3.26.4 Maple [F]

$$\int x(a+b\,\mathrm{arccsch}\,(cx))^3\,dx$$

input
```
int(x*(a+b*arccsch(c*x))^3,x)
```

output
```
int(x*(a+b*arccsch(c*x))^3,x)
```

3.26.5 Fricas [F]

$$\int x\left(a+b\mathrm{csch}^{-1}(cx)\right)^3\,dx=\int(b\,\mathrm{arcsch}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arccsch(c*x)^3 + 3*a*b^2*x*arccsch(c*x)^2 + 3*a^2*b*x*arccs
ch(c*x) + a^3*x, x)
```

3.26.6 Sympy [F]

$$\int x\left(a+b\mathrm{csch}^{-1}(cx)\right)^3\,dx=\int x(a+b\,\mathrm{acsch}\,(cx))^3\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))**3,x)
```

output
```
Integral(x*(a + b*acsch(c*x))**3, x)
```

3.26.7 Maxima [F]

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=\int \left(b\,\text{arcsch}\,(cx)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arccsch(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsch(c*x) + x*sqr
t(1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c +
log(x)/c^2)*a*b^2 - 1/4*(24*c^2*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1
)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*c^2*integra
te(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 2*x^2*log(sqrt(c^2*x^2 + 1) + 1)^
3 + 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)
*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 48*c^2*integrate(
1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
+ 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integr
ate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 +
 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integra
te(1/2*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x)*log(c) - 48*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1)
 + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*lo
g(c) + 24*c^2*integrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2
 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 8*c^2*integr
ate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^
2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^
3*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*...
```

3.26.8 Giac [F]

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=\int \left(b\,\text{arcsch}\,(cx)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3*x, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int x\,{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^3\,dx$$

input
```
int(x*(a + b*asinh(1/(c*x)))^3,x)
```

output
```
int(x*(a + b*asinh(1/(c*x)))^3, x)
```

3.27 $\int (a+b\text{csch}^{-1}(cx))^3\, dx$

3.27.1 Optimal result

Integrand size = 10, antiderivative size = 120

$$\int (a+b\text{csch}^{-1}(cx))^3\, dx = x(a+b\text{csch}^{-1}(cx))^3 + \frac{6b(a+b\text{csch}^{-1}(cx))^2 \operatorname{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{6b^2(a+b\text{csch}^{-1}(cx))\,\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c} - \frac{6b^2(a+b\text{csch}^{-1}(cx))\,\text{PolyLog}\left(2,e^{\text{csch}^{-1}(cx)}\right)}{c} - \frac{6b^3\,\text{PolyLog}\left(3,-e^{\text{csch}^{-1}(cx)}\right)}{c} + \frac{6b^3\,\text{PolyLog}\left(3,e^{\text{csch}^{-1}(cx)}\right)}{c}$$

output
```
x*(a+b*arccsch(c*x))^3+6*b*(a+b*arccsch(c*x))^2*arctanh(1/c/x+(1+1/c^2/x^2
)^(1/2))/c+6*b^2*(a+b*arccsch(c*x))*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/
c-6*b^2*(a+b*arccsch(c*x))*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c-6*b^3*po
lylog(3,-1/c/x-(1+1/c^2/x^2)^(1/2))/c+6*b^3*polylog(3,1/c/x+(1+1/c^2/x^2)^
(1/2))/c
```

3.27.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 246 vs. 2(120) = 240.

Time = 0.42 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.05

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = a^3x+3a^2bx\text{csch}^{-1}(cx)+\frac{3a^2b\log\left(cx\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{c} + \frac{3ab^2\left(\text{csch}^{-1}(cx)\left(cx\text{csch}^{-1}(cx)-2\log\left(1-e^{-\text{csch}^{-1}(cx)}\right)+2\log\left(1+e^{-\text{csch}^{-1}(cx)}\right)\right)-2\,\text{PolyLog}\left(2,-\right.}{c} + \frac{b^3\left(cx\text{csch}^{-1}(cx)^3-3\text{csch}^{-1}(cx)^2\log\left(1-e^{-\text{csch}^{-1}(cx)}\right)+3\text{csch}^{-1}(cx)^2\log\left(1+e^{-\text{csch}^{-1}(cx)}\right)-6\text{csch}^{-1}\right.}{}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcCsch[c*x] + (3*a^2*b*Log[c*x*(1 + Sqrt[(1 + c^2*x^2)/
(c^2*x^2)])])/c + (3*a*b^2*(ArcCsch[c*x]*(c*x*ArcCsch[c*x] - 2*Log[1 - E^(
-ArcCsch[c*x])] + 2*Log[1 + E^(-ArcCsch[c*x])]) - 2*PolyLog[2, -E^(-ArcCsc
h[c*x])] + 2*PolyLog[2, E^(-ArcCsch[c*x])]))/c + (b^3*(c*x*ArcCsch[c*x]^3
- 3*ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] + 3*ArcCsch[c*x]^2*Log[1 + E
^(-ArcCsch[c*x])] - 6*ArcCsch[c*x]*PolyLog[2, -E^(-ArcCsch[c*x])] + 6*ArcC
sch[c*x]*PolyLog[2, E^(-ArcCsch[c*x])] - 6*PolyLog[3, -E^(-ArcCsch[c*x])]
+ 6*PolyLog[3, E^(-ArcCsch[c*x])]))/c
```

3.27.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {6834, 5975, 3042, 26, 4670, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$

$$\Big\downarrow \; 6834$$

$$-\frac{\int c^2\sqrt{1+\frac{1}{c^2x^2}}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3d\text{csch}^{-1}(cx)}{c}$$

$\Big\downarrow$ 5975

$$-\frac{3b\int cx\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-cx\left(a+b\text{csch}^{-1}(cx)\right)^3}{c}$$

$\Big\downarrow$ 3042

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^3+3b\int i\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)}{c}$$

$\Big\downarrow$ 26

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^3+3ib\int\left(a+b\text{csch}^{-1}(cx)\right)^2\csc\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)}{c}$$

$\Big\downarrow$ 4670

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^3+3ib\left(2ib\int\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-2ib\int\left(a+b\text{csch}^{-1}(cx\right.\right.}{c}$$

$\Big\downarrow$ 3011

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^3+3ib\left(-2ib\left(b\int\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)\left(a+\right.\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^3+3ib\left(-2ib\left(b\int e^{-\text{csch}^{-1}(cx)}\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)de^{\text{csch}^{-1}(cx)}-\text{PolyLog}\left(2,-e^{\text{csch}^{-}}\right.\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{-cx\left(a+b\text{csch}^{-1}(cx)\right)^3+3ib\left(2i\text{arctanh}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(b\,\text{PolyLog}\left(3,-e^{\text{csch}^{-1}(cx)}\right)-\text{P}\right.\right.}{}$$

input
```
Int[(a + b*ArcCsch[c*x])^3,x]
```

output
```
-((-(c*x*(a + b*ArcCsch[c*x])^3) + (3*I)*b*((2*I)*(a + b*ArcCsch[c*x])^2*A
rcTanh[E^ArcCsch[c*x]] - (2*I)*b*(-((a + b*ArcCsch[c*x])*PolyLog[2, -E^Arc
Csch[c*x]]) + b*PolyLog[3, -E^ArcCsch[c*x]]) + (2*I)*b*(-((a + b*ArcCsch[c
*x])*PolyLog[2, E^ArcCsch[c*x]]) + b*PolyLog[3, E^ArcCsch[c*x]])))/c)
```

3.27. $\int\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

3.27.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4670
```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x
)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e
+ f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 5975
```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /;
FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 6834
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-c^(-1)   S
ubst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
 b, c, n}, x] && IGtQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.27.4 Maple [F]

$$\int (a + b\ \mathrm{arccsch}\,(cx))^3\, dx$$

input
```
int((a+b*arccsch(c*x))^3,x)
```

output
```
int((a+b*arccsch(c*x))^3,x)
```

3.27.5 Fricas [F]

$$\int \left(a + b\mathrm{csch}^{-1}(cx)\right)^3\, dx = \int \left(b\,\mathrm{arcsch}\,(cx) + a\right)^3 dx$$

input
```
integrate((a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3, x)
```

3.27.6 Sympy [F]

$$\int \left(a + b\mathrm{csch}^{-1}(cx)\right)^3\, dx = \int \left(a + b\,\mathrm{acsch}\,(cx)\right)^3\, dx$$

input
```
integrate((a+b*acsch(c*x))**3,x)
```

output
```
Integral((a + b*acsch(c*x))**3, x)
```

3.27.7 Maxima [F]

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^3 dx = \int (b\,\text{arcsch}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

output
```
b^3*x*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*x + 3/2*(2*c*x*arccsch(c*x) + log
(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*a^2*b/c - in
tegrate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 +
(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^
2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)
*(b^3*log(c) - a*b^2 + (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 + (b^3*c^2*x
^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^2 - 2*a*b^
2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x) - 3*(b^3*lo
g(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b
^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b
^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 -
 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) -
a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(s
qrt(c^2*x^2 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 +
b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log
(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2
 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*s
qrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

3.27.8 Giac [F]

$$\int \left(a+b\text{csch}^{-1}(cx)\right)^3 dx = \int (b\,\text{arcsch}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=\int \left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^3,x)
```

output
```
int((a + b*asinh(1/(c*x)))^3, x)
```

3.28 $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx$

3.28.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx = \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right) + \frac{3}{2}b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{4}b^3\,\text{PolyLog}\left(4,e^{2\text{csch}^{-1}(cx)}\right)$$

output
```
1/4*(a+b*arccsch(c*x))^4/b-(a+b*arccsch(c*x))^3*ln(1-(1/c/x+(1+1/c^2/x^2)^
(1/2))^2)-3/2*b*(a+b*arccsch(c*x))^2*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))
^2)+3/2*b^2*(a+b*arccsch(c*x))*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)-3/
4*b^3*polylog(4,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.65

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^3}{x}\,dx = \frac{1}{4}\Big(6a^2b\operatorname{csch}^{-1}(cx)^2+4ab^2\operatorname{csch}^{-1}(cx)^3+b^3\operatorname{csch}^{-1}(cx)^4 - 12a^2b\operatorname{csch}^{-1}(cx)\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - 12ab^2\operatorname{csch}^{-1}(cx)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right) - 4b^3\operatorname{csch}^{-1}(cx)^3\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)+4a^3\log(cx) - 6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right) + 6b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right) - 3b^3\operatorname{PolyLog}\left(4,e^{2\operatorname{csch}^{-1}(cx)}\right)\Big)$$

input
```
Integrate[(a + b*ArcCsch[c*x])^3/x,x]
```

output
```
(6*a^2*b*ArcCsch[c*x]^2 + 4*a*b^2*ArcCsch[c*x]^3 + b^3*ArcCsch[c*x]^4 - 12
*a^2*b*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] - 12*a*b^2*ArcCsch[c*x]^2*
Log[1 - E^(2*ArcCsch[c*x])] - 4*b^3*ArcCsch[c*x]^3*Log[1 - E^(2*ArcCsch[c*
x])] + 4*a^3*Log[c*x] - 6*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch
[c*x])] + 6*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])] - 3*b^
3*PolyLog[4, E^(2*ArcCsch[c*x])])/4
```

3.28.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6840, 3042, 26, 4199, 25, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{csch}^{-1}(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{6840}$$

$$-\int c\sqrt{1+\frac{1}{c^2x^2}}x\left(a+b\text{csch}^{-1}(cx)\right)^3d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\int -i\left(a+b\text{csch}^{-1}(cx)\right)^3\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$i\int \left(a+b\text{csch}^{-1}(cx)\right)^3\tan\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{4199}$$

$$i\left(2i\int -\frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)^3}{1-e^{2\text{csch}^{-1}(cx)}}d\text{csch}^{-1}(cx)-\frac{i\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b}\right)$$

$$\Big\downarrow \text{25}$$

$$i\left(-2i\int \frac{e^{2\text{csch}^{-1}(cx)}\left(a+b\text{csch}^{-1}(cx)\right)^3}{1-e^{2\text{csch}^{-1}(cx)}}d\text{csch}^{-1}(cx)-\frac{i\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b}\right)$$

$$\Big\downarrow \text{2620}$$

$$i\left(-2i\left(\frac{3}{2}b\int \left(a+b\text{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-\frac{1}{2}\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^3\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$i\left(-2i\left(\frac{3}{2}b\left(b\int \left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)-\frac{1}{2}\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$i\left(-2i\left(\frac{3}{2}b\left(b\left(\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{2}b\int \text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)d\text{csch}^{-1}(cx)\right)-\frac{1}{2}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$i\left(-2i\left(\frac{3}{2}b\left(b\left(\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)-\frac{1}{4}b\int e^{-2\text{csch}^{-1}(cx)}\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)de^{2\text{csch}}\right.\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

3.28. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}dx$

$$i\left(-2i\left(\frac{3}{2}b\left(b\left(\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{1}{4}b\operatorname{PolyLog}\left(4,e^{2\operatorname{csch}^{-1}(cx)}\right)\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,e^{2\operatorname{cs}}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^3/x,x]
```

output
```
I*(((-1/4*I)*(a + b*ArcCsch[c*x])^4)/b - (2*I)*(-1/2*((a + b*ArcCsch[c*x])
^3*Log[1 - E^(2*ArcCsch[c*x])]) + (3*b*(-1/2*((a + b*ArcCsch[c*x])^2*PolyL
og[2, E^(2*ArcCsch[c*x])]) + b*(((a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcC
sch[c*x])])/2 - (b*PolyLog[4, E^(2*ArcCsch[c*x])])/4)))/2))
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.28.4 Maple [F]

$$\int \frac{(a + b\ \mathrm{arccsch}\,(cx))^3}{x}dx$$

input
```
int((a+b*arccsch(c*x))^3/x,x)
```

output
```
int((a+b*arccsch(c*x))^3/x,x)
```

3.28.5 Fricas [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(b\operatorname{arcsch}\left(cx\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*
x) + a^3)/x, x)
```

3.28.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(a+b\operatorname{acsch}\left(cx\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*acsch(c*x))**3/x,x)
```

output
```
Integral((a + b*acsch(c*x))**3/x, x)
```

3.28.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx=\int \frac{\left(b\operatorname{arcsch}\left(cx\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x,x, algorithm="maxima")
```

output
```
b^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*log(x) - integrate((b^3*log(
c)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x^2 + b^3)*log(x)^3 +
(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2*log(c))*x^2 + 3*(b^
3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log
(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x
) + sqrt(c^2*x^2 + 1)*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x
^2 + (2*b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*
log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c)
 + a^2*b*c^2)*x^2)*log(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^
3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)
*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(
x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*
c^2*log(c) + a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c)
 - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*lo
g(sqrt(c^2*x^2 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(
c) + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)
^2 + 3*a^2*b*c^2*log(c))*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a
*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*
c^2*log(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b*c^2)*x^2)*log(x))*sqrt(c^2*x^2 +
 1))/(c^2*x^3 + (c^2*x^3 + x)*sqrt(c^2*x^2 + 1) + x), x)
```

3.28.8 Giac [F]

$$\int \frac{\left(a + b\text{csch}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(b\,\text{arcsch}\left(cx\right) + a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3/x, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\mathrm{csch}^{-1}(cx)\right)^3}{x}\,dx = \int \frac{\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^3/x,x)
```

output
```
int((a + b*asinh(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx$

3.29.1 Optimal result

Integrand size = 14, antiderivative size = 78

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx = 6b^3c\sqrt{1+\frac{1}{c^2x^2}}-\frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}+3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}$$

output
```
-6*b^2*(a+b*arccsch(c*x))/x-(a+b*arccsch(c*x))^3/x+6*b^3*c*(1+1/c^2/x^2)^(
1/2)+3*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{a^3+6ab^2-3a^2bc\sqrt{1+\frac{1}{c^2x^2}}x-6b^3c\sqrt{1+\frac{1}{c^2x^2}}x+3b\left(a^2+2b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{csch}^{-1}(cx)+3b^2\Big(}{x}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^3/x^2,x]
```

output
```
-((a^3 + 6*a*b^2 - 3*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 + 1/
(c^2*x^2)]*x + 3*b*(a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch
[c*x] + 3*b^2*(a - b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x]^2 + b^3*ArcCs
ch[c*x]^3)/x)
```

3.29.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6840, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}dx$$

$$\Big\downarrow \text{6840}$$

$$-c\int \sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^3d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-c\int \left(a+b\text{csch}^{-1}(cx)\right)^3\sin\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{3777}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}-3ib\int -\frac{i\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{26}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}-3b\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{cx}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}-3b\int -i\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{26}$$

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\int \left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx)\right)$$

$\downarrow$ 3777

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\int \sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}(cx\right.\right.$$

$\downarrow$ 3042

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\int \left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)\right.\right.$$

$\downarrow$ 3777

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-ib\int -\frac{i}{cx}d\text{csch}^{-1}(cx)\right)\right.\right.$$

$\downarrow$ 26

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-b\int \frac{1}{cx}d\text{csch}^{-1}(cx)\right)\right)\right.$$

$\downarrow$ 3042

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-b\int -i\sin\left(i\text{csch}^{-1}(cx\right.\right.\right.\right.$$

$\downarrow$ 26

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}+ib\int \sin\left(i\text{csch}^{-1}(cx)\right)d\right.\right.\right.$$

$\downarrow$ 3118

$$-c\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}+3ib\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-b\sqrt{\frac{1}{c^2x^2}+1}\right)\right)\right)$$

input
```
Int[(a + b*ArcCsch[c*x])^3/x^2,x]
```

3.29. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx$

output
```
-(c*((a + b*ArcCsch[c*x])^3/(c*x) + (3*I)*b*(I*Sqrt[1 + 1/(c^2*x^2)]*(a +
b*ArcCsch[c*x])^2 - (2*I)*b*(-(b*Sqrt[1 + 1/(c^2*x^2)]) + (a + b*ArcCsch[c
*x])/(c*x)))))
```

3.29.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.29.4 Maple [F]

$$\int \frac{(a+b\,\text{arccsch}\,(cx))^3}{x^2}dx$$

input
```
int((a+b*arccsch(c*x))^3/x^2,x)
```

output
```
int((a+b*arccsch(c*x))^3/x^2,x)
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.85

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 3\,(a^2b+2\,b^3)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + a^3 + 6\,ab^2 - 3\left(b^3cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - ab^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{cx}\right.}{x}$$

input
```
integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
-(b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2*
b^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c*x))^2 - 3*(2*a*b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - a^2*b - 2*b^3
)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

3.29.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))^3}{x^2}\,dx$$

input
```
integrate((a+b*acsch(c*x))**3/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))**3/x**2, x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.85

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx$$
$$= -\frac{b^3 \operatorname{arcsch}(cx)^3}{x} + 3\left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsch}(cx)}{x}\right)a^2b$$
$$+ 6\left(c\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx) - \frac{1}{x}\right)ab^2$$
$$+ 3\left(c\sqrt{\frac{1}{c^2x^2}+1}\operatorname{arcsch}(cx)^2 + 2\,c\sqrt{\frac{1}{c^2x^2}+1} - \frac{2\operatorname{arcsch}(cx)}{x}\right)b^3$$
$$- \frac{3\,ab^2 \operatorname{arcsch}(cx)^2}{x} - \frac{a^3}{x}$$

input
```
integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-b^3*arccsch(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a^2*b
 + 6*(c*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2
*x^2) + 1)*arccsch(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) + 1) - 2*arccsch(c*x)/x)*
b^3 - 3*a*b^2*arccsch(c*x)^2/x - a^3/x
```

3.29.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3/x^2, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^3/x^2,x)
```

output
```
int((a + b*asinh(1/(c*x)))^3/x^2, x)
```

3.30 $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 123

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx=\frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x}-\frac{3}{8}b^3c^2\text{csch}^{-1}(cx)-\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2}+\frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x}-\frac{1}{4}c^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2}$$

output
```
-3/8*b^3*c^2*arccsch(c*x)-3/4*b^2*(a+b*arccsch(c*x))/x^2-1/4*c^2*(a+b*arcc
sch(c*x))^3-1/2*(a+b*arccsch(c*x))^3/x^2+3/8*b^3*c*(1+1/c^2/x^2)^(1/2)/x+3
/4*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x
```

3.30.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.48

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx=-\frac{4a^3+6ab^2-6a^2bc\sqrt{1+\frac{1}{c^2x^2}}x-3b^3c\sqrt{1+\frac{1}{c^2x^2}}x+6b\left(2a^2+b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{csch}^{-1}(cx)+6b^2\cdots}{8\cdots}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^3/x^3,x]
```

output
```
-1/8*(4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1
 + 1/(c^2*x^2)]*x + 6*b*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*Ar
cCsch[c*x] + 6*b^2*(-(b*c*Sqrt[1 + 1/(c^2*x^2)]*x) + a*(2 + c^2*x^2))*ArcC
sch[c*x]^2 + 2*b^3*(2 + c^2*x^2)*ArcCsch[c*x]^3 + 3*b*(2*a^2 + b^2)*c^2*x^
2*ArcSinh[1/(c*x)])/x^2
```

3.30.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6840, 5969, 3042, 25, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 6840}$$

$$-c^2\int \frac{\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^3}{cx}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{ 5969}$$

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^2x^2}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2c^2x^2}-\frac{3}{2}b\int -\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 25}$$

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2c^2x^2}+\frac{3}{2}b\int \left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3792}$$

$$-c^2\left(\frac{3}{2}b\left(\frac{1}{2}\int\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{2}b^2\int-\frac{1}{c^2x^2}d\text{csch}^{-1}(cx)+\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{2c^2x^2}-\sqrt{\frac{1}{c^2x^2}+1}\right.\right.$$

$\downarrow$ 17

$$-c^2\left(\frac{3}{2}b\left(\frac{1}{2}b^2\int-\frac{1}{c^2x^2}d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{6b}\right.\right.$$

$\downarrow$ 25

$$-c^2\left(\frac{3}{2}b\left(-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{6b}\right.\right.$$

$\downarrow$ 3042

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2c^2x^2}+\frac{3}{2}b\left(-\frac{1}{2}b^2\int-\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b(\right.\right.$$

$\downarrow$ 25

$$-c^2\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2c^2x^2}+\frac{3}{2}b\left(\frac{1}{2}b^2\int\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b(a+\right.\right.$$

$\downarrow$ 3115

$$-c^2\left(\frac{3}{2}b\left(\frac{1}{2}b^2\left(\frac{1}{2}\int1d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}}{2cx}\right)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{2c^2x^2}+\frac{(a+}{}\right.\right.$$

$\downarrow$ 24

$$-c^2\left(\frac{3}{2}b\left(-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{6b}+\frac{1}{2}b^2\left(\frac{1}{2}\text{csch}^{-1}(cx)-\right.\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^3/x^3,x]
```

3.30. $\int\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}dx$

output
```
-(c^2*((a + b*ArcCsch[c*x])^3/(2*c^2*x^2) + (3*b*((b^2*(-1/2*Sqrt[1 + 1/(c
^2*x^2)]/(c*x) + ArcCsch[c*x]/2))/2 + (b*(a + b*ArcCsch[c*x]))/(2*c^2*x^2)
 - (Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(2*c*x) + (a + b*ArcCsch
[c*x])^3/(6*b)))/2))
```

3.30.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.30. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx$

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.30.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsch}\,(cx))^3}{x^3}dx$$

input
```
int((a+b*arccsch(c*x))^3/x^3,x)
```

output
```
int((a+b*arccsch(c*x))^3/x^3,x)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(107) = 214$.

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.17

$$\int \frac{(a+b\mathrm{csch}^{-1}(cx))^3}{x^3}\,dx =$$

$$-\frac{2\,(b^3c^2x^2+2\,b^3)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-3\,(2\,a^2b+b^3)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+4\,a^3+6\,ab^2+6\left(ab^2c^2x^2-b^3cx\sqrt{\frac{c^2}{c}}\right.}{}$$

input
```
integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
-1/8*(2*(b^3*c^2*x^2 + 2*b^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^3 - 3*(2*a^2*b + b^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 4*a^3 + 6
*a*b^2 + 6*(a*b^2*c^2*x^2 - b^3*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b^
2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(4*a*b^2*c*x*s
qrt((c^2*x^2 + 1)/(c^2*x^2)) - (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

3.30.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\operatorname{acsch}\left(cx\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*acsch(c*x))**3/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))**3/x**3, x)
```

3.30.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(b\operatorname{arcsch}\left(cx\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1)
- c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) +
1) - 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^
2 - 1/2*a^3/x^2 - integrate(1/2*(2*b^3*log(c)^3 - 6*a*b^2*log(c)^2 + 2*(b^
3*c^2*x^2 + b^3)*log(x)^3 + 2*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^
2 + 6*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3
*(2*b^3*log(c) - 2*a*b^2 + 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 2*(b^3*c^2
*x^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(2*b^3*log(c) - 2*a*b^2 + (b^3*c^2*
(2*log(c) - 1) - 2*a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqr
t(c^2*x^2 + 1) + 1)^2 + 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)
^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x) - 6*(b^3*log(c)^2 - 2*a*b^2*log(c) +
(b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2
 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3
*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 +
 (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) -
a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*
(b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2
*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*l
og(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3
*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^...
```

3.30.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\text{arcsch}\,(cx)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3/x^3, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^3/x^3,x)
```

output
```
int((a + b*asinh(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^4} \, dx$

3.31.1 Optimal result

Integrand size = 14, antiderivative size = 166

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^4} \, dx = -\frac{14}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}}+\frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} -\frac{2b^2\left(a+b\text{csch}^{-1}(cx)\right)}{9x^3}+\frac{4b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x} -\frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2 +\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^2}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3x^3}$$

output
```
2/27*b^3*c^3*(1+1/c^2/x^2)^(3/2)-2/9*b^2*(a+b*arccsch(c*x))/x^3+4/3*b^2*c^
2*(a+b*arccsch(c*x))/x-1/3*(a+b*arccsch(c*x))^3/x^3-14/9*b^3*c^3*(1+1/c^2/
x^2)^(1/2)-2/3*b*c^3*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)+1/3*b*c*(a+b
*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^2
```

3.31.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.20

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx$$
$$= \frac{-9a^3+2b^3c\sqrt{1+\frac{1}{c^2x^2}}x(1-20c^2x^2)+9a^2bc\sqrt{1+\frac{1}{c^2x^2}}x(1-2c^2x^2)+6ab^2(-1+6c^2x^2)+3b\Big(-9a^2+6c\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^3/x^4,x]
```

output
```
(-9*a^3 + 2*b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 20*c^2*x^2) + 9*a^2*b*c*Sqr
t[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 6*a*b^2*(-1 + 6*c^2*x^2) + 3*b*(-9*
a^2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c^2*
x^2))*ArcCsch[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*
x^2))*ArcCsch[c*x]^2 - 9*b^3*ArcCsch[c*x]^3)/(27*x^3)
```

3.31.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6840, 5969, 3042, 26, 3792, 26, 3042, 26, 3113, 2009, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx$$
$$\Big\downarrow \text{ 6840}$$
$$-c^3\int \frac{\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{c^2x^2}d\operatorname{csch}^{-1}(cx)$$
$$\Big\downarrow \text{ 5969}$$
$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-b\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{c^3x^3}d\operatorname{csch}^{-1}(cx)\right)$$

$$\downarrow 3042$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-b\int i\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\sin\left(i\operatorname{csch}^{-1}(cx)\right)^3d\operatorname{csch}^{-1}(cx)\right)$$

$$\downarrow 26$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\int\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\sin\left(i\operatorname{csch}^{-1}(cx)\right)^3d\operatorname{csch}^{-1}(cx)\right)$$

$$\downarrow 3792$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\int\frac{i\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{cx}d\operatorname{csch}^{-1}(cx)+\frac{2}{9}b^2\int-\frac{i}{c^3x^3}d\operatorname{csch}^{-1}(cx)+\frac{2ib\left(a+b\operatorname{csc}\right.}{9c^3x}\right.\right.$$

$$\downarrow 26$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}i\int\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{cx}d\operatorname{csch}^{-1}(cx)-\frac{2}{9}ib^2\int\frac{1}{c^3x^3}d\operatorname{csch}^{-1}(cx)+\frac{2ib\left(a+b\operatorname{csch}\right.}{9c^3x^3}\right.\right.$$

$$\downarrow 3042$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}i\int-i\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\sin\left(i\operatorname{csch}^{-1}(cx)\right)d\operatorname{csch}^{-1}(cx)-\frac{2}{9}ib^2\int i\sin\left(i\operatorname{csch}^{-1}\right.\right.\right.$$

$$\downarrow 26$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\int\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\sin\left(i\operatorname{csch}^{-1}(cx)\right)d\operatorname{csch}^{-1}(cx)+\frac{2}{9}b^2\int\sin\left(i\operatorname{csch}^{-1}(cx)\right)^3\right.\right.$$

$$\downarrow 3113$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\int\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\sin\left(i\operatorname{csch}^{-1}(cx)\right)d\operatorname{csch}^{-1}(cx)+\frac{2}{9}ib^2\int-\frac{1}{c^2x^2}d\sqrt{1+\frac{1}{c^2}}\right.\right.$$

$$\downarrow 2009$$

$$-c^3\left(\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\int\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\sin\left(i\operatorname{csch}^{-1}(cx)\right)d\operatorname{csch}^{-1}(cx)+\frac{2ib\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9c^3x^3}-\right.\right.$$

3.31. $\int\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}dx$

$$\Big\downarrow \text{3777}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\int\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)d\text{csch}^{-1}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\int\left(a+b\text{csch}^{-1}(cx)\right)\sin\left(i\text{csch}^{-1}(cx)+\right.\right.\right.\right.$$

$$\Big\downarrow \text{3777}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-ib\int-\frac{i}{cx}d\text{csch}^{-1}(c\right.\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-b\int\frac{1}{cx}d\text{csch}^{-1}(cx)\right.\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}-b\int-i\sin\left(i\text{csch}^{-1}\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-2ib\left(\frac{a+b\text{csch}^{-1}(cx)}{cx}+ib\int\sin\left(i\text{csch}^{-1}(c\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{3118}$$

$$-c^3\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3c^3x^3}-ib\left(\frac{2ib\left(a+b\text{csch}^{-1}(cx)\right)}{9c^3x^3}-\frac{i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{3c^2x^2}+\frac{2}{3}\left(i\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{c}\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^3/x^4,x]
```

3.31. $\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^4}\,dx$

output
```
-(c^3*((a + b*ArcCsch[c*x])^3/(3*c^3*x^3) - I*b*(((2*I)/9)*b^2*(Sqrt[1 + 1
/(c^2*x^2)] - (1 + 1/(c^2*x^2))^(3/2)/3) + (((2*I)/9)*b*(a + b*ArcCsch[c*x
]))/(c^3*x^3) - ((I/3)*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(c^2*
x^2) + (2*(I*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2 - (2*I)*b*(-(b*S
qrt[1 + 1/(c^2*x^2)]) + (a + b*ArcCsch[c*x])/(c*x))))/3)))
```

3.31.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.31. $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx$

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.31.4 Maple [F]

$$\int \frac{(a + b\,\mathrm{arccsch}\,(cx))^3}{x^4} dx$$

input
```
int((a+b*arccsch(c*x))^3/x^4,x)
```

output
```
int((a+b*arccsch(c*x))^3/x^4,x)
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(144) = 288.

Time = 0.26 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.81

$$\int \frac{\left(a + b\mathrm{csch}^{-1}(cx)\right)^3}{x^4}\, dx$$
$$= \frac{36\,ab^2c^2x^2 - 9\,b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 9\,a^3 - 6\,ab^2 - 9\left(3\,ab^2 + (2\,b^3c^3x^3 - b^3cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2}{c^2x}}}{cx}\right.}{}$$

input
```
integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
1/27*(36*a*b^2*c^2*x^2 - 9*b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x))^3 - 9*a^3 - 6*a*b^2 - 9*(3*a*b^2 + (2*b^3*c^3*x^3 - b^3*c*x)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
))^2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b - 2*b^3 - 6*(2*a*b^2*c^3*x^3 - a*b^2*c*
x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 - (9*a^2*b + 2*b^3)*c*x)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)))/x^3
```

3.31.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{\left(a+b\operatorname{acsch}(cx)\right)^3}{x^4}\,dx$$

input
```
integrate((a+b*acsch(c*x))**3/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))**3/x**4, x)
```

3.31.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{\left(b\operatorname{arcsch}(cx)+a\right)^3}{x^4}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
1/3*a^2*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c -
 3*arccsch(c*x)/x^3) - 1/3*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^3 - 1/3*a^3/
x^3 - integrate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log
(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*
b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (3*b^3*log(c) - 3*a*b^2
 + 3*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 3*(b^3*c^2*x^2 + b^3)*log(x) + sqr
t(c^2*x^2 + 1)*(3*b^3*log(c) - 3*a*b^2 + (b^3*c^2*(3*log(c) - 1) - 3*a*b^2
*c^2)*x^2 + 3*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 +
3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))
*x^2)*log(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*
b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2
 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(
c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log
(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*
sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*lo
g(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*lo
g(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*lo
g(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^
2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)
*sqrt(c^2*x^2 + 1)), x)
```

3.31.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx=\int \frac{(b\operatorname{arcsch}(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3/x^4, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\text{csch}^{-1}(cx)\right)^3}{x^4}\,dx = \int \frac{\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^3/x^4,x)
```

output
```
int((a + b*asinh(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx$

3.32.1 Optimal result

Integrand size = 14, antiderivative size = 204

$$\begin{aligned}\int \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx &= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3}-\frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x}+\frac{45}{256}b^3c^4\text{csch}^{-1}(cx)\\
&\quad-\frac{3b^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{32x^4}+\frac{9b^2c^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{32x^2}\\
&\quad+\frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{16x^3}\\
&\quad-\frac{9bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\,\text{csch}^{-1}(cx)\right)^2}{32x}\\
&\quad+\frac{3}{32}c^4\left(a+b\,\text{csch}^{-1}(cx)\right)^3-\frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^3}{4x^4}\end{aligned}$$

output
```
45/256*b^3*c^4*arccsch(c*x)-3/32*b^2*(a+b*arccsch(c*x))/x^4+9/32*b^2*c^2*(
a+b*arccsch(c*x))/x^2+3/32*c^4*(a+b*arccsch(c*x))^3-1/4*(a+b*arccsch(c*x))
^3/x^4+3/128*b^3*c*(1+1/c^2/x^2)^(1/2)/x^3-45/256*b^3*c^3*(1+1/c^2/x^2)^(1
/2)/x+3/16*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+
b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x
```

3.32.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.36

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$=\frac{-64a^3-24ab^2+48a^2bc\sqrt{1+\frac{1}{c^2x^2}}x+6b^3c\sqrt{1+\frac{1}{c^2x^2}}x+72ab^2c^2x^2-72a^2bc^3\sqrt{1+\frac{1}{c^2x^2}}x^3-45b^3c^3\sqrt{1+}}{}$$

input
```
Integrate[(a + b*ArcCsch[c*x])^3/x^5,x]
```

output
```
(-64*a^3 - 24*a*b^2 + 48*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1
+ 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x
^3 - 45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 24*b*(8*a^2 + b^2*(1 - 3*c^2*x
^2) + 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3*c^2*x^2))*ArcCsch[c*x] + 24*
b^2*(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(2 - 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*Arc
Csch[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^3 + 9*b*(8*a^2 + 5*b^2)*
c^4*x^4*ArcSinh[1/(c*x)])/(256*x^4)
```

3.32.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.29, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {6840, 5969, 3042, 3792, 25, 3042, 25, 3115, 25, 3042, 25, 3115, 24, 3792, 17, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$\Big\downarrow \text{6840}$$
$$-c^4\int \frac{\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^3}{c^3x^3}d\text{csch}^{-1}(cx)$$
$$\Big\downarrow \text{5969}$$
$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^4x^4}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^4d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3792}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^2x^2}d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\int\frac{1}{c^4x^4}d\text{csch}^{-1}(cx)-\frac{b(a+b\text{csch}\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(-\frac{3}{4}\int\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^2x^2}d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\int\frac{1}{c^4x^4}d\text{csch}^{-1}(cx)-\frac{b(a+b\text{csch}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(-\frac{3}{4}\int-\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\int\sin\left(i\text{csch}^{-1}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\int\sin\left(i\text{csch}^{-1}(cx\right.\right.\right.$$

$$\Big\downarrow \text{3115}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{3}{4}\int-\frac{1}{c^2x^2}d\text{csch}\right.\right.\right.$$

$$\Big\downarrow \text{25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{\sqrt{\frac{1}{c^2x^2}+1}}{4c^3x^3}-\frac{3}{4}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.32. $\int\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}dx$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{\sqrt{\frac{1}{c^2x^2}+1}}{4c^3x^3}-\frac{3}{4}\right.\right.\right.$$

$$\Big\downarrow \text{ 25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{\sqrt{\frac{1}{c^2x^2}+1}}{4c^3x^3}+\frac{3}{4}\right.\right.\right.$$

$$\Big\downarrow \text{ 3115}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{8}b^2\left(\frac{3}{4}\left(\frac{1}{2}\int 1d\text{csch}^{-}\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 24}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\int\left(a+b\text{csch}^{-1}(cx)\right)^2\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{8c^4x^4}+\right.\right.$$

$$\Big\downarrow \text{ 3792}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}\int\left(a+b\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)+\frac{1}{2}b^2\int-\frac{1}{c^2x^2}d\text{csch}^{-1}(cx)+\frac{b\left(a+bc\right.}{2c}\right.\right.\right.$$

$$\Big\downarrow \text{ 17}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}b^2\int-\frac{1}{c^2x^2}d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}\right.}{2c^2x^2}\right.\right.\right.$$

$$\Big\downarrow \text{ 25}$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int\frac{1}{c^2x^2}d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}\right.}{2c^2x^2}\right.\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

3.32. $\int\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}dx$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{1}{2}b^2\int -\sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\right.\right.\right.$$

$$\Big\downarrow 25$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}b^2\int \sin\left(i\text{csch}^{-1}(cx)\right)^2d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b(a\right.\right.\right.$$

$$\Big\downarrow 3115$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(\frac{1}{2}b^2\left(\frac{1}{2}\int 1d\text{csch}^{-1}(cx)-\frac{\sqrt{\frac{1}{c^2x^2}+1}}{2cx}\right)-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\right.\right.\right.$$

$$\Big\downarrow 24$$

$$-c^4\left(\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4c^4x^4}-\frac{3}{4}b\left(\frac{3}{4}\left(-\frac{\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2cx}+\frac{b\left(a+b\text{csch}^{-1}(cx)\right)}{2c^2x^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{6b}\right.\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])^3/x^5,x]
```

output
```
-(c^4*((a + b*ArcCsch[c*x])^3/(4*c^4*x^4) - (3*b*((b^2*(Sqrt[1 + 1/(c^2*x^
2)]/(4*c^3*x^3) + (3*(-1/2*Sqrt[1 + 1/(c^2*x^2)]/(c*x) + ArcCsch[c*x]/2))/
4))/8 - (b*(a + b*ArcCsch[c*x]))/(8*c^4*x^4) + (Sqrt[1 + 1/(c^2*x^2)]*(a +
 b*ArcCsch[c*x])^2)/(4*c^3*x^3) + (3*((b^2*(-1/2*Sqrt[1 + 1/(c^2*x^2)]/(c*
x) + ArcCsch[c*x]/2))/2 + (b*(a + b*ArcCsch[c*x]))/(2*c^2*x^2) - (Sqrt[1 +
 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(2*c*x) + (a + b*ArcCsch[c*x])^3/(6*
b)))/4))/4))
```

3.32.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5969
```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.32.4 Maple [F]

$$\int \frac{(a+b\ \mathrm{arccsch}\,(cx))^3}{x^5} dx$$

input
```
int((a+b*arccsch(c*x))^3/x^5,x)
```

output
```
int((a+b*arccsch(c*x))^3/x^5,x)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx$$
$$= \frac{72\,ab^2c^2x^2 + 8\left(3\,b^3c^4x^4 - 8\,b^3\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 64\,a^3 - 24\,ab^2 + 24\left(3\,ab^2c^4x^4 - 8\,ab^2 - \left(3\,b^3c^3x^3 \right.\right.}{}$$

input
```
integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x))^3 - 64*a^3 - 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 -
 8*a*b^2 - (3*b^3*c^3*x^3 - 2*b^3*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log(
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(3*(8*a^2*b + 5*b^3)*
c^4*x^4 + 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 - 16*(3*a*b^2*c^3*x^3 - 2*a*b^
2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2
)) + 1)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 - 2*(8*a^2*b + b^3)*c*x)*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))/x^4
```

3.32.6 Sympy [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx = \int \frac{\left(a+b\operatorname{acsch}(cx)\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*acsch(c*x))**3/x**5,x)
```

output
```
Integral((a + b*acsch(c*x))**3/x**5, x)
```

3.32.7 Maxima [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(b\,\text{arcsch}\,(cx)+a\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt
(1/(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sq
rt(1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2)
 + 1) + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^3*log(sqrt(c^2*x^2 + 1) + 1)^
3/x^4 - 1/4*a^3/x^4 - integrate(1/4*(4*b^3*log(c)^3 - 12*a*b^2*log(c)^2 +
4*(b^3*c^2*x^2 + b^3)*log(x)^3 + 4*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^
2)*x^2 + 12*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)
^2 + 3*(4*b^3*log(c) - 4*a*b^2 + 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 4*(b
^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(4*b^3*log(c) - 4*a*b^2 + (b^
3*c^2*(4*log(c) - 1) - 4*a*b^2*c^2)*x^2 + 4*(b^3*c^2*x^2 + b^3)*log(x)))*l
og(sqrt(c^2*x^2 + 1) + 1)^2 + 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2
*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x) - 12*(b^3*log(c)^2 - 2*a*b^2*l
og(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*
log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x
) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c
))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*l
og(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) +
 1) + 4*(b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 +
(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*lo...
```

3.32.8 Giac [F]

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(b\,\text{arcsch}\,(cx)+a\right)^3}{x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3/x^5, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^5}\,dx=\int \frac{\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))^3/x^5,x)
```

output
```
int((a + b*asinh(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$

3.33.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx=\text{Int}\left(\frac{x}{a+b\text{csch}^{-1}(cx)},x\right)$$

output
```
Unintegrable(x/(a+b*arccsch(c*x)),x)
```

3.33.2 Mathematica [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$$

input
```
Integrate[x/(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[x/(a + b*ArcCsch[c*x]), x]
```

3.33.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$$
$$\Big\downarrow \text{6866}$$
$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}dx$$

input
```
Int[x/(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.33.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.33.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+b\,\operatorname{arccsch}\left(cx\right)}dx$$

input
```
int(x/(a+b*arccsch(c*x)),x)
```

output
```
int(x/(a+b*arccsch(c*x)),x)
```

3.33.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{x}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral(x/(b*arccsch(c*x) + a), x)
```

3.33.6 Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{x}{a+b\,\text{acsch}\,(cx)}\,dx$$

input
```
integrate(x/(a+b*acsch(c*x)),x)
```

output
```
Integral(x/(a + b*acsch(c*x)), x)
```

3.33.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{x}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate(x/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*arccsch(c*x) + a), x)
```

3.33.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{x}{a + b\text{csch}^{-1}(cx)} \, dx = \int \frac{x}{b \operatorname{arcsch}(cx) + a} \, dx$$

input
```
integrate(x/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate(x/(b*arccsch(c*x) + a), x)
```

3.33.9 Mupad [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{x}{a + b\text{csch}^{-1}(cx)} \, dx = \int \frac{x}{a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)} \, dx$$

input
```
int(x/(a + b*asinh(1/(c*x))),x)
```

output
```
int(x/(a + b*asinh(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx$

3.34.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\text{Int}\left(\frac{1}{a+b\text{csch}^{-1}(cx)},x\right)$$

output
```
Unintegrable(1/(a+b*arccsch(c*x)),x)
```

3.34.2 Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])^(-1),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])^(-1), x]
```

3.34.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx$$

$$\Big\downarrow \; 6866$$

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}dx$$

input
```
Int[(a + b*ArcCsch[c*x])^(-1),x]
```

output
```
$Aborted
```

3.34.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.34.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\,\operatorname{arccsch}\left(cx\right)}dx$$

input
```
int(1/(a+b*arccsch(c*x)),x)
```

output
```
int(1/(a+b*arccsch(c*x)),x)
```

3.34.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccsch(c*x) + a), x)
```

3.34.6 Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\text{acsch}\,(cx)}\,dx$$

input
```
integrate(1/(a+b*acsch(c*x)),x)
```

output
```
Integral(1/(a + b*acsch(c*x)), x)
```

3.34.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccsch(c*x) + a), x)
```

3.34.8 Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate(1/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccsch(c*x) + a), x)
```

3.34.9 Mupad [N/A]

Not integrable

Time = 4.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{1}{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int(1/(a + b*asinh(1/(c*x))),x)
```

output
```
int(1/(a + b*asinh(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$

3.35.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)},x\right)$$

output
```
Unintegrable(1/x/(a+b*arccsch(c*x)),x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$$

input
```
Integrate[1/(x*(a + b*ArcCsch[c*x])),x]
```

output
```
Integrate[1/(x*(a + b*ArcCsch[c*x])), x]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$$
$$\Big\downarrow \text{6866}$$
$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}dx$$

input
```
Int[1/(x*(a + b*ArcCsch[c*x])),x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\,\operatorname{arccsch}\left(cx\right)\right)}dx$$

input
```
int(1/x/(a+b*arccsch(c*x)),x)
```

output
```
int(1/x/(a+b*arccsch(c*x)),x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arcsch}(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*arccsch(c*x) + a*x), x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\,(a+b\operatorname{acsch}(cx))}\,dx$$

input
```
integrate(1/x/(a+b*acsch(c*x)),x)
```

output
```
Integral(1/(x*(a + b*acsch(c*x))), x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arcsch}(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x), x)
```

3.35.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\operatorname{arcsch}(cx)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x), x)
```

3.35.9 Mupad [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x*(a + b*asinh(1/(c*x)))),x)
```

output
```
int(1/(x*(a + b*asinh(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$

3.36.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=-\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}+\frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}$$

output
```
-c*Chi(a/b+arccsch(c*x))*cosh(a/b)/b+c*Shi(a/b+arccsch(c*x))*sinh(a/b)/b
```

3.36.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=-\frac{c\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)}{b}$$

input
```
Integrate[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

output
```
-((c*(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]] - Sinh[a/b]*SinhIntegral[
a/b + ArcCsch[c*x]]))/b)
```

3.36.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6840, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}dx$$

$$\Big\downarrow \text{ 6840}$$

$$-c\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\int \frac{\sin\left(i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{ 3784}$$

$$-c\left(\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)+i\sinh\left(\frac{a}{b}\right)\int \frac{i\sinh\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 26}$$

$$-c\left(\cosh\left(\frac{a}{b}\right)\int \frac{\cosh\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)-\sinh\left(\frac{a}{b}\right)\int \frac{\sinh\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$-c\left(\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{ia}{b}+i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)-\sinh\left(\frac{a}{b}\right)\int -\frac{i\sin\left(\frac{ia}{b}+i\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 26}$$

$$-c\left(\cosh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{ia}{b}+i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)+i\sinh\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{ia}{b}+i\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 3779}$$

$$-c\left(-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}+\cosh\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{ia}{b}+i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{3782}$$

$$-c\left(\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

output
```
-(c*((Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/b - (Sinh[a/b]*SinhInteg
ral[a/b + ArcCsch[c*x]])/b))
```

3.36.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.36.4 Maple [F]

$$\int \frac{1}{x^2\left(a+b\ \mathrm{arccsch}\left(cx\right)\right)}dx$$

input
```
int(1/x^2/(a+b*arccsch(c*x)),x)
```

output
```
int(1/x^2/(a+b*arccsch(c*x)),x)
```

3.36.5 Fricas [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsch}\left(cx\right)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*arccsch(c*x) + a*x^2), x)
```

3.36.6 Sympy [F]

$$\int \frac{1}{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\left(a+b\,\mathrm{acsch}\left(cx\right)\right)}\,dx$$

input
```
integrate(1/x**2/(a+b*acsch(c*x)),x)
```

output
```
Integral(1/(x**2*(a + b*acsch(c*x))), x)
```

3.36.7 Maxima [F]

$$\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arcsch}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

3.36.8 Giac [F]

$$\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\text{arcsch}\,(cx)+a)x^2}\,dx$$

input
```
integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x^2*(a + b*asinh(1/(c*x)))),x)
```

output
```
int(1/(x^2*(a + b*asinh(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3\left(a+b\mathbf{csch}^{-1}(cx)\right)}\,dx$

3.37.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{2b}$$

output
```
-1/2*c^2*cosh(2*a/b)*Shi(2*a/b+2*arccsch(c*x))/b+1/2*c^2*Chi(2*a/b+2*arccs
ch(c*x))*sinh(2*a/b)/b
```

3.37.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\frac{c^2\left(\text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\right)}{2b}$$

input
```
Integrate[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

output
```
(c^2*(CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]
*SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]]))/(2*b)
```

3.37.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6840, 5971, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+b\text{csch}^{-1}(cx)\right)} \, dx$$

$$\Big\downarrow \text{6840}$$

$$-c^2 \int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{cx \left(a+b\text{csch}^{-1}(cx)\right)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{5971}$$

$$-c^2 \int \frac{\sinh\left(2\text{csch}^{-1}(cx)\right)}{2\left(a+b\text{csch}^{-1}(cx)\right)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}c^2 \int \frac{\sinh\left(2\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}c^2 \int -\frac{i\sin\left(2i\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ic^2 \int \frac{\sin\left(2i\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}ic^2\left(\cosh\left(\frac{2a}{b}\right)\int \frac{i\sinh\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right.$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}ic^2\left(i\cosh\left(\frac{2a}{b}\right)\int \frac{\sinh\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int \frac{\cosh\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}ic^2\left(i\cosh\left(\frac{2a}{b}\right)\int -\frac{i\sin\left(\frac{2ia}{b}+2i\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csc}\right.$$

$$\Big\downarrow \; 26$$

$$\frac{1}{2}ic^2\left(\cosh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{csch}^{-1}(cx)\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(\right.$$

$$\Big\downarrow \; 3779$$

$$\frac{1}{2}ic^2\left(\frac{i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{b}-i\sinh\left(\frac{2a}{b}\right)\int\frac{\sin\left(\frac{2ia}{b}+2i\text{csch}^{-1}(cx)+\frac{\pi}{2}\right)}{a+b\text{csch}^{-1}(cx)}d\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 3782$$

$$\frac{1}{2}ic^2\left(\frac{i\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{b}-\frac{i\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{b}\right)$$

input
```
Int[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

output
```
(I/2)*c^2*(((-I)*CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b])/b +
 (I*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]])/b)
```

3.37.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.37.4 Maple [F]

$$\int \frac{1}{x^3 \left(a + b \operatorname{arccsch}(cx)\right)} dx$$

input
```
int(1/x^3/(a+b*arccsch(c*x)),x)
```

output
```
int(1/x^3/(a+b*arccsch(c*x)),x)
```

3.37.5 Fricas [F]

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsch}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^3*arccsch(c*x) + a*x^3), x)
```

3.37.6 Sympy [F]

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x^3\,(a+b\,\mathrm{acsch}\,(cx))}\,dx$$

input
```
integrate(1/x**3/(a+b*acsch(c*x)),x)
```

output
```
Integral(1/(x**3*(a + b*acsch(c*x))), x)
```

3.37.7 Maxima [F]

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{(b\,\mathrm{arcsch}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

3.37.8 Giac [F]

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{(b\,\text{arcsch}\,(cx)+a)x^3}\,dx$$

input
```
integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \int \frac{1}{x^3\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

input
```
int(1/(x^3*(a + b*asinh(1/(c*x)))),x)
```

output
```
int(1/(x^3*(a + b*asinh(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4\left(a+b\text{csch}^{-1}(cx)\right)}\,dx$

3.38.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{x^4\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b} - \frac{c^3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b} - \frac{c^3\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b} + \frac{c^3\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b}$$

output
```
1/4*c^3*Chi(a/b+arccsch(c*x))*cosh(a/b)/b-1/4*c^3*Chi(3*a/b+3*arccsch(c*x)
)*cosh(3*a/b)/b-1/4*c^3*Shi(a/b+arccsch(c*x))*sinh(a/b)/b+1/4*c^3*Shi(3*a/
b+3*arccsch(c*x))*sinh(3*a/b)/b
```

3.38.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\left(a+b\text{csch}^{-1}(cx)\right)}\,dx = -\frac{c^3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx\right.}{4b}$$

input
```
Integrate[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

output
```
-1/4*(c^3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]]) + Cosh[(3*a)/b]*C
oshIntegral[3*(a/b + ArcCsch[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcCsch
[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCsch[c*x])]))/b
```

3.38.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6840, 5971, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+b\text{csch}^{-1}(cx)\right)} \, dx$$

$$\Big\downarrow \text{6840}$$

$$-c^3 \int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2 \left(a+b\text{csch}^{-1}(cx)\right)}d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{5971}$$

$$-c^3 \int \left(\frac{\cosh\left(3\text{csch}^{-1}(cx)\right)}{4\left(a+b\text{csch}^{-1}(cx)\right)}-\frac{\sqrt{1+\frac{1}{c^2x^2}}}{4\left(a+b\text{csch}^{-1}(cx)\right)}\right)d\text{csch}^{-1}(cx)$$

$$\Big\downarrow \text{2009}$$

$$-c^3\left(-\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b}+\frac{\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b}+\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b}-\text{sinl}\right.$$

input
```
Int[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

output
```
-(c^3*(-1/4*(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/b + (Cosh[(3*a)/b
]*CoshIntegral[(3*a)/b + 3*ArcCsch[c*x]])/(4*b) + (Sinh[a/b]*SinhIntegral[
a/b + ArcCsch[c*x]])/(4*b) - (Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCs
ch[c*x]])/(4*b)))
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5971
```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

rule 6840
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
-(c^(m + 1))^(-1)   Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, A
rcCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (G
tQ[n, 0] || LtQ[m, -1])
```

3.38.4 Maple [F]

$$\int \frac{1}{x^4 (a + b\, \text{arccsch}(cx))} dx$$

input
```
int(1/x^4/(a+b*arccsch(c*x)),x)
```

output
```
int(1/x^4/(a+b*arccsch(c*x)),x)
```

3.38.5 Fricas [F]

$$\int \frac{1}{x^4 \left(a + b\text{csch}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b\, \text{arcsch}\,(cx) + a)x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^4*arccsch(c*x) + a*x^4), x)
```

3.38.6 Sympy [F]

$$\int \frac{1}{x^4 \left(a + b\text{csch}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{acsch}\left(cx\right)\right)} \, dx$$

input
```
integrate(1/x**4/(a+b*acsch(c*x)),x)
```

output
```
Integral(1/(x**4*(a + b*acsch(c*x))), x)
```

3.38.7 Maxima [F]

$$\int \frac{1}{x^4 \left(a + b\text{csch}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arcsch}\left(cx\right) + a)x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

3.38.8 Giac [F]

$$\int \frac{1}{x^4 \left(a + b\text{csch}^{-1}(cx)\right)} \, dx = \int \frac{1}{(b \operatorname{arcsch}\left(cx\right) + a)x^4} \, dx$$

input
```
integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a + b\text{csch}^{-1}(cx)\right)} \, dx = \int \frac{1}{x^4 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)} \, dx$$

input
```
int(1/(x^4*(a + b*asinh(1/(c*x)))),x)
```

output
```
int(1/(x^4*(a + b*asinh(1/(c*x)))), x)
```

3.39 $\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$

3.39.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\text{Int}\left((dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

3.39.2 Mathematica [N/A]

Not integrable

Time = 5.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

3.39.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

output
```
$Aborted
```

3.39.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.39.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m (a+b \operatorname{arccsch}(cx))^3 dx$$

input
```
int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

3.39.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arcsch}\,(cx)+a)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*
x) + a^3)*(d*x)^m, x)
```

3.39.6 Sympy [N/A]

Not integrable

Time = 22.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (dx)^m (a+b\,\text{acsch}\,(cx))^3 \, dx$$

input
```
integrate((d*x)**m*(a+b*acsch(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*acsch(c*x))**3, x)
```

3.39.7 Maxima [N/A]

Not integrable

Time = 7.73 (sec) , antiderivative size = 1351, normalized size of antiderivative = 84.44

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (b\,\text{arcsch}\,(cx)+a)^3(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

output
```
b^3*d^m*x*x^m*log(sqrt(c^2*x^2 + 1) + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*
(m + 1)) - integrate((3*((b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (a*
b^2*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^3*c^2)*x^2 + (b^3*c^2*d
^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^3*d^m
*(m + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*
c^2*d^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x))
*x^m)*log(sqrt(c^2*x^2 + 1) + 1)^2 + (b^3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d
^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) + (b^3*c^2*d^m*(m + 1)*x^
2 + b^3*d^m*(m + 1))*log(x)^3 + (b^3*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^
2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*d^m*(m + 1)*log(c))*x^2 + 3*(b^3*d^m*
(m + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c
^2*d^m*(m + 1))*x^2)*log(x)^2 + 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*
(m + 1)*log(c) + a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b
^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*log(x))*sqrt(c^2*x
^2 + 1)*x^m + (b^3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3
*a^2*b*d^m*(m + 1)*log(c) + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*lo
g(x)^3 + (b^3*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2
+ 3*a^2*b*c^2*d^m*(m + 1)*log(c))*x^2 + 3*(b^3*d^m*(m + 1)*log(c) - a*b^2*
d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*lo
g(x)^2 + 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^2...
```

3.39.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx = \int (b \,\text{arcsch}\,(cx) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^3*(d*x)^m, x)
```

3.39.9 Mupad [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int (d\,x)^m\,{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^3 \,d x$$

input
```
int((d*x)^m*(a + b*asinh(1/(c*x)))^3,x)
```

output
```
int((d*x)^m*(a + b*asinh(1/(c*x)))^3, x)
```

3.40 $\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = \text{Int}\left((dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

3.40.2 Mathematica [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = \int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

3.40.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$
$$\Big\downarrow \text{6866}$$
$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
```

output
```
$Aborted
```

3.40.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.40.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\operatorname{arccsch}(cx)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

3.40.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (b\,\text{arcsch}\,(cx)+a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)*(d*x)^m, x)
```

3.40.6 Sympy [N/A]

Not integrable

Time = 10.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (dx)^m \left(a+b\,\text{acsch}\,(cx)\right)^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*acsch(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*acsch(c*x))**2, x)
```

3.40.7 Maxima [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 644, normalized size of antiderivative = 40.25

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (b\,\text{arcsch}\,(cx)+a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

output
```
b^2*d^m*x*x^m*log(sqrt(c^2*x^2 + 1) + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*
(m + 1)) - integrate(-((b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c
) + (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2 + (b
^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x)^2 + 2*(b^2*d^m*(m + 1)*lo
g(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1)
)*x^2)*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d
^m*(m + 1)*log(c) + (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*
log(c))*x^2 + (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x)^2 + 2*(b^
2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b
*c^2*d^m*(m + 1))*x^2)*log(x))*x^m - 2*((b^2*d^m*(m + 1)*log(c) - a*b*d^m*
(m + 1) - (a*b*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^2*c^2)*x^2 +
 (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x))*sqrt(c^2*x^2 + 1)*x^m
 + (b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c)
 - a*b*c^2*d^m*(m + 1))*x^2 + (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*
log(x))*x^m)*log(sqrt(c^2*x^2 + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x
^2 + m + 1)*sqrt(c^2*x^2 + 1) + m + 1), x)
```

3.40.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (b \operatorname{arcsch}(cx) + a)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)^2*(d*x)^m, x)
```

3.40.9 Mupad [N/A]

Not integrable

Time = 5.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx = \int (d\,x)^m \left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a + b*asinh(1/(c*x)))^2,x)
```

output
```
int((d*x)^m*(a + b*asinh(1/(c*x)))^2, x)
```

3.40. $\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right)^2 \, dx$

3.41 $\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.41.1 Optimal result

Integrand size = 14, antiderivative size = 67

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \frac{(dx)^{1+m} \left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},-\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

output
```
(d*x)^(1+m)*(a+b*arccsch(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[
1-1/2*m],-1/c^2/x^2)/c/m/(1+m)
```

3.41.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.21

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \frac{x(dx)^m \left((1+m)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{\sqrt{1+c^2x^2}}\right)}{(1+m)^2}$$

input
```
Integrate[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

output
```
(x*(d*x)^m*((1 + m)*(a + b*ArcCsch[c*x]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/Sqrt[1 + c^2*x^2])
)/(1 + m)^2
```

3.41.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6838, 862, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \ 6838$$
$$\frac{bd\int \frac{(dx)^{m-1}}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{c(m+1)}+\frac{(dx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{d(m+1)}$$
$$\Big\downarrow \ 862$$
$$\frac{(dx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{d(m+1)}-\frac{b\left(\frac{1}{x}\right)^m (dx)^m \int \frac{\left(\frac{1}{x}\right)^{-m-1}}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c(m+1)}$$
$$\Big\downarrow \ 278$$
$$\frac{(dx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m \text{Hypergeometric2F1}\left(\frac{1}{2},-\frac{m}{2},1-\frac{m}{2},-\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

input
```
Int[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometr
ic2F1[1/2, -1/2*m, 1 - m/2, -(1/(c^2*x^2))])/(c*m*(1 + m))
```

3.41.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 6838
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Simp[b*(d/(c*(m +
 1)))   Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c,
d, m}, x] && NeQ[m, -1]
```

3.41.4 Maple [F]

$$\int (dx)^m (a + b \operatorname{arccsch}(cx)) \, dx$$

input
```
int((d*x)^m*(a+b*arccsch(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arccsch(c*x)),x)
```

3.41.5 Fricas [F]

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right) \, dx = \int (b \operatorname{arcsch}(cx) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)*(d*x)^m, x)
```

3.41.6 Sympy [F]

$$\int (dx)^m \left(a + b\operatorname{csch}^{-1}(cx)\right) \, dx = \int (dx)^m (a + b \operatorname{acsch}(cx)) \, dx$$

input
```
integrate((d*x)**m*(a+b*acsch(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*acsch(c*x)), x)
```

3.41.7 Maxima [F]

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right) dx = \int (b\,\text{arcsch}\,(cx) + a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
(c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sq
rt(c^2*x^2 + 1) + m + 1), x) - (d^m*x*x^m*log(x) - d^m*x*x^m*log(sqrt(c^2*
x^2 + 1) + 1))/(m + 1) - integrate((c^2*d^m*(m + 1)*x^2*log(c) + d^m*(m +
1)*log(c) - d^m)*x^m/(c^2*(m + 1)*x^2 + m + 1), x))*b + (d*x)^(m + 1)*a/(d
*(m + 1))
```

3.41.8 Giac [F]

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right) dx = \int (b\,\text{arcsch}\,(cx) + a)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*(d*x)^m, x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b\text{csch}^{-1}(cx)\right) dx = \int (d\,x)^m \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right) dx$$

input
```
int((d*x)^m*(a + b*asinh(1/(c*x))),x)
```

output
```
int((d*x)^m*(a + b*asinh(1/(c*x))), x)
```

3.42 $\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\text{csch}^{-1}(cx)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arccsch(c*x)),x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)} \, dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\,\text{arccsch}\,(cx)}dx$$

input
```
int((d*x)^m/(a+b*arccsch(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arccsch(c*x)),x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arccsch(c*x) + a), x)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\,\text{acsch}\,(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*acsch(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*acsch(c*x)), x)
```

3.42.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

3.42.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{b\,\text{arcsch}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

3.42.9 Mupad [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{(d\,x)^m}{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}\,dx$$

input
```
int((d*x)^m/(a + b*asinh(1/(c*x))),x)
```

output
```
int((d*x)^m/(a + b*asinh(1/(c*x))), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx$

3.43.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\text{Int}\left(\frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2} \, dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{\left(a+b \operatorname{arccsch}\left(cx\right)\right)^2}dx$$

input
```
int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

output
```
int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

3.43.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{(b\,\text{arcsch}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2), x)
```

3.43.6 Sympy [N/A]

Not integrable

Time = 5.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{(a+b\,\text{acsch}\,(cx))^2}\,dx$$

input
```
integrate((d*x)**m/(a+b*acsch(c*x))**2,x)
```

output
```
Integral((d*x)**m/(a + b*acsch(c*x))**2, x)
```

3.43.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 566, normalized size of antiderivative = 35.38

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{(b\,\text{arcsch}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

output
```
-((c^2*d^m*x^3 + d^m*x)*sqrt(c^2*x^2 + 1)*x^m + (c^2*d^m*x^3 + d^m*x)*x^m)
/((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^2)*
log(x) - (b^2*c^2*x^2 + sqrt(c^2*x^2 + 1)*b^2 + b^2)*log(sqrt(c^2*x^2 + 1)
 + 1) + sqrt(c^2*x^2 + 1)*(b^2*log(c) + b^2*log(x) - a*b)) - integrate(-((
c^2*d^m*(m + 3)*x^2 + d^m*(m + 1))*(c^2*x^2 + 1)*x^m + (c^4*d^m*(m + 2)*x^
4 + c^2*d^m*(3*m + 5)*x^2 + 2*d^m*(m + 1))*sqrt(c^2*x^2 + 1)*x^m + (c^4*d^
m*(m + 1)*x^4 + 2*c^2*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c)
 - a*b*c^4)*x^4 + 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) + (c^2*x^2
 + 1)*(b^2*log(c) + b^2*log(x) - a*b) - a*b + (b^2*c^4*x^4 + 2*b^2*c^2*x^2
 + b^2)*log(x) - (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + (c^2*x^2 + 1)*b^2 + b^2 +
2*(b^2*c^2*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*sq
rt(c^2*x^2 + 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*
c^2*x^2 + b^2)*log(x))), x)
```

3.43.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{arcsch}\,(cx)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arccsch(c*x) + a)^2, x)
```

3.43.9 Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{(dx)^m}{\left(a+b\text{csch}^{-1}(cx)\right)^2}\,dx=\int \frac{(d\,x)^m}{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)^2}\,dx$$

input
```
int((d*x)^m/(a + b*asinh(1/(c*x)))^2,x)
```

output
```
int((d*x)^m/(a + b*asinh(1/(c*x)))^2, x)
```

3.44 $\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.44.1 Optimal result

Integrand size = 16, antiderivative size = 167

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}x}{6c^3} + \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{2c} + \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}x^3}{12c} - \frac{bd^4\text{csch}^{-1}(cx)}{4e} + \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e} + \frac{bd(2c^2d^2-e^2)\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{2c^3}$$

output
```
-1/4*b*d^4*arccsch(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsch(c*x))/e+1/2*b*d*(2*c^
2*d^2-e^2)*arctanh((1+1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2-e^2)*x*(1+1
/c^2/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1+1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(
1+1/c^2/x^2)^(1/2)/c
```

3.44.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= \frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+be\sqrt{1+\frac{1}{c^2x^2}}x(-2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2}{12c^3}$$

input
```
Integrate[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

output
```
(3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 + 1/(c
^2*x^2)]*x*(-2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3
+ 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsch[c*x] + 6*b*d*(2*c^2*d^2 - e^2
)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(12*c^3)
```

3.44.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6844, 1892, 1803, 540, 25, 2338, 27, 2338, 27, 538, 222, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \text{ 6844}$$
$$\frac{b\int \frac{(d+ex)^4}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{4ce}+\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}$$
$$\Big\downarrow \text{ 1892}$$
$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{4ce}+\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}$$
$$\Big\downarrow \text{ 1803}$$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^4x^4}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{4ce}$$

$$\Big\downarrow \text{540}$$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(-\frac{1}{3}\int -\frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2-\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{\frac{1}{c^2x^2}+1}\right)}{4ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\int \frac{\left(\frac{3d^4}{x^3}+\frac{12ed^3}{x^2}+12e^3d+\frac{2e^2\left(9d^2-\frac{e^2}{c^2}\right)}{x}\right)x^3}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{3}e^4x^3\sqrt{\frac{1}{c^2x^2}+1}\right)}{4ce}$$

$$\Big\downarrow \text{2338}$$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(-\frac{1}{2}\int -\frac{2\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2-\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2-\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{3}e^4x^3\sqrt{\frac{1}{c^2x^2}+1}\right)}{4ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(\int \frac{\left(\frac{3d^4}{x^2}+\frac{6e\left(2d^2-\frac{e^2}{c^2}\right)d}{x}+2e^2\left(9d^2-\frac{e^2}{c^2}\right)\right)x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{3}e^4x^3\sqrt{\frac{1}{c^2x^2}+1}\right)}{4ce}$$

$$\Big\downarrow \text{2338}$$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(-\int -\frac{3d\left(\frac{d^3}{x}+2e\left(2d^2-\frac{e^2}{c^2}\right)\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{3}e^4x^3\sqrt{\frac{1}{c^2x^2}+1}\right)}{4ce}$$

$$\Big\downarrow \text{27}$$

3.44. $\int (d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\int\frac{\left(\frac{d^3}{x}+2e\left(2d^2-\frac{e^2}{c^2}\right)\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{3}e^4x^3\sqrt{\frac{1}{c^2x^2}+1}\right)}{4ce}$$

$\big\downarrow$ 538

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(d^3\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\left(2d^2-\frac{e^2}{c^2}\right)\int\frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\ldots}{4ce}$$

$\big\downarrow$ 222

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(2e\left(2d^2-\frac{e^2}{c^2}\right)\int\frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}+cd^3\text{arcsinh}\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{3}\ldots}{4ce}$$

$\big\downarrow$ 243

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(e\left(2d^2-\frac{e^2}{c^2}\right)\int\frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd^3\text{arcsinh}\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{3}\ldots}{4ce}$$

$\big\downarrow$ 73

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(2c^2e\left(2d^2-\frac{e^2}{c^2}\right)\int\frac{1}{c^2\sqrt{1+\frac{1}{c^2x^2}}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}+cd^3\text{arcsinh}\left(\frac{1}{cx}\right)\right)-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\ldots}{4ce}$$

$\big\downarrow$ 221

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\left(\frac{1}{3}\left(3d\left(cd^3\text{arcsinh}\left(\frac{1}{cx}\right)-2e\,\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)\left(2d^2-\frac{e^2}{c^2}\right)\right)-2e^2x\sqrt{\frac{1}{c^2x^2}+1}\left(9d^2-\frac{e^2}{c^2}\right)-6de^3x^2\sqrt{\frac{1}{c^2x^2}+1}\ldots}{4ce}$$

input
```
Int[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

3.44. $\int(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

output
```
((d + e*x)^4*(a + b*ArcCsch[c*x]))/(4*e) - (b*(-1/3*(e^4*Sqrt[1 + 1/(c^2*x
^2)]*x^3) + (-2*e^2*(9*d^2 - e^2/c^2)*Sqrt[1 + 1/(c^2*x^2)]*x - 6*d*e^3*Sq
rt[1 + 1/(c^2*x^2)]*x^2 + 3*d*(c*d^3*ArcSinh[1/(c*x)] - 2*e*(2*d^2 - e^2/c
^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]]))/3))/(4*c*e)
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.44. $\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.44.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.50

method	result
parts	$\frac{a(ex+d)^4}{4e}+\frac{b\left(\frac{c\,e^3\,\mathrm{arccsch}(cx)x^4}{4}+c\,e^2\,\mathrm{arccsch}(cx)x^3d+\frac{3ce\,\mathrm{arccsch}(cx)x^2d^2}{2}+\mathrm{arccsch}(cx)xc\,d^3+\frac{c\,\mathrm{arccsch}(cx)d^4}{4e}-\frac{\sqrt{c^2x^2}}{}\right.}{}$
derivativedivides	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\mathrm{arccsch}(cx)c^4d^4}{4e}+\mathrm{arccsch}(cx)c^4d^3x+\frac{3e\,\mathrm{arccsch}(cx)c^4d^2x^2}{2}+e^2\,\mathrm{arccsch}(cx)c^4d\,x^3+\frac{e^3\,\mathrm{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+}}{}\right.}{}}{}$
default	$\frac{\frac{a(cex+cd)^4}{4c^3e}+\frac{b\left(\frac{\mathrm{arccsch}(cx)c^4d^4}{4e}+\mathrm{arccsch}(cx)c^4d^3x+\frac{3e\,\mathrm{arccsch}(cx)c^4d^2x^2}{2}+e^2\,\mathrm{arccsch}(cx)c^4d\,x^3+\frac{e^3\,\mathrm{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+}}{}\right.}{}}{}$

input
```
int((e*x+d)^3*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x+d)^4/e+b/c*(1/4*c*e^3*arccsch(c*x)*x^4+c*e^2*arccsch(c*x)*x^3*d
+3/2*c*e*arccsch(c*x)*x^2*d^2+arccsch(c*x)*x*c*d^3+1/4*c/e*arccsch(c*x)*d^
4-1/12/c^4/e*(c^2*x^2+1)^(1/2)*(3*c^4*d^4*arctanh(1/(c^2*x^2+1)^(1/2))-12*
c^3*d^3*e*arcsinh(c*x)-e^4*c^2*x^2*(c^2*x^2+1)^(1/2)-6*c^2*d*e^3*x*(c^2*x^
2+1)^(1/2)-18*c^2*d^2*e^2*(c^2*x^2+1)^(1/2)+6*c*d*e^3*arcsinh(c*x)+2*e^4*(
c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.44.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(147) = 294$.

Time = 0.33 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.51

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^3e^3x^4+12\,ac^3de^2x^3+18\,ac^3d^2ex^2+12\,ac^3d^3x+3\left(4\,bc^3d^3+6\,bc^3d^2e+4\,bc^3de^2+bc^3e^3\right)\log\left(cx\sqrt{\frac{c^2}{}}\right.}{}$$

input
```
integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3
*d^3*x + 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*e^3)*log(c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 6*(2*b*c^2*d^3 - b*d*e^2)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*
e + 4*b*c^3*d*e^2 + b*c^3*e^3)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
 - 1) + 3*(b*c^3*e^3*x^4 + 4*b*c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3
*d^3*x - 4*b*c^3*d^3 - 6*b*c^3*d^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e^3*x^3 + 6*b*c^2*d*e^
2*x^2 + 2*(9*b*c^2*d^2*e - b*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

3.44.6 Sympy [F]

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (a+b\operatorname{acsch}(cx))\,(d+ex)^3\,dx$$

input
```
integrate((e*x+d)**3*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x)**3, x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.56

$$\begin{aligned}
&\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= \frac{1}{4}ae^3x^4 + ade^2x^3 + \frac{3}{2}ad^2ex^2 + \frac{3}{2}\left(x^2\operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2e \\
&\quad + \frac{1}{4}\left(4x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2 \\
&\quad + \frac{1}{12}\left(3x^4\operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be^3 + ad^3x \\
&\quad + \frac{\left(2cx\operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^3}{2c}
\end{aligned}$$

input
```
integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*arccsch(c*x) + x*
sqrt(1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/4*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^
2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)
/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e^2 + 1/12*(3*x^4*arccsc
h(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3
)*b*e^3 + a*d^3*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) +
1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^3/c
```

3.44.8 Giac [F]

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex+d)^3 (b \operatorname{arcsch}(cx)+a) \, dx$$

input
```
integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^3*(b*arccsch(c*x) + a), x)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(a+b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^3 \, dx$$

input
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^3,x)
```

output
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^3, x)
```

3.45 $\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx$

3.45.1 Optimal result

Integrand size = 16, antiderivative size = 122

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{bde\sqrt{1+\frac{1}{c^2x^2}}x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b(6c^2d^2-e^2)\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

output
```
-1/3*b*d^3*arccsch(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsch(c*x))/e+1/6*b*(6*c^2*
d^2-e^2)*arctanh((1+1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1+1/c^2/x^2)^(1/2)/c+1/
6*b*e^2*x^2*(1+1/c^2/x^2)^(1/2)/c
```

3.45.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{c^2x\left(be\sqrt{1+\frac{1}{c^2x^2}}(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\,\text{csch}^{-1}(cx)+b(6c^2d^2}{6c^3}$$

input
```
Integrate[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(c^2*x*(b*e*Sqrt[1 + 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e
^2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsch[c*x] + b*(6*c^2*d
^2 - e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3)
```

3.45.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6844, 1892, 1803, 540, 25, 2338, 25, 538, 222, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6844}$$

$$\frac{b\int \frac{(d+ex)^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{3ce}+\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{1892}$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3x}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{3ce}+\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{1803}$$

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^3x^3}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{3ce}$$

$$\Big\downarrow \text{540}$$

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(-\frac{1}{2}\int -\frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2-\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$$\Big\downarrow \text{25}$$

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\int \frac{\left(\frac{2d^3}{x^2}+6e^2d+\frac{e\left(6d^2-\frac{e^2}{c^2}\right)}{x}\right)x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$\Big\downarrow$ 2338

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(-\int -\frac{\left(\frac{2d^3}{x}+e\left(6d^2-\frac{e^2}{c^2}\right)\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$\Big\downarrow$ 25

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(\int \frac{\left(\frac{2d^3}{x}+e\left(6d^2-\frac{e^2}{c^2}\right)\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$\Big\downarrow$ 538

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(2d^3\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}+e\left(6d^2-\frac{e^2}{c^2}\right)\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$\Big\downarrow$ 222

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(e\left(6d^2-\frac{e^2}{c^2}\right)\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}+2cd^3\text{arcsinh}\left(\frac{1}{cx}\right)-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$\Big\downarrow$ 243

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(\frac{1}{2}e\left(6d^2-\frac{e^2}{c^2}\right)\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}+2cd^3\text{arcsinh}\left(\frac{1}{cx}\right)-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$\Big\downarrow$ 73

3.45. $\int (d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(c^2e\left(6d^2-\frac{e^2}{c^2}\right)\int\frac{1}{c^2\sqrt{1+\frac{1}{c^2x^2}}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}+2cd^3\text{arcsinh}\left(\frac{1}{cx}\right)-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

$$\Big\downarrow \; 221$$

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{b\left(\frac{1}{2}\left(2cd^3\text{arcsinh}\left(\frac{1}{cx}\right)-e\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)\left(6d^2-\frac{e^2}{c^2}\right)-6de^2x\sqrt{\frac{1}{c^2x^2}+1}\right)-\frac{1}{2}e^3x^2\sqrt{\frac{1}{c^2x^2}+1}\right)}{3ce}$$

input
```
Int[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
((d + e*x)^3*(a + b*ArcCsch[c*x]))/(3*e) - (b*(-1/2*(e^3*Sqrt[1 + 1/(c^2*x
^2)]*x^2) + (-6*d*e^2*Sqrt[1 + 1/(c^2*x^2)]*x + 2*c*d^3*ArcSinh[1/(c*x)] -
 e*(6*d^2 - e^2/c^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/2))/(3*c*e)
```

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.45. $\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.45.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.52

method	result
parts	$\frac{a(ex+d)^3}{3e}+\frac{b\left(\frac{c\,e^2\,\text{arccsch}(cx)x^3}{3}+ce\,\text{arccsch}(cx)x^2d+\text{arccsch}(cx)xc\,d^2+\frac{c\,\text{arccsch}(cx)d^3}{3e}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\,\text{arctanh}\left(\frac{-}{\sqrt{}}\right.\right.}{}\right.}{c}$
derivativedivides	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\text{arccsch}(cx)c^3d^3}{3e}+\text{arccsch}(cx)c^3d^2x+e\,\text{arccsch}(cx)c^3d\,x^2+\frac{e^2\,\text{arccsch}(cx)c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\,\text{arctanh}\left(\frac{}{\sqrt{c^2}}\right.\right.}{}\right.}{c^2}}{c}$
default	$\frac{\frac{a(cex+cd)^3}{3c^2e}+\frac{b\left(\frac{\text{arccsch}(cx)c^3d^3}{3e}+\text{arccsch}(cx)c^3d^2x+e\,\text{arccsch}(cx)c^3d\,x^2+\frac{e^2\,\text{arccsch}(cx)c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\,\text{arctanh}\left(\frac{}{\sqrt{c^2}}\right.\right.}{}\right.}{c^2}}{c}$

input
```
int((e*x+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*(e*x+d)^3/e+b/c*(1/3*c*e^2*arccsch(c*x)*x^3+c*e*arccsch(c*x)*x^2*d+a
rccsch(c*x)*x*c*d^2+1/3*c/e*arccsch(c*x)*d^3+1/6/c^3/e*(c^2*x^2+1)^(1/2)*(
-2*c^3*d^3*arctanh(1/(c^2*x^2+1)^(1/2))+6*c^2*d^2*e*arcsinh(c*x)+6*c*d*e^2
*(c^2*x^2+1)^(1/2)+e^3*c*x*(c^2*x^2+1)^(1/2)-e^3*arcsinh(c*x))/((c^2*x^2+1
)/c^2/x^2)^(1/2)/x)
```

3.45.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(108) = 216.

Time = 0.31 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.69

$$\int (d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{2\,ac^3e^2x^3+6\,ac^3dex^2+6\,ac^3d^2x+2\left(3\,bc^3d^2+3\,bc^3de+bc^3e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-\left(6\,bc^2d^2-\right.}{}$$

input
```
integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^3 + 6*a*c^3*d*e*x^2 + 6*a*c^3*d^2*x + 2*(3*b*c^3*d^2 +
3*b*c^3*d*e + b*c^3*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1)
- (6*b*c^2*d^2 - b*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(
3*b*c^3*d^2 + 3*b*c^3*d*e + b*c^3*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2
)) - c*x - 1) + 2*(b*c^3*e^2*x^3 + 3*b*c^3*d*e*x^2 + 3*b*c^3*d^2*x - 3*b*c
^3*d^2 - 3*b*c^3*d*e - b*c^3*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + (b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
)/c^3
```

3.45.6 Sympy [F]

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (a+b\,\text{acsch}\,(cx))\,(d+ex)^2 \, dx$$

input
```
integrate((e*x+d)**2*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x)**2, x)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.57

$$\begin{aligned}
&\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx \\
&= \frac{1}{3}ae^2x^3 + adex^2 + \left(x^2\,\text{arcsch}\,(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bde \\
&\quad + \frac{1}{12}\left(4\,x^3\,\text{arcsch}\,(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2 \\
&\quad + ad^2x + \frac{\left(2\,cx\,\text{arcsch}\,(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^2}{2\,c}
\end{aligned}$$

input
```
integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + a*d*e*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)
*b*d*e + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*
x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^
2) + 1) - 1)/c^2)/c)*b*e^2 + a*d^2*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(
1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^2/c
```

3.45.8 Giac [F]

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex+d)^2 (b \operatorname{arcsch}(cx)+a) \, dx$$

input
```
integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(b*arccsch(c*x) + a), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(a+b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^2 \, dx$$

input
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^2,x)
```

output
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^2, x)
```

3.46 $\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 81

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c}-\frac{bd^2\text{csch}^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{bd\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

output
```
-1/2*b*d^2*arccsch(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsch(c*x))/e+b*d*arctanh((
1+1/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1+1/c^2/x^2)^(1/2)/c
```

3.46.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=adx+\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c}+bdx\,\text{csch}^{-1}(cx)+\frac{1}{2}bex^2\text{csch}^{-1}(cx)+\frac{2bd\sqrt{1+\frac{1}{c^2x^2}}\,x\,\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

input
```
Integrate[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

output
```
a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*
ArcCsch[c*x] + (b*e*x^2*ArcCsch[c*x])/2 + (2*b*d*Sqrt[1 + 1/(c^2*x^2)]*x*A
rcTanh[(-1 + Sqrt[1 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

3.46.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6844, 1892, 1730, 540, 25, 27, 538, 222, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex) \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 6844$$

$$\frac{b\int \frac{(d+ex)^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{2ce}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \; 1892$$

$$\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{2ce}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}$$

$$\Big\downarrow \; 1730$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\int \frac{\left(\frac{d}{x}+e\right)^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{2ce}$$

$$\Big\downarrow \; 540$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(e^2x\left(-\sqrt{\frac{1}{c^2x^2}+1}\right)-\int -\frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)}{2ce}$$

$$\Big\downarrow \; 25$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(\int \frac{d\left(\frac{d}{x}+2e\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

$$\Big\downarrow \ 27$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(d\int \frac{\left(\frac{d}{x}+2e\right)x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

$$\Big\downarrow \ 538$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(d\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}+2e\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

$$\Big\downarrow \ 222$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(2e\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}+cd\text{arcsinh}\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

$$\Big\downarrow \ 243$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(e\int \frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x^2}+cd\text{arcsinh}\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

$$\Big\downarrow \ 73$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(2c^2e\int \frac{1}{c^2\sqrt{1+\frac{1}{c^2x^2}}-c^2}d\sqrt{1+\frac{1}{c^2x^2}}+cd\text{arcsinh}\left(\frac{1}{cx}\right)\right)-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

$$\Big\downarrow \ 221$$

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}-\frac{b\left(d\left(cd\text{arcsinh}\left(\frac{1}{cx}\right)-2e\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)\right)-e^2x\sqrt{\frac{1}{c^2x^2}+1}\right)}{2ce}$$

input
```
Int[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

output
```
((d + e*x)^2*(a + b*ArcCsch[c*x]))/(2*e) - (b*(-(e^2*Sqrt[1 + 1/(c^2*x^2)]
*x) + d*(c*d*ArcSinh[1/(c*x)] - 2*e*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])))/(2*c
*e)
```

3.46. $\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 540
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[(c + d*x)^n, x, x], R = PolynomialRemain
der[(c + d*x)^n, x, x]}, Simp[R*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1)))
, x] + Simp[1/(a*(m + 1))   Int[x^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*(m +
1)*Qx - b*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IG
tQ[n, 1] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 1730
```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.46.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

method	result	size
parts	$a\left(\frac{1}{2}e\,x^2+dx\right)+\frac{b\left(\frac{c\ \mathrm{arccsch}(cx)e\,x^2}{2}+\mathrm{arccsch}(cx)dxc+\frac{\sqrt{c^2x^2+1}\left(2dc\ \mathrm{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2c^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x}\right)}{c}$	98
derivativedivides	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\mathrm{arccsch}(cx)d\,c^2x+\frac{\mathrm{arccsch}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2+1}\left(2dc\ \mathrm{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c}}{c}$	115
default	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\mathrm{arccsch}(cx)d\,c^2x+\frac{\mathrm{arccsch}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2+1}\left(2dc\ \mathrm{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c}}{c}$	115

input
```
int((e*x+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e*x^2+d*x)+b/c*(1/2*c*arccsch(c*x)*e*x^2+arccsch(c*x)*d*x*c+1/2/c^2
/((c^2*x^2+1)/c^2/x^2)^(1/2)/x*(c^2*x^2+1)^(1/2)*(2*d*c*arcsinh(c*x)+e*(c^
2*x^2+1)^(1/2)))
```

3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(71) = 142.

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.56

$$\int (d+ex) \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= \frac{acex^2 + 2\,acdx + bex\sqrt{\frac{c^2x^2+1}{c^2x^2}} - 2\,bd\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (2\,bcd + bce)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) -}{2\,c}$$

input
```
integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/2*(a*c*e*x^2 + 2*a*c*d*x + b*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 2*b*d*l
og(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + (2*b*c*d + b*c*e)*log(c*x*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (2*b*c*d + b*c*e)*log(c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*e*x^2 + 2*b*c*d*x - 2*b*c*d - b*c
*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/c
```

3.46.6 Sympy [F]

$$\int (d+ex) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (a + b\,\text{acsch}\,(cx))\,(d+ex) \, dx$$

input
```
integrate((e*x+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x), x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int (d+ex) \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= \frac{1}{2}aex^2 + \frac{1}{2}\left(x^2\,\text{arcsch}\,(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)be + adx$$
$$+ \frac{\left(2\,cx\,\text{arcsch}\,(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd}{2\,c}$$

input
```
integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b*e + a*d
*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b*d/c
```

3.46.8 Giac [F]

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (ex+d)(b\,\text{arcsch}\,(cx)+a)\,dx$$

input
```
integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)*(b*arccsch(c*x) + a), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,(d+e\,x)\,dx$$

input
```
int((a + b*asinh(1/(c*x)))*(d + e*x),x)
```

output
```
int((a + b*asinh(1/(c*x)))*(d + e*x), x)
```

3.47 $\int (a + b\text{csch}^{-1}(cx)) \, dx$

3.47.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a + b\text{csch}^{-1}(cx)) \, dx = ax + bx\text{csch}^{-1}(cx) + \frac{b\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

output
```
a*x+b*x*arccsch(c*x)+b*arctanh((1+1/c^2/x^2)^(1/2))/c
```

3.47.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int (a + b\text{csch}^{-1}(cx)) \, dx = ax + bx\text{csch}^{-1}(cx) + \frac{2b\sqrt{1+\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

input
```
Integrate[a + b*ArcCsch[c*x],x]
```

output
```
a*x + b*x*ArcCsch[c*x] + (2*b*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(-1 + Sqrt[1
 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

3.47.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\text{csch}^{-1}(cx)\right) dx$$

$$\Big\downarrow \; 2009$$

$$ax + \frac{b\text{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c} + bx\text{csch}^{-1}(cx)$$

input
```
Int[a + b*ArcCsch[c*x],x]
```

output
```
a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.47.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

method	result	size
default	$ax + bx \operatorname{arccsch}(cx) + \frac{b\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
parts	$ax + bx \operatorname{arccsch}(cx) + \frac{b\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
derivativedivides	$\frac{acx+b\left(cx \operatorname{arccsch}(cx)+\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)\right)}{c}$	39

input
```
int(a+b*arccsch(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

3.47.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.77

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{acx + bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) - b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (bcx -}{c}$$

input
```
integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

output
```
(a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)))/c
```

3.47.6 Sympy [F]

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx = \int \left(a + b\,\text{acsch}\,(cx)\right)\,dx$$

input
```
integrate(a+b*acsch(c*x),x)
```

output
```
Integral(a + b*acsch(c*x), x)
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

$$\int \left(a + b\text{csch}^{-1}(cx)\right)\,dx$$
$$= ax + \frac{\left(2\,cx\,\text{arcsch}\,(cx) + \log\left(\sqrt{\frac{1}{c^2x^2} + 1} + 1\right) - \log\left(\sqrt{\frac{1}{c^2x^2} + 1} - 1\right)\right)b}{2\,c}$$

input
```
integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(
1/(c^2*x^2) + 1) - 1))*b/c
```

3.47.8 Giac [F]

$$\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int b\,\text{arcsch}\,(cx)+a\,dx$$

input
```
integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

output
```
integrate(b*arccsch(c*x) + a, x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\,dx$$

input
```
int(a + b*asinh(1/(c*x)),x)
```

output
```
int(a + b*asinh(1/(c*x)), x)
```

3.48 $\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx$

3.48.1 Optimal result

Integrand size = 16, antiderivative size = 215

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\,\text{PolyLog}\left(2,\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2e}$$

output
```
-(a+b*arccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsch(c*x
))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))*(e-(c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*ar
ccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))*(e+(c^2*d^2+e^2)^(1/2))/c/d)/
e-1/2*b*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e+b*polylog(2,(1/c/x+(1+1
/c^2/x^2)^(1/2))*(e-(c^2*d^2+e^2)^(1/2))/c/d)/e+b*polylog(2,(1/c/x+(1+1/c^
2/x^2)^(1/2))*(e+(c^2*d^2+e^2)^(1/2))/c/d)/e
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.35

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \frac{a\log(d+ex)}{e} + \frac{b\left(\pi^2 - 4i\pi\text{csch}^{-1}(cx) - 8\text{csch}^{-1}(cx)^2 - 32\arcsin\left(\frac{\sqrt{1+\frac{ie}{cd}}}{\sqrt{2}}\right)\arctan\left(\frac{(icd+e)\cot\left(\frac{1}{4}\left(\pi+2i\text{csch}^{-1}(cx)\right)\right)}{\sqrt{c^2d^2+e^2}}\right) - 8c\right.}{}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

output
```
(a*Log[d + e*x])/e + (b*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 -
 32*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*ArcTan[((I*c*d + e)*Cot[(Pi + (2
*I)*ArcCsch[c*x])/4])/Sqrt[c^2*d^2 + e^2]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*
ArcCsch[c*x])] + (4*I)*Pi*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*
x])/(c*d)] + 8*ArcCsch[c*x]*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[
c*x])/(c*d)] + (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*Log[1 + ((-e +
 Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + (4*I)*Pi*Log[1 - ((e + Sqrt
[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*ArcCsch[c*x]*Log[1 - ((e + Sqr
t[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (16*I)*ArcSin[Sqrt[1 + (I*e)/(c
*d)]/Sqrt[2]]*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] -
(4*I)*Pi*Log[e + d/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (
(e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*PolyLog[2, ((e + Sqrt
[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)]))/(8*e)
```

3.48.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6843, 2998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx$$

$$\Big\downarrow \text{6843}$$

$$\frac{b\int \frac{\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{b\int \frac{\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{ce}-\frac{b\int \frac{\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \text{2998}$$

$$\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{b\,\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{b\,\text{PolyLog}\left(2,\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{b\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)}{2e}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

output
```
((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(
c*d)])/e + ((a + b*ArcCsch[c*x])*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcC
sch[c*x])/(c*d)])/e - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e
 + (b*PolyLog[2, ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e + (b
*PolyLog[2, ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e - (b*Poly
Log[2, E^(2*ArcCsch[c*x])])/(2*e)
```

3.48.3.1 Defintions of rubi rules used

rule 2998
```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

rule 6843
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[(a + b*ArcCsch[c*x])*(Log[1 - (e - Sqrt[c^2*d^2 + e^2])*(E^ArcCsch[c*x
]/(c*d))]/e), x] + (Simp[(a + b*ArcCsch[c*x])*(Log[1 - (e + Sqrt[c^2*d^2 +
e^2])*(E^ArcCsch[c*x]/(c*d))]/e), x] - Simp[(a + b*ArcCsch[c*x])*(Log[1 - E
^(2*ArcCsch[c*x])]/e), x] + Simp[b/(c*e)   Int[Log[1 - (e - Sqrt[c^2*d^2 +
e^2])*(E^ArcCsch[c*x]/(c*d))]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Simp[b/
(c*e)   Int[Log[1 - (e + Sqrt[c^2*d^2 + e^2])*(E^ArcCsch[c*x]/(c*d))]/(x^2*
Sqrt[1 + 1/(c^2*x^2)]), x], x] - Simp[b/(c*e)   Int[Log[1 - E^(2*ArcCsch[c*
x])]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]) /; FreeQ[{a, b, c, d, e}, x]
```

3.48.4 Maple [F]

$$\int \frac{a + b\ \mathrm{arccsch}\left(cx\right)}{ex + d}dx$$

input
```
int((a+b*arccsch(c*x))/(e*x+d),x)
```

output
```
int((a+b*arccsch(c*x))/(e*x+d),x)
```

3.48.5 Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e*x + d), x)
```

3.48.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x), x)
```

3.48.7 Maxima [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x + d), x) + a*log(e*x
 + d)/e
```

3.48.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{d + ex} \, dx = \int \frac{b \,\text{arcsch}\,(cx) + a}{ex + d} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x + d), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{d + ex} \, dx = \int \frac{a + b \,\text{asinh}\left(\frac{1}{c\,x}\right)}{d + e\,x} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x), x)
```

3.49 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx$

3.49.1 Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx=\frac{b\text{csch}^{-1}(cx)}{de}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}+\frac{b\text{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2+e^2}}$$

output
```
b*arccsch(c*x)/d/e+(-a-b*arccsch(c*x))/e/(e*x+d)+b*arctanh((c^2*d-e/x)/c/(
c^2*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d/(c^2*d^2+e^2)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx=-\frac{a}{e(d+ex)}-\frac{b\text{csch}^{-1}(cx)}{e(d+ex)}+\frac{b\text{arcsinh}\left(\frac{1}{cx}\right)}{de}+\frac{b\log(d+ex)}{d\sqrt{c^2d^2+e^2}}-\frac{b\log\left(e+c\left(-cd+\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2+e^2}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

output
```
-(a/(e*(d + e*x))) - (b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcSinh[1/(c*x)])
/(d*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 + e^2]) - (b*Log[e + c*(-(c*d) +
 Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 + e^2])
```

3.49.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6844, 1892, 1803, 605, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 6844}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)}dx}{ce}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 1892}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x^3}dx}{ce}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)x}d\frac{1}{x}}{ce}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 605}$$

$$\frac{b\left(\frac{\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{d}-\frac{e\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{b\left(\frac{c\,\text{arcsinh}\left(\frac{1}{cx}\right)}{d}-\frac{e\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{d}\right)}{ce}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{b\left(\frac{e\int \frac{1}{d^2+\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d-\frac{e}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}}{d}+\frac{c\,\text{arcsinh}\left(\frac{1}{cx}\right)}{d}\right)}{ce}-\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\left(\frac{c\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d}+\frac{ce\operatorname{arctanh}\left(\frac{c\left(d-\frac{e}{c^2x}\right)}{\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{d\sqrt{c^2d^2+e^2}}\right)}{ce}-\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

output
```
-((a + b*ArcCsch[c*x])/(e*(d + e*x))) + (b*((c*ArcSinh[1/(c*x)])/d + (c*e*
ArcTanh[(c*(d - e/(c^2*x)))/(Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])/
(d*Sqrt[c^2*d^2 + e^2])))/(c*e)
```

3.49.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 605
```
Int[((x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)), x_Symbol]
:> Simp[1/d   Int[x^(m - 1)*(a + b*x^2)^p, x], x] - Simp[c/d   Int[x^(m - 1
)*((a + b*x^2)^p/(c + d*x)), x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m,
 0] && LtQ[-1, p, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.49.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.88

method	result
parts	$-\frac{a}{(ex+d)e}+\frac{b\left(-\frac{c^{2}\,\mathrm{arccsch}(cx)}{(cex+cd)e}+\frac{\sqrt{c^{2}x^{2}+1}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}-\ln\left(\frac{2\sqrt{c^{2}x^{2}+1}\,\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}\,e-2d\,c^{2}x+2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,xd\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}}\right)}{c}$
derivativedivides	$\frac{-\frac{a\,c^{2}}{(cex+cd)e}+b\,c^{2}\left(-\frac{\mathrm{arccsch}(cx)}{(cex+cd)e}+\frac{\sqrt{c^{2}x^{2}+1}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}-\ln\left(\frac{2\sqrt{c^{2}x^{2}+1}\,\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}\,e-2d\,c^{2}x+2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,c^{2}xd\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}}\right)}{c}$
default	$\frac{-\frac{a\,c^{2}}{(cex+cd)e}+b\,c^{2}\left(-\frac{\mathrm{arccsch}(cx)}{(cex+cd)e}+\frac{\sqrt{c^{2}x^{2}+1}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}-\ln\left(\frac{2\sqrt{c^{2}x^{2}+1}\,\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}\,e-2d\,c^{2}x+2e}{cex+cd}\right)\right)}{e\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,c^{2}xd\sqrt{\frac{c^{2}d^{2}+e^{2}}{e^{2}}}}\right)}{c}$

input
```
int((a+b*arccsch(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/(e*x+d)/e+b/c*(-c^2/(c*e*x+c*d)/e*arccsch(c*x)+1/e*(c^2*x^2+1)^(1/2)*(a
rctanh(1/(c^2*x^2+1)^(1/2))*((c^2*d^2+e^2)/e^2)^(1/2)-ln(2*((c^2*x^2+1)^(1
/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d)))/((c^2*x^2+1)/c^2/
x^2)^(1/2)/x/d/((c^2*d^2+e^2)/e^2)^(1/2))
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(92) = 184$.

Time = 0.29 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.61

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx =$$

$$-\frac{ac^2d^3+ade^2-\sqrt{c^2d^2+e^2}(be^2x+bde)\log\left(-\frac{c^3d^2x-cde+(c^3d^2+ce^2)x\sqrt{\frac{c^2x^2+1}{c^2x^2}}+\left(c^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+c^2dx-e\right)\sqrt{c^2d^2+e^2}}{ex+d}\right.}{}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

output
```
-(a*c^2*d^3 + a*d*e^2 - sqrt(c^2*d^2 + e^2)*(b*e^2*x + b*d*e)*log(-(c^3*d^
2*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + (c^2*d*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 + e^2))/(e*x +
d)) - (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3)*x)*log(c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^3 + b*d*e^2 + (b*c^2*d^2*e + b*e^3
)*x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^2*d^3 + b*d*e
^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e + d^2*e
^3 + (c^2*d^3*e^2 + d*e^4)*x)
```

3.49.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{(d+ex)^2}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d)**2,x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x)**2, x)
```

3.49.7 Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex + d)^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c^2*integrate(x/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e + (c^2*e^
2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt(c^2*x^2 + 1)), x) + I*c*(log(I*c*x
 + 1) - log(-I*c*x + 1))/(c^2*d^2 + e^2) - 2*e*log(e*x + d)/(c^2*d^3 + d*e
^2) - (2*c^2*d^3*log(c) + 2*d*e^2*log(c) - 2*(c^2*d^2*e + e^3)*x*log(x) +
(c^2*d^2*e*x + c^2*d^3)*log(c^2*x^2 + 1) - 2*(c^2*d^3 + d*e^2)*log(sqrt(c^
2*x^2 + 1) + 1))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d*e^4)*x))*b - a/(e
^2*x + d*e)
```

3.49.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex + d)^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x + d)^2, x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^2}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^2}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^2,x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^2, x)
```

3.50 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3} dx$

3.50.1 Optimal result

Integrand size = 16, antiderivative size = 163

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3} dx = -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}+\frac{b\operatorname{csch}^{-1}(cx)}{2d^2e}-\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2}+\frac{b(2c^2d^2+e^2)\operatorname{arctanh}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}$$

output
```
1/2*b*arccsch(c*x)/d^2/e+1/2*(-a-b*arccsch(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*
d^2+e^2)*arctanh((c^2*d-e/x)/c/(c^2*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^
2/(c^2*d^2+e^2)^(3/2)-1/2*b*c*e*(1+1/c^2/x^2)^(1/2)/d/(c^2*d^2+e^2)/(e+d/x
)
```

3.50.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.25

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx = \frac{1}{2}\left(-\frac{a}{e(d+ex)^2}-\frac{bce\sqrt{1+\frac{1}{c^2x^2}}x}{d\left(c^2d^2+e^2\right)(d+ex)}-\frac{b\operatorname{csch}^{-1}(cx)}{e(d+ex)^2}+\frac{b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d^2e}+\frac{b(2c^2d^2+e^2)\log(d+ex)}{d^2\left(c^2d^2+e^2\right)^{3/2}}-\frac{b(2c^2d^2+e^2)\log\left(e+c\left(-cd+\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{d^2\left(c^2d^2+e^2\right)^{3/2}}\right)$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

output
```
(-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d^2 + e^2)
*(d + e*x)) - (b*ArcCsch[c*x])/(e*(d + e*x)^2) + (b*ArcSinh[1/(c*x)])/(d^2
*e) + (b*(2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c^2*d^2 + e^2)^(3/2)) - (b*
(2*c^2*d^2 + e^2)*Log[e + c*(-(c*d) + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*
x^2)])*x])/(d^2*(c^2*d^2 + e^2)^(3/2)))/2
```

3.50.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6844, 1892, 1803, 603, 719, 222, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{6844}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^2}dx}{2ce}-\frac{a+b\operatorname{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1892}$$

$$-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^4}dx}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 1803}$$

$$\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)^2x^2}d\frac{1}{x}}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 603}$$

$$\frac{b\left(-\frac{c^2\int \frac{e-\frac{\frac{e^2}{c^2d}+d}{x}}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}}{c^2d^2+e^2}-\frac{c^2e^2\sqrt{\frac{1}{c^2x^2}+1}}{d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{b\left(-\frac{c^2\left(e\left(\frac{e^2}{c^2d^2}+2\right)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-\left(\frac{e^2}{c^2d^2}+1\right)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}\right)}{c^2d^2+e^2}-\frac{c^2e^2\sqrt{\frac{1}{c^2x^2}+1}}{d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{b\left(-\frac{c^2\left(e\left(\frac{e^2}{c^2d^2}+2\right)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(\frac{d}{x}+e\right)}d\frac{1}{x}-c\,\text{arcsinh}\left(\frac{1}{cx}\right)\left(\frac{e^2}{c^2d^2}+1\right)\right)}{c^2d^2+e^2}-\frac{c^2e^2\sqrt{\frac{1}{c^2x^2}+1}}{d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{b\left(-\frac{c^2\left(-e\left(\frac{e^2}{c^2d^2}+2\right)\int \frac{1}{d^2+\frac{e^2}{c^2}-\frac{1}{x^2}}d\frac{d-\frac{e}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}-c\,\text{arcsinh}\left(\frac{1}{cx}\right)\left(\frac{e^2}{c^2d^2}+1\right)\right)}{c^2d^2+e^2}-\frac{c^2e^2\sqrt{\frac{1}{c^2x^2}+1}}{d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 219}$$

3.50. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx$

$$\frac{b\left(-\frac{c^2\left(-c\operatorname{arcsinh}\left(\frac{1}{cx}\right)\left(\frac{e^2}{c^2d^2}+1\right)-\frac{ce\left(\frac{e^2}{c^2d^2}+2\right)\operatorname{arctanh}\left(\frac{c\left(d-\frac{e}{c^2x}\right)}{\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{\sqrt{c^2d^2+e^2}}\right)}{c^2d^2+e^2}-\frac{c^2e^2\sqrt{\frac{1}{c^2x^2}+1}}{d(c^2d^2+e^2)\left(\frac{d}{x}+e\right)}\right)}{2ce}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

output
```
-1/2*(a + b*ArcCsch[c*x])/(e*(d + e*x)^2) + (b*(-((c^2*e^2*Sqrt[1 + 1/(c^2
*x^2)])/(d*(c^2*d^2 + e^2)*(e + d/x))) - (c^2*(-(c*(1 + e^2/(c^2*d^2))*Arc
Sinh[1/(c*x)]) - (c*e*(2 + e^2/(c^2*d^2))*ArcTanh[(c*(d - e/(c^2*x)))/(Sqr
t[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])/Sqrt[c^2*d^2 + e^2]))/(c^2*d^2 +
 e^2)))/(2*c*e)
```

3.50.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 603
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> With[{Qx = PolynomialQuotient[x^m, c + d*x, x], R = PolynomialRemainde
r[x^m, c + d*x, x]}, Simp[d*R*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n +
1)*(b*c^2 + a*d^2))), x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)
^(n + 1)*(a + b*x^2)^p*ExpandToSum[(n + 1)*(b*c^2 + a*d^2)*Qx + b*c*R*(n +
1) - b*d*R*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, p}, x] && IGt
Q[m, 1] && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1803
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x
)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
 EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1892
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.50. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3} dx$

3.50.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(150) = 300$.

Time = 2.62 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.34

method	result
parts	$-\frac{a}{2(ex+d)^2 e}+\frac{b\left(-\frac{c^3\operatorname{arccsch}(cx)}{2(cex+cd)^2 e}-\frac{\sqrt{c^2x^2+1}\left(-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^3-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3 \cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\operatorname{arccsch}(cx)}{2(cex+cd)^2 e}-\frac{\sqrt{c^2x^2+1}\left(-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^3-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3 \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a c^3}{2(cex+cd)^2 e}+b c^3\left(-\frac{\operatorname{arccsch}(cx)}{2(cex+cd)^2 e}-\frac{\sqrt{c^2x^2+1}\left(-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3d^3-\sqrt{\frac{c^2d^2+e^2}{e^2}}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^3 \cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((a+b*arccsch(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/(e*x+d)^2/e+b/c*(-1/2*c^3/(c*e*x+c*d)^2/e*arccsch(c*x)-1/2/e*(c^2*x
^2+1)^(1/2)*(-((c^2*d^2+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*c^3*d
^3-((c^2*d^2+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*c^3*d^2*e*x+2*ln
(2*((c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))*
c^3*d^3+2*ln(2*((c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(
c*e*x+c*d))*c^3*d^2*e*x+(c^2*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*c*d*e^
2-((c^2*d^2+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*c*d*e^2-((c^2*d^2
+e^2)/e^2)^(1/2)*arctanh(1/(c^2*x^2+1)^(1/2))*e^3*c*x+ln(2*((c^2*x^2+1)^(1
/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))*c*d*e^2+ln(2*((c^2
*x^2+1)^(1/2)*((c^2*d^2+e^2)/e^2)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))*e^3*c*x)
/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d^2/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)
/(c*e*x+c*d))
```

3.50.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(147) = 294$.

Time = 0.45 (sec) , antiderivative size = 745, normalized size of antiderivative = 4.57

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^3}\, dx =$$

$$-\frac{ac^4d^6 + bc^3d^5e + 2\,ac^2d^4e^2 + bcd^3e^3 + ad^2e^4 + (bc^3d^3e^3 + bcde^5)x^2 - (2\,bc^2d^4e + bd^2e^3 + (2\,bc^2d^2e^3 + b}{}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/2*(a*c^4*d^6 + b*c^3*d^5*e + 2*a*c^2*d^4*e^2 + b*c*d^3*e^3 + a*d^2*e^4
+ (b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 - (2*b*c^2*d^4*e + b*d^2*e^3 + (2*b*c^2*
d^2*e^3 + b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 + b*d*e^4)*x)*sqrt(c^2*d^2 + e^2
)*log(-(c^3*d^2*x - c*d*e + (c^3*d^2 + c*e^2)*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + (c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + c^2*d*x - e)*sqrt(c^2*d^2 +
 e^2))/(e*x + d)) + 2*(b*c^3*d^4*e^2 + b*c*d^2*e^4)*x - (b*c^4*d^6 + 2*b*c
^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2
*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*log(c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) - c*x + 1) + (b*c^4*d^6 + 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d
^4*e^2 + 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e + 2*b*c^2*d^3*e^3 +
 b*d*e^5)*x)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^4*d^6
 + 2*b*c^2*d^4*e^2 + b*d^2*e^4)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) + ((b*c^3*d^3*e^3 + b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 + b*c*d^2*e^4
)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4*e^5 +
 (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*
e^4 + d^3*e^6)*x)
```

3.50.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^3}\, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{(d + ex)^3}\, dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d)**3,x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x)**3, x)
```

3.50.7 Maxima [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*I*c^3*d*(log(I*c*x + 1) - log(-I*c*x + 1))/(c^4*d^4 + 2*c^2*d^2*e^
2 + e^4) + 4*c^2*integrate(1/2*x/(c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*
x + d^2*e + (c^2*d^2*e + e^3)*x^2 + (c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e
^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(c^2*x^2 + 1)), x) - 2*(3*c^2*d^
2*e + e^3)*log(e*x + d)/(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4) - (2*c^4*d^6*l
og(c) + 2*d^2*e^4*log(c) - 2*d^2*e^4 + 2*(2*d^4*e^2*log(c) - d^4*e^2)*c^2
- 2*(c^2*d^3*e^3 + d*e^5)*x + (c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*
d^2*e^4)*x^2 + 2*(c^4*d^5*e - c^2*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*((c^4*d
^4*e^2 + 2*c^2*d^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*
x)*log(x) - 2*(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4)*log(sqrt(c^2*x^2 + 1) +
1))/(c^4*d^8*e + 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 +
d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x))*b - 1/2*a/(e^
3*x^2 + 2*d*e^2*x + d^2*e)
```

3.50.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x + d)^3, x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{\left(d+e\,x\right)^3}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^3,x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^3, x)
```

3.51 $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 918

$$\begin{aligned}
&\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx\\
&=-\frac{4bd\sqrt{d+ex}(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{4b(d+ex)^{3/2}\left(1+c^2x^2\right)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}\\
&-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}\\
&-\frac{32bcd^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}\\
&-\frac{4bc(c^2d^2-3e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}\\
&+\frac{32bcd^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&-\frac{4bcd(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}\\
&-\frac{32bd^4\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arcc
sch(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsch(c*x))/e^3+4/35*b*(c^2*x^2+1)*
(e*x+d)^(1/2)/c^3/(1+1/c^2/x^2)^(1/2)+8/105*b*d*(c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/e/x/(1+1/c^2/x^2)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(1-x*(-c^2)^(1/2))
^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*(
(e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^2/x^2)^(1/2)
/(e*x+d)^(1/2)-4/35*b*c*d^2*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2)
,(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+
1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^
2)^(1/2)))^(1/2)+4/105*b*c*(2*c^2*d^2+9*e^2)*EllipticE(1/2*(1-x*(-c^2)^(1/
2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d
)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(5/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x
+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+32/105*b*c*d^3*EllipticF(1/2*(1-x*(-c^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c
^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^
2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/105*b*c*d*(c^2*d^2+e^2)*EllipticF(
1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/
(-c^2)^(5/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.51.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 45.60 (sec) , antiderivative size = 1094, normalized size of antiderivative = 1.19

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=-\frac{ad^3\sqrt{d+ex}B_{-\frac{ex}{d}}\left(3,\frac{3}{2}\right)}{e^3\sqrt{1+\frac{ex}{d}}}$$

$$+ b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4\left(5c^2d^2+9e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{105e^2}-\frac{16c^3d^3\operatorname{csch}^{-1}(cx)}{105e^3}-\frac{2}{7}c^3x^3\operatorname{csch}^{-1}(cx)-\frac{2c^2x^2\left(2e\sqrt{1+\frac{1}{c^2x^2}}+cd\operatorname{csch}^{-1}(cx)\right)}{35e}-\frac{8cx\left(cde\sqrt{1+\frac{1}{c^2x^2}}\right.}{1}\right.}{\sqrt{d+ex}}\right.$$

input
```
Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

output
```
-((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d]))
+ (b*(-((c*(e + d/x)*x*((4*(5*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105
*e^2) - (16*c^3*d^3*ArcCsch[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsch[c*x])/7 -
 (2*c^2*x^2*(2*e*Sqrt[1 + 1/(c^2*x^2)] + c*d*ArcCsch[c*x]))/(35*e) - (8*c*
x*(c*d*e*Sqrt[1 + 1/(c^2*x^2)] - c^2*d^2*ArcCsch[c*x]))/(105*e^2)))/Sqrt[d
 + e*x]) - (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(9*c^3*d^3*e + c*d*e^3)*
Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin
[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2
*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*
Sqrt[2]*(c*d - I*e)*(8*c^4*d^4 - 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[1 + I*c*x]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Ar
cSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 +
1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-5*c^3*d^3*e - 9*c*d*e^3)*Co
sh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2
*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x)
)/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ell
ipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] -
 I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d +
 I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I...
```

3.51.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2125 vs. 2(918) = 1836.

Time = 4.22 (sec) , antiderivative size = 2125, normalized size of antiderivative = 2.31, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.952$, Rules used = {6864, 27, 7272, 2351, 634, 599, 27, 631, 687, 27, 687, 27, 599, 27, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6864}$$

$$\frac{b\int \frac{2(d+ex)^{3/2}\left(8d^2-12exd+15e^2x^2\right)}{105e^3\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\int \frac{(d+ex)^{3/2}\left(8d^2-12exd+15e^2x^2\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{105ce^3}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 7272$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{(d+ex)^{3/2}\left(8d^2-12exd+15e^2x^2\right)}{x\sqrt{c^2x^2+1}}dx}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 2351$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx+\int \frac{(d+ex)^{3/2}(15e^2x-12de)}{\sqrt{c^2x^2+1}}dx\right)}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 634$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx-\int \frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)+\int \frac{(d+ex)^{3/2}(15e^2x-12de)}{\sqrt{c^2x^2+1}}dx\right)}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 599$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\frac{2\int \frac{e^2(2d+ex)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e^2}+d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)+\int \frac{(d+ex)^{3/2}(15e^2x-12de)}{\sqrt{c^2x^2+1}}dx\right)}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 27$$

3.51. $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)+\int \frac{(d+ex)^{3/2}(15e^2x-12de)}{\sqrt{c^2x^2+1}}d\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 631

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-2d^2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 687

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{2\int -\frac{15e\sqrt{d+ex}\left(4d^2c^2+dexc^2+3e^2\right)}{2\sqrt{c^2x^2+1}}dx}{5c^2}+8d^2\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-2d^2\int -\frac{}{ex\sqrt{\frac{(d+ex)^2}{e^2}}}\right.\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 27

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{3e\int \frac{\sqrt{d+ex}\left(4d^2c^2+dexc^2+3e^2\right)}{\sqrt{c^2x^2+1}}dx}{c^2}+8d^2\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-2d^2\int -\frac{}{ex\sqrt{\frac{(d+ex)^2c}{e^2}}}\right.\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 687

3.51. $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{3e\left(\frac{2\int\frac{c^2\left(4d\left(3c^2d^2+2e^2\right)+e\left(13c^2d^2+9e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{2}{3}de\sqrt{c^2x^2+1}\sqrt{d+ex}\right)}{c^2}+8d^2\left(2\int\frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\right.\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{3e\left(\frac{1}{3}\int\frac{4d\left(3c^2d^2+2e^2\right)+e\left(13c^2d^2+9e^2\right)x}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx+\frac{2}{3}de\sqrt{c^2x^2+1}\sqrt{d+ex}\right)}{c^2}+8d^2\left(2\int\frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d}\right.\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{599}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{3e\left(\frac{2}{3}de\sqrt{c^2x^2+1}\sqrt{d+ex}-\frac{2\int\frac{e\left(d\left(c^2d^2+e^2\right)-\left(13c^2d^2+9e^2\right)(d+ex)\right)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{3e^2}\right)}{c^2}+8d^2\left(2\int\frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}}}\right.\right.}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \text{27}$$

3.51. $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{3e\left(\frac{2}{3}de\sqrt{c^2x^2+1}\sqrt{d+ex}-\frac{2\int \frac{d\left(c^2d^2+e^2\right)-\left(13c^2d^2+9e^2\right)(d+ex)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{3e}\right)}{c^2}+8d^2\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+\cdots}}\cdots\right)\cdots\right)}{105ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 1511

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{3e\left(\frac{2}{3}de\sqrt{c^2x^2+1}\sqrt{d+ex}-\frac{2\left(\frac{\sqrt{c^2d^2+e^2}\left(13c^2d^2+9e^2\right)\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\left(-cd\sqrt{c^2d^2+e^2}\cdots\right)}{\cdots}\cdots\right)}{3e}\cdots\right)}{c^2}\cdots\right)}{\cdots}$$

$$\frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 1416

3.51. $\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^3}-\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+\ldots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.}{\ldots}$$

$\downarrow$ 1509

3.51. $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))(d+ex)^{7/2}}{7e^3}-\frac{4d(a+b\operatorname{csch}^{-1}(cx))(d+ex)^{5/2}}{5e^3}+\frac{2d^2(a+b\operatorname{csch}^{-1}(cx))(d+ex)^{3/2}}{3e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(8\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\ \text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

$\downarrow$ 1540

3.51. $\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^3}-\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(8\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

$\downarrow$ 1416

3.51. $\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^3}-\frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}+\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.}{\cdots}$$

$\downarrow$ 2222

3.51. $\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^3}-\frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}+\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(8\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+}\cdots}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

3.51. $\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

input
```
Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

output
```
(2*d^2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2
)*(a + b*ArcCsch[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))
/(7*e^3) + (2*b*Sqrt[1 + c^2*x^2]*((6*e^2*(d + e*x)^(3/2)*Sqrt[1 + c^2*x^2
])/c^2 - (3*e*((2*d*e*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2])/3 - (2*((Sqrt[c^2*d
^2 + e^2]*(13*c^2*d^2 + 9*e^2)*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 -
(2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2)*(1
+ (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d
 + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x)
)/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqr
t[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2
+ e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2*
d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/c - ((c^2*d
^2 + e^2)^(3/4)*(13*c^2*d^2 + 9*e^2 - c*d*Sqrt[c^2*d^2 + e^2])*(1 + (c*(d
+ e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))
/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt
[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 +
 e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[1 + (c^2
*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/(3*e)))/c^
2 + 8*d^2*(2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/
e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)...
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.51. $\int x^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.51.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.94 (sec) , antiderivative size = 2515, normalized size of antiderivative = 2.74

method	result	size
derivativedivides	Expression too large to display	2515
default	Expression too large to display	2515
parts	Expression too large to display	2518

input
```
int(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(
1/7*arccsch(c*x)*(e*x+d)^(7/2)-2/5*arccsch(c*x)*d*(e*x+d)^(5/2)+1/3*arccsc
h(c*x)*d^2*(e*x+d)^(3/2)+2/105/c^4*(-7*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)
*c^3*d*e*(e*x+d)^(5/2)-3*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d*(e*x+d)^(
7/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e*(e*x+d)^(1/2)+7*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^2*(e*x+d)^(5/2)+5*I*((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2)*c^3*d^2*e*(e*x+d)^(3/2)-4*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*
d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-5*(-(I*c*e*(e*
x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d
*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*c^4*d^4-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^3*(e*x+d)^(1/2)+8*(-(I*
c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d
)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)
*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*
d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4+I*(-(
I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x
+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(...
```

3.51.5 Fricas [F]

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x + d), x)
```

3.51.6 Sympy [F]

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))*sqrt(d + e*x), x)
```

3.51.7 Maxima [F]

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
-1/11025*(1157625*c^2*e^3*integrate(1/105*sqrt(e*x + d)*x^4*log(x)/(c^2*e^
3*x^2 + e^3), x) + 1680*c^2*d^3*(integrate(((e*x + d)*c^2*d - c^2*d^2 - e^
2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)),
x)/c^2 + 2*sqrt(e*x + d)/(c^2*e))/e^2 + 1157625*e^3*integrate(1/105*sqrt(e
*x + d)*x^2*log(x)/(c^2*e^3*x^2 + e^3), x) + 280*c^2*d^2*(3*e^2*integrate(
sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^
2 - 2*(e*x + d)^(3/2)/(c^2*e))/e^2 - 42*c^2*d*(15*e^2*integrate(((e*x + d)
*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 +
e^2)*sqrt(e*x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2 - 5*(e*x + d)^(3/2)
*c^2*d - 15*sqrt(e*x + d)*e^2)/(c^4*e))/e^2 - 3675*(3*e^2*integrate(sqrt(e
*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*
(e*x + d)^(3/2)/(c^2*e))*log(c) + 105*c^2*(105*e^4*integrate(sqrt(e*x + d)
/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^4 + 2*(15*(e*
x + d)^(7/2)*c^2 - 42*(e*x + d)^(5/2)*c^2*d + 35*(c^2*d^2 - e^2)*(e*x + d)
^(3/2))/(c^4*e))*log(c)/e^2 + 30*c^2*(105*e^4*integrate(sqrt(e*x + d)/((e*
x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^4 + 2*(15*(e*x + d
)^(7/2)*c^2 - 42*(e*x + d)^(5/2)*c^2*d + 35*(c^2*d^2 - e^2)*(e*x + d)^(3/2
))/(c^4*e))/e^2 - 210*(15*e^3*x^3 + 3*d*e^2*x^2 - 4*d^2*e*x + 8*d^3)*sqrt(
e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e^3 - 11025*integrate(2/105*(15*c^2*e^
3*x^4 + 3*c^2*d*e^2*x^3 - 4*c^2*d^2*e*x^2 + 8*c^2*d^3*x)*sqrt(e*x + d)/...
```

3.51.8 Giac [F]

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x^2, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int(x^2*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int(x^2*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.52 $\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

3.52.1 Optimal result

Integrand size = 19, antiderivative size = 679

$$\begin{aligned}
&\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} \\
&+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2} \\
&+\frac{8bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&-\frac{8bcd^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+\frac{4bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&+\frac{8bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
-2/3*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsch
(c*x))/e^2+4/15*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1+1/c^2/x^2)^(1/2)+8/15
*b*d^3*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c
^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/
2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*c*d*Elliptic
E(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^
(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e/x/(1+1/c^2/x
^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/15*b*c*d^2*Elliptic
F(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^
(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2
)/(-c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*c*(c^2*d^2+e^2
)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d
-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2)/(-c^2)^(5/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.33 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.62

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{15}\left(\frac{4b\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}+\frac{2a\sqrt{d+ex}(-2d^2+dex+3e^2x^2)}{e^2}+\frac{2b\sqrt{d+ex}(-2d^2+dex+3e^2x^2)\,\text{csch}^{-1}(cx)}{e^2}+\frac{4ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(2cd(cd+ie)E\left(i\text{arcsinh}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+(c^2d^2-2icde+e^2)\,\text{E}\cdots\right.}{c^3\sqrt{-\frac{\cdots}{cd\cdots}}\cdots}\right.$$

input
```
Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

output
```
((4*b*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^
2 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x
^2)*ArcCsch[c*x])/e^2 + ((4*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[
-((e*(I + c*x))/(c*d - I*e))]*(2*c*d*(c*d + I*e)*EllipticE[I*ArcSinh[Sqrt[
-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (c^2*d^2 - (2
*I)*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]]
, (c*d - I*e)/(c*d + I*e)] - 2*c^2*d^2*EllipticPi[1 - (I*e)/(c*d), I*ArcSi
nh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^3*
Sqrt[-(c/(c*d - I*e))]*e^2*Sqrt[1 + 1/(c^2*x^2)]*x))/15
```

3.52.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2043 vs. $2(679) = 1358$.

Time = 3.82 (sec) , antiderivative size = 2043, normalized size of antiderivative = 3.01, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {6864, 27, 7272, 2351, 27, 497, 27, 599, 27, 634, 599, 27, 631, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 6864}$$

$$\frac{b\int -\frac{2(2d-3ex)(d+ex)^{3/2}}{15e^2\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2b\int \frac{(2d-3ex)(d+ex)^{3/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{15ce^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{ 7272}$$

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{(2d-3ex)(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{ 2351}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int -\frac{3e(d+ex)^{3/2}}{\sqrt{c^2x^2+1}}dx+2d\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx-3e\int \frac{(d+ex)^{3/2}}{\sqrt{c^2x^2+1}}dx\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 497

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx-3e\left(\frac{2\int \frac{3d^2c^2+4dexc^2-e^2}{2\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}\right)\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx-3e\left(\frac{\int \frac{3d^2c^2+4dexc^2-e^2}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{3c^2}+\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}\right)\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 599

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx-3e\left(\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}-\frac{2\int \frac{e\left(d^2c^2-4d(d+ex)c^2+e^2\right)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{3c^2e^2}\right)\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\downarrow$ 27

3.52. $\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{(d+ex)^{3/2}}{x\sqrt{c^2x^2+1}}dx-3e\left(\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}-\frac{2\int \frac{d^2c^2-4d(d+ex)c^2+e^2}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{3c^2e}\right)\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{634}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx-\int \frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)-3e\left(\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}-\frac{2\int \frac{d^2c^2-4d(d+ex)c^2+e^2}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\dots}}\dots}{3c^2e}\right.\right.}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}\dots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{599}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(\frac{2\int \frac{e^2(2d+ex)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e^2}+d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)-3e\left(\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}-\frac{2\int\dots}{\dots}\right.\right.}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}\dots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)-3e\left(\frac{2e\sqrt{c^2x^2+1}\sqrt{d+ex}}{3c^2}-\dots\right.\right.}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}\dots$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{631}$$

3.52. $\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(2\int \frac{2d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-2d^2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)\right.}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{1511}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(2\left(\frac{\left(\sqrt{c^2d^2+e^2}+cd\right)\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}}{c}\right.\right.\right.}{}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^2}-\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(2d\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt[4]{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.}{}$$

$$\Big\downarrow \text{1509}$$

3.52. $\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^2}-\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-$$

$$2b\sqrt{c^2x^2+1}\left(2d\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt[4]{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

$\downarrow$ 1540

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^2}-\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-$$

$$2b\sqrt{c^2x^2+1}\left(2d\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt[4]{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

$\downarrow$ 1416

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^2}-\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-$$

$$2b\sqrt{c^2x^2+1}\left(2d\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt[4]{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

$\downarrow$ 2222

3.52. $\int x\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^2}-\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+}}{\sqrt[4]{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.}{}$$

input
```
Int[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

output
```
(-2*d*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(
a + b*ArcCsch[c*x]))/(5*e^2) - (2*b*Sqrt[1 + c^2*x^2]*(-3*e*((2*e*Sqrt[d +
 e*x]*Sqrt[1 + c^2*x^2])/(3*c^2) - (2*(4*c*d*Sqrt[c^2*d^2 + e^2]*(-((Sqrt[
d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)
^2)/e^2])/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) +
 ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 +
(c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^
2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan
[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 +
 e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^
2*(d + e*x)^2)/e^2])) - ((c^2*d^2 + e^2)^(3/4)*(4*c*d - Sqrt[c^2*d^2 + e^2
])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^
2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*
x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*Sqrt[c]
*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2]
)))/(3*c^2*e)) + 2*d*(2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 +
 (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (
c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)
^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 ...
```

3.52. $\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.52. $\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.52.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.06 (sec) , antiderivative size = 1963, normalized size of antiderivative = 2.89

method	result	size
derivativedivides	Expression too large to display	1963
default	Expression too large to display	1963
parts	Expression too large to display	1967

input
```
int(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(-a*(-1/5*(e*x+d)^(5/2)+1/3*(e*x+d)^(3/2)*d)-b*(-1/5*arccsch(c*x)*(e
*x+d)^(5/2)+1/3*arccsch(c*x)*(e*x+d)^(3/2)*d+2/15/c^3*(-2*I*(-(I*c*e*(e*x+
d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(
e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2
*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2)*e^3*(e*x+d)^(1/2)-I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*e*(e*x+d)^(5
/2)+2*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^(3/2)-(-(I*c*e*(
e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2
*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
))*c^3*d^3-2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1
/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipt
icE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2
),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3-2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)...
```

3.52.5 Fricas [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.52.6 Sympy [F]

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex}\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x), x)
```

3.52.7 Maxima [F]

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
-1/225*(3375*c^2*e^2*integrate(1/15*sqrt(e*x + d)*x^3*log(x)/(c^2*e^2*x^2
+ e^2), x) - 60*c^2*d^2*(e*integrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((
e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^2
 + 2*sqrt(e*x + d)/c^2)/e^2 + 3375*e^2*integrate(1/15*sqrt(e*x + d)*x*log(
x)/(c^2*e^2*x^2 + e^2), x) - 10*(3*e^3*integrate(sqrt(e*x + d)/((e*x + d)^
2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*(e*x + d)^(3/2)/c^2
)*c^2*d/e^2 + 225*(e*integrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x +
d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^2 + 2*s
qrt(e*x + d)/c^2)*log(c) - 15*c^2*(15*e^3*integrate(((e*x + d)*c^2*d - c^2
*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*
x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2 - 5*(e*x + d)^(3/2)*c^2*d - 15*
sqrt(e*x + d)*e^2)/c^4)*log(c)/e^2 - 6*c^2*(15*e^3*integrate(((e*x + d)*c^
2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2
)*sqrt(e*x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2 - 5*(e*x + d)^(3/2)*c^
2*d - 15*sqrt(e*x + d)*e^2)/c^4)/e^2 - 30*(3*e^2*x^2 + d*e*x - 2*d^2)*sqrt
(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e^2 - 225*integrate(2/15*(3*c^2*e^2*x
^3 + c^2*d*e*x^2 - 2*c^2*d^2*x)*sqrt(e*x + d)/(c^2*e^2*x^2 + e^2 + (c^2*e^
2*x^2 + e^2)*sqrt(c^2*x^2 + 1)), x))*b + 2/15*a*(3*(e*x + d)^(5/2)/e^2 - 5
*(e*x + d)^(3/2)*d/e^2)
```

3.52.8 Giac [F]

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int(x*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int(x*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.53 $\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

3.53.1 Optimal result

Integrand size = 18, antiderivative size = 429

$$\begin{aligned}
&\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx \\
&= \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e} \\
&+ \frac{4bc\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\
&+ \frac{4bcd\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{4bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output

```
2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e+4/3*b*c*EllipticE(1/2*(1-x*(-c^2)^(
1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x
+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d
+e/(-c^2)^(1/2)))^(1/2)+4/3*b*c*d*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2
^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)
*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)-4/3*b*d^2*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(
1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)
/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.53.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.85 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.16

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{2a(d+ex)^{3/2}}{3e} + \frac{b\left(-\frac{(cd+cex)\left(-\frac{4}{3}\sqrt{1+\frac{1}{c^2x^2}}-\frac{2cd\operatorname{csch}^{-1}(cx)}{3e}-\frac{2}{3}cx\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}} - \frac{2(cd+cex)\left(-\frac{\sqrt{2}cde\sqrt{1+icx}(i+cx)\sqrt{\frac{cd+cex}{cd-ie}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{e(i+\cdots}{cd-\cdots}}\right.\right.}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{e+\frac{d}{x}}(cx)^{3/2}\sqrt{\frac{e(1-icx)}{icd+e}}}\cdots\right)}{\cdots}\right)}{\cdots}$$

input
```
Integrate[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

output
```
(2*a*(d + e*x)^(3/2))/(3*e) + (b*(-(((c*d + c*e*x)*((-4*Sqrt[1 + 1/(c^2*x^
2)])/3 - (2*c*d*ArcCsch[c*x])/(3*e) - (2*c*x*ArcCsch[c*x])/3))/Sqrt[d + e*
x]) - (2*(c*d + c*e*x)*(-((Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c
*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e)
)]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*
Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(c^2*d^2 + e^
2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipti
cPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cosh[2
*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*
c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*
(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c
*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellipti
cE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e
*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*S
qrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, A
rcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-(
(e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2))))/(3*e*Sqrt[e + d/x]*Sqrt[c*x]*Sqrt[d + e*x])))/c^2
```

3.53.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1313 vs. $2(429) = 858$.

Time = 1.89 (sec) , antiderivative size = 1313, normalized size of antiderivative = 3.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6844, 1898, 634, 599, 27, 631, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 6844}$$

$$\frac{2b\int \frac{(d+ex)^{3/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{3ce}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{ 1898}$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\int \frac{(d+ex)^{3/2}}{x\sqrt{x^2+\frac{1}{c^2}}}dx}{3cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{ 634}$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx-\int \frac{-xe^2-2de}{\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{ 599}$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{2\int \frac{e^2(2d+ex)}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}+d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(2\int \frac{2d+ex}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}+d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 631

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(2\int \frac{2d+ex}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}-2d^2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 1511

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(2\left(\frac{\left(\sqrt{c^2d^2+e^2}+cd\right)\int \frac{1}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{c}\right)-2d\ldots}{3cex\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 1416

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\sqrt{c^2d^2+e^2}+cd\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}{\left(\frac{1}{c^2}+\frac{d^2}{e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\ldots\right.\right.}{2c^{3/2}\sqrt{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}}\ldots\right.\right.}{3cex\sqrt{\frac{1}{c^2x\ldots}}}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$\downarrow$ 1509

3.53. $\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$2b\sqrt{\frac{1}{c^2}+x^2}\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\sqrt{c^2d^2+e^2}+cd\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}{\left(\frac{1}{c^2}+\frac{d^2}{e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\right.\right.}{2c^{3/2}\sqrt{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}}\right.\right.$$

$$\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}$$

$$\Big\downarrow \text{ 1540}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e}+$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\right.\right.}{2c^{3/2}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}\right.\right.$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e}+$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\right.\right.}{2c^{3/2}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}\right.\right.$$

3.53. $\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\Big\downarrow \text{2222}$$

$$\frac{2(a+b\operatorname{csch}^{-1}(cx))\,(d+ex)^{3/2}}{3e}+$$

$$\frac{2b\sqrt{x^2+\frac{1}{c^2}}\left(2\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\right.\right.}{2c^{3/2}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}\right.\right.}{}$$

input
```
Int[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

output
```
(2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (2*b*Sqrt[c^(-2) + x^2]*(
2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(
d + e*x))/e^2 + (d + e*x)^2/e^2])/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/S
qrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2
*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^
2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*Ellipti
cE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqr
t[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2
 + (d + e*x)^2/e^2])))/c) + ((c^2*d^2 + e^2)^(1/4)*(c*d + Sqrt[c^2*d^2 + e
^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d
*(d + e*x))/e^2 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/
Sqrt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d
^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[c^(-
2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])) - 2*d^2*(-1/2*(Sqr
t[c]*(c^2*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/
Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e
*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]
*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (
c*d)/Sqrt[c^2*d^2 + e^2])/2])/(e^2*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))
/e^2 + (d + e*x)^2/e^2]) + ((c^2*d^2 + e^2)*(1 - (c*d)/Sqrt[c^2*d^2 + e...
```

3.53. $\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

3.53.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.53.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.60 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{\frac{2a(ex+d)^{\frac{3}{2}}}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(i\operatorname{EllipticF}\left(\sqrt{\right.\right.}{}\right.}{}$
default	$\frac{\frac{2a(ex+d)^{\frac{3}{2}}}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(i\operatorname{EllipticF}\left(\sqrt{\right.\right.}{}\right.}{}$
parts	$\frac{2a(ex+d)^{\frac{3}{2}}}{3e}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(i\operatorname{Elliptic}\right.}{}\right.}{}$

input
```
int((a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/3*a*(e*x+d)^(3/2)+b*(1/3*(e*x+d)^(3/2)*arccsch(c*x)+2/3/c^2*(-(I*c*
e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(I*EllipticF((e*x+d)^(1/2)
*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2))*c*d*e-I*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2
),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2))*c*d*e-2*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(
c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^
2+EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticPi((e*x+d)^(1/2)*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^
2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticF((e*
x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2))*e^2+EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))
^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^
2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2)/(I*e-c*d)))
```

3.53. $\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

3.53.5 Fricas [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.53.6 Sympy [F]

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \left(a+b\operatorname{acsch}\left(cx\right)\right)\sqrt{d+ex}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x), x)
```

3.53.7 Maxima [F]

$$\int \sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\operatorname{arcsch}\left(cx\right)+a)\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*(27*c^2*e*integrate(1/3*sqrt(e*x + d)*x^2*log(x)/(c^2*e*x^2 + e), x)
+ 9*e^2*integrate(sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2
*d^2 + e^2), x)*log(c) + 6*c^2*d*(e^2*integrate(((e*x + d)*c^2*d - c^2*d^2
 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x +
d)), x)/c^2 + 2*sqrt(e*x + d)*e/c^2)/e^2 + 27*e*integrate(1/3*sqrt(e*x + d
)*log(x)/(c^2*e*x^2 + e), x) - 3*(3*e^4*integrate(sqrt(e*x + d)/((e*x + d)
^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c^2 - 2*(e*x + d)^(3/2)*e/
c^2)*c^2*log(c)/e^2 - 6*(e*x + d)^(3/2)*log(sqrt(c^2*x^2 + 1) + 1)/e - 2*(
3*e^4*integrate(sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d
^2 + e^2), x)/c^2 - 2*(e*x + d)^(3/2)*e/c^2)*c^2/e^2 - 9*integrate(2/3*(c^
2*e*x^2 + c^2*d*x)*sqrt(e*x + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^2
 + 1) + e), x))*b + 2/3*(e*x + d)^(3/2)*a/e
```

3.53.8 Giac [F]

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.54 $\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\text{Int}\left(\frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

3.54.2 Mathematica [N/A]

Not integrable

Time = 43.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

3.54.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x}\,dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x}dx$$

input
```
Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
$Aborted
```

3.54.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.54.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\,\text{arccsch}\,(cx))\,\sqrt{ex+d}}{x}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

3.54.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

3.54.6 Sympy [N/A]

Not integrable

Time = 13.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\operatorname{acsch}(cx))\sqrt{d+ex}}{x}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x)/x, x)
```

3.54.7 Maxima [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.57

$$\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
(sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d))) + 2*sqrt
(e*x + d))*a - ((sqrt(d)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sq
rt(d))) + 2*sqrt(e*x + d))*log(c) + integrate(sqrt(e*x + d)*log(x)/x, x) -
 integrate(sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/x, x))*b
```

3.54.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

3.54.9 Mupad [N/A]

Not integrable

Time = 5.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x}\,dx$$

input
```
int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x,x)
```

output
```
int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

3.55 $\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \text{Int}\left(\frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

3.55.2 Mathematica [N/A]

Not integrable

Time = 6.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

3.55.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2} \, dx$$

$$\Big\downarrow \ 6866$$

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}dx$$

input
```
Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.55.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.55.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a+b\,\operatorname{arccsch}(cx))\sqrt{ex+d}}{x^2}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

3.55.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.55.6 Sympy [N/A]

Not integrable

Time = 14.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\sqrt{d+ex}}{x^2}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x)/x**2, x)
```

3.55.7 Maxima [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 7.10

$$\int \frac{\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex+d}(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*(e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d) -
2*sqrt(e*x + d)/x)*a - 1/2*((e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d
) + sqrt(d)))/sqrt(d) - 2*sqrt(e*x + d)/x)*log(c) + 2*integrate(sqrt(e*x +
 d)*log(x)/x^2, x) - 2*integrate(sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/
x^2, x))*b
```

3.55.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{ex+d}(b\operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.55.9 Mupad [N/A]

Not integrable

Time = 4.84 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x^2}\,dx$$

input
```
int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
```

output
```
int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

3.56 $\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$

3.56.1 Optimal result

Integrand size = 18, antiderivative size = 486

$$\begin{aligned}\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx &= \frac{4be\sqrt{d+ex}(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}x} \\ &+ \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e} \\ &+ \frac{28bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\ &- \frac{4bc(2c^2d^2-e^2)\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{5/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &- \frac{4bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output
```
2/5*(e*x+d)^(5/2)*(a+b*arccsch(c*x))/e+4/15*b*e*(c^2*x^2+1)*(e*x+d)^(1/2)/
c^3/x/(1+1/c^2/x^2)^(1/2)+28/15*b*c*d*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/
2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)
*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/(-c^2)
^(1/2)))^(1/2)-4/15*b*c*(2*c^2*d^2-e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)
^(1/2)*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/x/(1+1/c^2/x^2)^(1/
2)/(e*x+d)^(1/2)-4/5*b*d^3*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2)
,2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)
^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.78

$$\int (d+ex)^{3/2} \left(a + b\operatorname{csch}^{-1}(cx)\right) dx = \frac{2\left(\frac{2be^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}{c} + 3a(d+ex)^{5/2} + 3b(d+ex)^{5/2}\operatorname{csch}^{-1}(cx) + \frac{2ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\cdots}{\cdots}\right)}{\cdots}$$

input
```
Integrate[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(2*((2*b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + 3*a*(d + e*x)^(5/2
) + 3*b*(d + e*x)^(5/2)*ArcCsch[c*x] + ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(7*c*d*(c*d + I*e)*EllipticE[
I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]
+ (-9*c^2*d^2 - (7*I)*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e
))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + 3*c^2*d^2*EllipticPi[1 - (I
*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c
*d + I*e)]))/(c^3*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(15*e)
```

3.56.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1357 vs. 2(486) = 972.

Time = 2.42 (sec) , antiderivative size = 1357, normalized size of antiderivative = 2.79, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6844, 1898, 634, 631, 1540, 1416, 2185, 27, 599, 25, 27, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \ 6844$$

$$\frac{2b\int \frac{(d+ex)^{5/2}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{5ce}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 1898$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\int \frac{(d+ex)^{5/2}}{x\sqrt{x^2+\frac{1}{c^2}}}dx}{5cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 634$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(d^3\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 631$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx-2d^3\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 1540$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\int \frac{-x^2e^3-3dxe^2-3d^2e}{\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{c\left(cd-\sqrt{c\right.}{}\right.\right.}{5cex\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 1416$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(}{\sqrt{c^2}\right.}{}\right.\right.}{5cex\sqrt{\frac{1}{c^2x}}}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 2185$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(}{\sqrt{c^2}\right.}{}\right.\right.}{}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(}{\sqrt{c^2}\right.}{}\right.\right.}{}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \; 599$$

3.56. $\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(\cdots}{\sqrt{c^{\cdots}}}\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 25$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(\cdots}{\sqrt{c^{\cdots}}}\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(\cdots}{\sqrt{c^{\cdots}}}\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 1511$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-2d^3\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(\cdots}{\sqrt{c^{\cdots}}}\right.}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}$$

$$\Big\downarrow \ 1416$$

3.56. $\int (d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e}+$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(-2\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int-\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+e}{\sqrt{c^2d^2}}\right.}{}\right.\right.$$

$$\downarrow \text{1509}$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e}+$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(-2\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int-\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+e}{\sqrt{c^2d^2}}\right.}{}\right.\right.$$

$$\downarrow \text{2222}$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e}+$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(-2\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{c\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{c\sqrt{d}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}\right)}{2\sqrt{d}}+\frac{\sqrt[4]{c^2d^2+e^2}\left(1-\frac{c}{\sqrt{c^2d}}\right.}{}\right.}{}\right.\right.$$

input
```
Int[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

3.56. $\int(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

output
```
(2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (2*b*Sqrt[c^(-2) + x^2]*(
(2*e^2*Sqrt[d + e*x]*Sqrt[c^(-2) + x^2])/3 + (2*((-7*d*Sqrt[c^2*d^2 + e^2]
*(-((Sqrt[d + e*x]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)
^2/e^2])/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + (
(c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2)
 + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1
 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt
[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sq
rt[c]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])))/c
+ ((c^2*d^2 + e^2)^(1/4)*(2*c^2*d^2 - e^2 + 7*c*d*Sqrt[c^2*d^2 + e^2])*(1
+ (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*
x))/e^2 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2
*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2
)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(5/2)*Sqrt[c^(-2) + d^2
/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])))/3 - 2*d^3*(-1/2*(Sqrt[c]*
(c^2*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[
c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2
/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*Elli
pticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/
Sqrt[c^2*d^2 + e^2])/2])/(e^2*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e...
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 634
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] - Int[(
1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(c^(n + 1/2) - (c + d*x)^(n
+ 1/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n - 1/2, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

3.56. $\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.56.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.23 (sec) , antiderivative size = 1939, normalized size of antiderivative = 3.99

method	result	size
derivativedivides	Expression too large to display	1939
default	Expression too large to display	1939
parts	Expression too large to display	1941

input
```
int((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/e*(1/5*a*(e*x+d)^(5/2)+b*(1/5*arccsch(c*x)*(e*x+d)^(5/2)+2/15/c^3*(-2*I*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-((c*d+I*e)*c/(c^2*
d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+2*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(
1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-3*I*(-(I*c*
e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)
*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+I*((c
*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(1/2)*e^3+2*((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2)*c^3*d^2*(e*x+d)^(3/2)-9*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),
(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+7*(-(I*c*e*(e*x+d)
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*
x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*
c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3
*d^3+3*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((
I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticP...
```

3.56.5 Fricas [F]

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex+d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) \, dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x + a*d + (b*e*x + b*d)*arccsch(c*x))*sqrt(e*x + d), x)
```

3.56.6 Sympy [F]

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (a+b\,\text{acsch}\,(cx))\,(d+ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x+d)**(3/2)*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x)**(3/2), x)
```

3.56.7 Maxima [F]

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex+d)^{\frac{3}{2}} (b\,\text{arcsch}\,(cx)+a) \, dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
2/5*(e*x + d)^(5/2)*a/e - 1/75*(375*c^2*e^2*integrate(1/5*sqrt(e*x + d)*x^
3*log(x)/(c^2*e*x^2 + e), x) + 375*c^2*d*e*integrate(1/5*sqrt(e*x + d)*x^2
*log(x)/(c^2*e*x^2 + e), x) + 75*d*e^2*integrate(sqrt(e*x + d)/((e*x + d)^
2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)*log(c) + 30*c^2*d^2*(e^2*in
tegrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*
c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^2 + 2*sqrt(e*x + d)*e/c^2)/e^2
 + 375*e^2*integrate(1/5*sqrt(e*x + d)*x*log(x)/(c^2*e*x^2 + e), x) + 375*
d*e*integrate(1/5*sqrt(e*x + d)*log(x)/(c^2*e*x^2 + e), x) - 25*(3*e^4*int
egrate(sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)
, x)/c^2 - 2*(e*x + d)^(3/2)*e/c^2)*c^2*d*log(c)/e^2 - 20*(3*e^4*integrate
(sqrt(e*x + d)/((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2), x)/c
^2 - 2*(e*x + d)^(3/2)*e/c^2)*c^2*d/e^2 + 75*(e^2*integrate(((e*x + d)*c^2
*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)
*sqrt(e*x + d)), x)/c^2 + 2*sqrt(e*x + d)*e/c^2)*log(c) - 5*c^2*(15*e^4*in
tegrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 - 2*(e*x + d)*
c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^4 - 2*(3*(e*x + d)^(5/2)*c^2*e
 - 5*(e*x + d)^(3/2)*c^2*d*e - 15*sqrt(e*x + d)*e^3)/c^4)*log(c)/e^2 - 30*
(e^2*x^2 + 2*d*e*x + d^2)*sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e - 2*c
^2*(15*e^4*integrate(((e*x + d)*c^2*d - c^2*d^2 - e^2)/(((e*x + d)^2*c^2 -
 2*(e*x + d)*c^2*d + c^2*d^2 + e^2)*sqrt(e*x + d)), x)/c^4 - 2*(3*(e*x ...
```

3.56.8 Giac [F]

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex+d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) \, dx$$

input
```
integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right) (d+e\,x)^{3/2} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.57 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 939

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$$
$$= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x}$$
$$- \frac{2d^3\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}$$
$$- \frac{6d(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4} + \frac{2(d+ex)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^4}$$
$$+ \frac{24bcd^2\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$+ \frac{4bc(2c^2d^2+9e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$
$$- \frac{64bcd^3\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$- \frac{32bcd(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{105\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$
$$+ \frac{64bd^4\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{35ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
2*d^2*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsc
h(c*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsch(c*x))/e^4-2*d^3*(a+b*arccsch(c*
x))*(e*x+d)^(1/2)/e^4+4/35*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1+1/c^2/x^2)
^(1/2)-4/21*b*d*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1+1/c^2/x^2)^(1/2)+64
/35*b*d^4*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*
(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^
(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+24/35*b*c*d^2*El
lipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(
-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^3/x/(1
+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+4/105*b*c*(2*
c^2*d^2+9*e^2)*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)
^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^
2)^(5/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1
/2)-64/35*b*c*d^3*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c
^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c
^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)
^(1/2)-32/105*b*c*d*(c^2*d^2+e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2
^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)
*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^3/x/(1+1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)
```

3.57.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.76 (sec) , antiderivative size = 1098, normalized size of antiderivative = 1.17

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx=\frac{ad^4\sqrt{1+\frac{ex}{d}}B_{-\frac{ex}{d}}\left(4,\frac{1}{2}\right)}{e^4\sqrt{d+ex}}$$

$$+b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4\left(-16c^2d^2+9e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{105e^3}+\frac{32c^3d^3\operatorname{csch}^{-1}(cx)}{35e^4}-\frac{2c^3x^3\operatorname{csch}^{-1}(cx)}{7e}-\frac{4c^2x^2\left(e\sqrt{1+\frac{1}{c^2x^2}}-3cd\operatorname{csch}^{-1}(cx)\right)}{35e^2}+\frac{4cx\left(5cde\sqrt{1+\cdots}\right.}{\cdots}\right.}{\sqrt{d+ex}}\right.$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

3.57. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

output
```
(a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (
b*(-((c*(e + d/x)*x*((4*(-16*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*
e^3) + (32*c^3*d^3*ArcCsch[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsch[c*x])/(7*e)
 - (4*c^2*x^2*(e*Sqrt[1 + 1/(c^2*x^2)] - 3*c*d*ArcCsch[c*x]))/(35*e^2) + (
4*c*x*(5*c*d*e*Sqrt[1 + 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsch[c*x]))/(105*e^3)
))/Sqrt[d + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(40*c^3*d^3*e -
 8*c*d*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*Elli
pticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqr
t[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d +
 e)])) + (I*Sqrt[2]*(c*d - I*e)*(48*c^4*d^4 - 16*c^2*d^2*e^2 + 9*e^4)*Sqrt
[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
 (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]
)/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-16*c^3*d^3*e
+ 9*c*d*e^3)*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(
c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*Elliptic
F[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[
-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*(
(c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)
/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*
d - I*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I +...
```

3.57.3 Rubi [A] (warning: unable to verify)

Time = 3.96 (sec) , antiderivative size = 1548, normalized size of antiderivative = 1.65, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6864, 27, 7272, 2351, 630, 1656, 1416, 2185, 27, 687, 27, 599, 25, 27, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}} \, dx$$

$$\Big\downarrow \ 6864$$

$$\frac{b\int -\frac{2\sqrt{d+ex}\left(16d^3-8exd^2+6e^2x^2d-5e^3x^3\right)}{35e^4\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\int \frac{\sqrt{d+ex}\left(16d^3-8exd^2+6e^2x^2d-5e^3x^3\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{35ce^4}-\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{\sqrt{d+ex}\left(16d^3-8exd^2+6e^2x^2d-5e^3x^3\right)}{x\sqrt{c^2x^2+1}}dx}{35ce^4x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(16d^3\int \frac{\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx+\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{c^2x^2+1}}dx\right)}{35ce^4x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{630}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{c^2x^2+1}}dx-32d^3\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{35ce^4x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{1656}$$

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{c^2x^2+1}}dx-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2}{e^2}}}\right.\right.}{35ce^4x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 1416

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{\sqrt{d+ex}\left(-5x^2e^3+6dxe^2-8d^2e\right)}{\sqrt{c^2x^2+1}}dx-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2}{e^2}}}\right.\right.}{}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 2185

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2}}{}\right.\right.}{}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 27

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2 \cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{687}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2 \cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2 \cdots}}{\cdots}\right.\right.\right.}{\cdots}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{599}$$

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2\cdots}}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 25

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2\cdots}}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 27

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-32d^3\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2 \cdots}}{\cdots}\right.\right.}{\cdots}$$

$$\frac{2d^3\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^4}-\frac{6d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\downarrow$ 1511

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^4}-\frac{6d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^4}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{e^4}-\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^4}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-32\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2d^2+\cdots}}{\cdots}\right.\right.}{\cdots}$$

$\downarrow$ 1416

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^4}-\frac{6d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^4}+\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{e^4}-\frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^4}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-32\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int-\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2d^2+\cdots}}{\cdots}\right.\right.}{\cdots}$$

$\downarrow$ 1509

3.57. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^4}-\frac{6d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^4}+\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{e^4}-\frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(-32\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int-\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2d^2+\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2222

3.57. $\int\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{7/2}}{7e^4}-\frac{6d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^4}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{e^4}-\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(-32\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right)}{2\sqrt{d}}+\frac{\sqrt[4]{c^2 \cdots}}{\cdots}\right.\right.\right.$$

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

output
```
(-2*d^3*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(
a + b*ArcCsch[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e
^4) + (2*(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^4) - (2*b*Sqrt[1 + c^2
*x^2]*((-2*e^2*(d + e*x)^(3/2)*Sqrt[1 + c^2*x^2])/c^2 - (e*((-16*d*e*Sqrt[
d + e*x]*Sqrt[1 + c^2*x^2])/3 + (2*(-(((16*c^2*d^2 - 9*e^2)*Sqrt[c^2*d^2 +
 e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 +
 (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*
d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e
^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)
/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*Ell
ipticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)
/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d +
e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/c) + ((c^2*d^2 + e^2)^(3/4)*(16*c^2*
d^2 - 9*e^2 + 8*c*d*Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 +
 e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^
2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*E
llipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*
d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(
d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/(3*e)))/c^2 - 32*d^3*(-1/2*((1 +
 (c^2*d^2)/e^2)*(c^2*d^2 + e^2)^(1/4)*(1 - (c*d)/Sqrt[c^2*d^2 + e^2])*(...
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 630
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2   Subst[Int[x^2/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1656
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e
^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 -
a*e^2, 0]
```

3.57. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.57. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

3.57.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 2545, normalized size of antiderivative = 2.71

method	result	size
derivativedivides	Expression too large to display	2545
default	Expression too large to display	2545
parts	Expression too large to display	2546

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^4*(-a*(-1/7*(e*x+d)^(7/2)+3/5*d*(e*x+d)^(5/2)-d^2*(e*x+d)^(3/2)+d^3*(e
*x+d)^(1/2))-b*(-1/7*arccsch(c*x)*(e*x+d)^(7/2)+3/5*arccsch(c*x)*d*(e*x+d)
^(5/2)-arccsch(c*x)*d^2*(e*x+d)^(3/2)+arccsch(c*x)*d^3*(e*x+d)^(1/2)+2/105
/c^4*(14*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d*e*(e*x+d)^(5/2)+3*((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d*(e*x+d)^(7/2)+40*I*(-(I*c*e*(e*x+d)+c^2
*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)
+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c
^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3
*e-14*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^2*(e*x+d)^(5/2)-3*I*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2)*c*e^3*(e*x+d)^(3/2)-3*I*((c*d+I*e)*c/(c^2*d^2+e^
2))^(1/2)*c^3*e*(e*x+d)^(7/2)-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e
^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(
-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3+19*((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2)*c^4*d^3*(e*x+d)^(3/2)-24*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-16*(-(I*c*
e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)...
```

3.57.5 Fricas [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^3}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsch(c*x) + a*x^3)/sqrt(e*x + d), x)
```

3.57.6 Sympy [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x^3(a + b\,\text{acsch}\,(cx))}{\sqrt{d+ex}}\,dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

3.57.7 Maxima [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^3}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/35*a*(5*(e*x + d)^(7/2)/e^4 - 21*(e*x + d)^(5/2)*d/e^4 + 35*(e*x + d)^(3
/2)*d^2/e^4 - 35*sqrt(e*x + d)*d^3/e^4) + 1/35*b*(2*(5*e^4*x^4 - d*e^3*x^3
 + 2*d^2*e^2*x^2 - 8*d^3*e*x - 16*d^4)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*
x + d)*e^4) + 35*integrate(2/35*(5*c^2*e^4*x^5 - c^2*d*e^3*x^4 + 2*c^2*d^2
*e^2*x^3 - 8*c^2*d^3*e*x^2 - 16*c^2*d^4*x)/((c^2*e^4*x^2 + e^4)*sqrt(c^2*x
^2 + 1)*sqrt(e*x + d) + (c^2*e^4*x^2 + e^4)*sqrt(e*x + d)), x) - 35*integr
ate(-1/35*(2*c^2*d*e^3*x^4 + 16*c^2*d^3*e*x^2 - 5*(7*e^4*log(c) + 2*e^4)*c
^2*x^5 + 32*c^2*d^4*x - (4*c^2*d^2*e^2 + 35*e^4*log(c))*x^3 - 35*(c^2*e^4*
x^5 + e^4*x^3)*log(x))/((c^2*e^4*x^2 + e^4)*sqrt(e*x + d)), x))
```

3.57.8 Giac [F]

$$\int \frac{x^3 \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex}}\, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^3}{\sqrt{ex + d}}\, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x + d), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \left(a + b\text{csch}^{-1}(cx)\right)}{\sqrt{d + ex}}\, dx = \int \frac{x^3 \left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d + e\,x}}\, dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.58 $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 707

$$\begin{aligned}
&\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx \\
&= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}x} + \frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} \\
&\quad - \frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3} \\
&\quad - \frac{4bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&\quad + \frac{32bcd^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad + \frac{4bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&\quad - \frac{32bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
-4/3*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsch
(c*x))/e^3+2*d^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+4/15*b*(c^2*x^2+1)*(
e*x+d)^(1/2)/c^3/e/x/(1+1/c^2/x^2)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(1-x*(
-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^
2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^
2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*c*d*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2
)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*
(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2)+32/15*b*c*d^2*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(
1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)
^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/
c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*c*(c^2*d^2+e^2)*EllipticF(1/2*(1-x*(-c
^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))
*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)
/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.58.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.98 (sec) , antiderivative size = 1012, normalized size of antiderivative = 1.43

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{ad^3\sqrt{1+\frac{ex}{d}}\,B_{-\frac{ex}{d}}\left(3,\frac{1}{2}\right)}{e^3\sqrt{d+ex}} + \frac{b\left(-\frac{c\left(e+\frac{d}{x}\right)x\left(\frac{4cd\sqrt{1+\frac{1}{c^2x^2}}}{5e^2}-\frac{16c^2d^2\operatorname{csch}^{-1}(cx)}{15e^3}-\frac{2c^2x^2\operatorname{csch}^{-1}(cx)}{5e}-\frac{4cx\left(e\sqrt{1+\frac{1}{c^2x^2}}-2cd\operatorname{csch}^{-1}(cx)\right)}{15e^2}\right)}{\sqrt{d+ex}} - \frac{2\sqrt{e+\frac{d}{x}}\sqrt{cx}\left(-\frac{\sqrt{2}\left(7c^2d\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

output
```
-((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x]))
+ (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^
2*ArcCsch[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsch[c*x])/(5*e) - (4*c*x*(e*Sqrt
[1 + 1/(c^2*x^2)] - 2*c*d*ArcCsch[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sq
rt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(7*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I +
 c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x)
)/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]
*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*
(8*c^3*d^3 - 3*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(
I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d -
 I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)
^(3/2)) + (6*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) +
(c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*El
lipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2
*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I
*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d
- I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]
], (c*d - I*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(
I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*El
lipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*...
```

3.58.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1458 vs. $2(707) = 1414$.

Time = 3.11 (sec) , antiderivative size = 1458, normalized size of antiderivative = 2.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {6864, 27, 7272, 2351, 630, 687, 27, 599, 25, 27, 1511, 1416, 1509, 1656, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}} \, dx$$

$$\Big\downarrow \text{ 6864}$$

$$\frac{b\int \frac{2\sqrt{d+ex}\left(8d^2-4exd+3e^2x^2\right)}{15e^3\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2d^2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2b\int \frac{\sqrt{d+ex}\left(8d^2-4exd+3e^2x^2\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{15ce^3}+\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{ 7272}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{\sqrt{d+ex}\left(8d^2-4exd+3e^2x^2\right)}{x\sqrt{c^2x^2+1}}dx}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{ 2351}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\int \frac{\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx+\int \frac{\sqrt{d+ex}\left(3e^2x-4de\right)}{\sqrt{c^2x^2+1}}dx\right)}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{ 630}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{\sqrt{d+ex}\left(3e^2x-4de\right)}{\sqrt{c^2x^2+1}}dx-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{ 687}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\frac{2\int -\frac{3e\left(4d^2c^2+3dexc^2+e^2\right)}{2\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{3c^2}-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+\frac{2e^2\sqrt{c^2x^2+1}\sqrt{d+ex}}{c^2}\right)}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{ 27}$$

3.58. $\displaystyle\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{e\int \frac{4d^2c^2+3dexc^2+e^2}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{c^2}-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+\frac{2e^2\sqrt{c^2x^2+1}\sqrt{d+ex}}{c^2}\right)}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\downarrow \text{599}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+\frac{2\int -\frac{e\left(d^2c^2+3d(d+ex)c^2+e^2\right)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c^2e}+2e^2\sqrt{}\right.}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\downarrow \text{25}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{2\int \frac{e\left(d^2c^2+3d(d+ex)c^2+e^2\right)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c^2e}+2e^2\sqrt{c^2}\right.}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\downarrow \text{27}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{2\int \frac{d^2c^2+3d(d+ex)c^2+e^2}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c^2}+2e^2\sqrt{c^2}\right.}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$
$$\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\downarrow \text{1511}$$

3.58. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(-16d^2\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{2\left(\sqrt{c^2d^2+e^2}\left(\sqrt{c^2d^2+e^2}+3cd\right)\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c}{e^2}}}\right.}{}\right.}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 1416

$$\frac{2b\sqrt{c^2x^2+1}\left(-\frac{2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\sqrt{c^2d^2+e^2}+3cd\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\right.\right.}{2\sqrt{c}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}\right.}{c^2}\right.}{}$$

$$\frac{2d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 1509

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}-\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-16\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\,d^2-\frac{2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(3cd+\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)}{e^2}}{\left(\frac{c^2}{e}\right.}}}{2\sqrt{c}\sqrt{}}\right.}{}\right.}{}$$

$\downarrow$ 1656

3.58. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}-\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(-16\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\right.\right.\right.$$

$$\Big\downarrow \text{1416}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}-\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(-16\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\left(\frac{c^2d^2}{e^2}+1\right)\sqrt[4]{c^2d^2+}}{}\right.\right.$$

$$\Big\downarrow \text{2222}$$

3.58. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{5/2}}{5e^3}-\frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^3}+\frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(-16\left(d\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right)}{2\sqrt{d}}+\frac{\sqrt[4]{c^2 \cdots}}{\cdots}\right.\right.\right.$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

output
```
(2*d^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a +
 b*ArcCsch[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^
3) + (2*b*Sqrt[1 + c^2*x^2]*((2*e^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2])/c^2 -
 (2*(-3*c*d*Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 -
 (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2)*(1
 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x
))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sq
rt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2
 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2
*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])) + ((c^2*d^2
 + e^2)^(3/4)*(3*c*d + Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d^
2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*
x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)
]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 +
(c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*
d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/c^2 - 16*d^2*(-1/2*((1 + (c^2
*d^2)/e^2)*(c^2*d^2 + e^2)^(1/4)*(1 - (c*d)/Sqrt[c^2*d^2 + e^2])*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x
))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))...
```

3.58. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 630
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2   Subst[Int[x^2/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.58. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1656
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e
^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 -
a*e^2, 0]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.58. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

3.58.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.41 (sec) , antiderivative size = 1991, normalized size of antiderivative = 2.82

method	result	size
derivativedivides	Expression too large to display	1991
default	Expression too large to display	1991
parts	Expression too large to display	1994

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*
arccsch(c*x)*(e*x+d)^(5/2)-2/3*arccsch(c*x)*(e*x+d)^(3/2)*d+arccsch(c*x)*d
^2*(e*x+d)^(1/2)+2/15/c^3*(7*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)
/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2
*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x
+d)^(5/2)*c^2*e-4*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*E
llipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-3*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^
2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^
(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-I*(-(I*c*e*(
e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2
*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
))*e^3+8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*
((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi
((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2...
```

3.58.5 Fricas [F]

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(e*x + d), x)
```

3.58.6 Sympy [F]

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x^2(a+b\,\text{acsch}\,(cx))}{\sqrt{d+ex}}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

3.58.7 Maxima [F]

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/15*a*(3*(e*x + d)^(5/2)/e^3 - 10*(e*x + d)^(3/2)*d/e^3 + 15*sqrt(e*x + d
)*d^2/e^3) + 1/15*b*(2*(3*e^3*x^3 - d*e^2*x^2 + 4*d^2*e*x + 8*d^3)*log(sqr
t(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^3) + 15*integrate(2/15*(3*c^2*e^3*x^4
 - c^2*d*e^2*x^3 + 4*c^2*d^2*e*x^2 + 8*c^2*d^3*x)/((c^2*e^3*x^2 + e^3)*sqr
t(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^2 + e^3)*sqrt(e*x + d)), x) - 15
*integrate(-1/15*(2*c^2*d*e^2*x^3 - 3*(5*e^3*log(c) + 2*e^3)*c^2*x^4 - 16*
c^2*d^3*x - (8*c^2*d^2*e + 15*e^3*log(c))*x^2 - 15*(c^2*e^3*x^4 + e^3*x^2)
*log(x))/((c^2*e^3*x^2 + e^3)*sqrt(e*x + d)), x))
```

3.58.8 Giac [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x + d), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx=\int \frac{x^2\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.59 $\int \frac{x\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

3.59.1 Optimal result

Integrand size = 19, antiderivative size = 474

$$\begin{aligned}
\int \frac{x\left(a+b\,\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx
&= -\frac{2d\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2} \\
&+\frac{4bc\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&-\frac{8bcd\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&+\frac{8bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}$$

output
```
2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^2-2*d*(a+b*arccsch(c*x))*(e*x+d)^(1
/2)/e^2+8/3*b*d^2*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2
)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d
*(-c^2)^(1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*
EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e/x/(1
+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/3*b*c*d*Ell
ipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-
c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))
^(1/2)/(-c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.88

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$$

$$= \frac{\frac{2a(-2d+ex)(d+ex)}{e^2} + \frac{2b(-2d+ex)(d+ex)\operatorname{csch}^{-1}(cx)}{e^2} + \frac{2b(icd+e)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\left(2i(cd+ie)e\sqrt{-\frac{e(-i+cx)}{cd+ie}}\,E\left(\arcsin\left(\sqrt{\frac{c(d-\cdots}{cd-\cdots}}\right.\right.\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

output
```
((2*a*(-2*d + e*x)*(d + e*x))/e^2 + (2*b*(-2*d + e*x)*(d + e*x)*ArcCsch[c*
x])/e^2 + (2*b*(I*c*d + e)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c*e*(I + c*x)*(d
+ e*x))/(I*c*d + e)^2]*((2*I)*(c*d + I*e)*e*Sqrt[-((e*(-I + c*x))/(c*d + I
*e))]*EllipticE[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d
+ I*e)] - I*c*d*e*Sqrt[2 + (2*I)*c*x]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x)
)/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*e^2*Sqrt[-((e*(-I + c*x))/(c*d +
I*e))]*EllipticF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d
 + I*e)] + 2*c^2*d^2*Sqrt[2 + (2*I)*c*x]*EllipticPi[1 + (I*c*d)/e, ArcSin[
Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(e^3*(c + c^3*x^
2)))/(3*Sqrt[d + e*x])
```

3.59.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1377 vs. 2(474) = 948.

Time = 2.93 (sec) , antiderivative size = 1377, normalized size of antiderivative = 2.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.737, Rules used = {6864, 27, 7272, 2351, 25, 27, 507, 630, 1459, 1416, 1509, 1656, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 6864}$$

$$\frac{b\int -\frac{2(2d-ex)\sqrt{d+ex}}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2b\int \frac{(2d-ex)\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{3ce^2}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 7272}$$

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{(2d-ex)\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 2351}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int -\frac{e\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx+2d\int \frac{\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx-\int \frac{e\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx-e\int \frac{\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$\downarrow$ 507

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\int \frac{\sqrt{d+ex}}{x\sqrt{c^2x^2+1}}dx-2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$\downarrow$ 630

$$-\frac{2b\sqrt{c^2x^2+1}\left(-2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-4d\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$\downarrow$ 1459

$$-\frac{2b\sqrt{c^2x^2+1}\left(-4d\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-2\left(\frac{\sqrt{c^2d^2+e^2}\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+e}}{c}\right.\right.}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$\downarrow$ 1416

$$-\frac{2b\sqrt{c^2x^2+1}\left(-2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+}}\right.\right.}{2c^{3/2}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}\right.\right.}{3ce^2x\sqrt{\frac{1}{c^2}}}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

3.59. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\Big\downarrow \text{1509}$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(-4d\int -\frac{d+ex}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-2\left(\frac{(c^2d^2+e^2)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c\cdots}{}}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+e\cdots}{\sqrt{c^2d^2\cdots}}\right)}}}{2c^{3/2}\sqrt{\frac{c^2d\cdots}{e^2}}}\right.\right.}{}$$

$$\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{1656}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^2}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(-2\left(\frac{(c^2d^2+e^2)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}\cdots\right.\right)}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{}$$

$$\Big\downarrow \text{1416}$$

3.59. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^2}-$$

$$2b\sqrt{c^2x^2+1}\left(-2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.$$

$\downarrow$ 2222

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)(d+ex)^{3/2}}{3e^2}-\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)\sqrt{d+ex}}{e^2}-$$

$$2b\sqrt{c^2x^2+1}\left(-2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

3.59. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

output
```
(-2*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*
ArcCsch[c*x]))/(3*e^2) - (2*b*Sqrt[1 + c^2*x^2]*(-2*(-((Sqrt[c^2*d^2 + e^2
]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^
2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2
+ e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])
*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2
)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*Ellipti
cE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqr
t[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x)
)/e^2 + (c^2*(d + e*x)^2)/e^2]))))/c) + ((c^2*d^2 + e^2)^(3/4)*(1 + (c*(d +
 e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/
e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[
c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 +
e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[1 + (c^2*
d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])) - 4*d*(-1/2*
((1 + (c^2*d^2)/e^2)*(c^2*d^2 + e^2)^(1/4)*(1 - (c*d)/Sqrt[c^2*d^2 + e^2])
*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*
d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d
+ e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x]
)/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*...
```

3.59.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 630
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2   Subst[Int[x^2/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

3.59. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1656
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e
^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 -
a*e^2, 0]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.59. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.59.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.00 (sec) , antiderivative size = 868, normalized size of antiderivative = 1.83

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\operatorname{arccsch}(cx)d\sqrt{ex+d}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{i\cdots}}{\cdots}\right.}{\cdots}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\operatorname{arccsch}(cx)d\sqrt{ex+d}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{i\cdots}}{\cdots}\right.}{\cdots}$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-d\sqrt{ex+d}\right)}{e^2}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{ice(\cdots}}{\cdots}\right.}{\cdots}$

input
```
int(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(-a*(-1/3*(e*x+d)^(3/2)+d*(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccs
ch(c*x)+arccsch(c*x)*d*(e*x+d)^(1/2)+2/3/c^2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d
)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2)*(2*I*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-2*I*E
llipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c
^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2))*c*d*e-EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),
(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^
(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2))*c^2*d^2+2*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticF((e*x+d)^(1/2)*((c*d+I
*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*e^2-EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*
e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+
c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c
*d)))
```

3.59.5 Fricas [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx=\int \frac{(b\operatorname{arcsch}(cx)+a)x}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x*arccsch(c*x) + a*x)/sqrt(e*x + d), x)
```

3.59.6 Sympy [F]

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx=\int \frac{x(a+b\operatorname{acsch}(cx))}{\sqrt{d+ex}}\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

3.59.7 Maxima [F]

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/3*a*((e*x + d)^(3/2)/e^2 - 3*sqrt(e*x + d)*d/e^2) + 1/3*b*(2*(e^2*x^2 -
d*e*x - 2*d^2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2) + 3*integrat
e(2/3*(c^2*e^2*x^3 - c^2*d*e*x^2 - 2*c^2*d^2*x)/((c^2*e^2*x^2 + e^2)*sqrt(
c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^2*x^2 + e^2)*sqrt(e*x + d)), x) - 3*in
tegrate(-1/3*(2*c^2*d*e*x^2 - (3*e^2*log(c) + 2*e^2)*c^2*x^3 + (4*c^2*d^2
- 3*e^2*log(c))*x - 3*(c^2*e^2*x^3 + e^2*x)*log(x))/((c^2*e^2*x^2 + e^2)*s
qrt(e*x + d)), x))
```

3.59.8 Giac [F]

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x}{\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/sqrt(e*x + d), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex}}\,dx = \int \frac{x\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.60 $\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

3.60.1 Optimal result

Integrand size = 18, antiderivative size = 284

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx &= \frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e} \\ &\quad + \frac{4bc\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\ &\quad - \frac{4bd\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}\end{aligned}$$

output
```
2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e+4*b*c*EllipticF(1/2*(1-x*(-c^2)^(1/2)
)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2
+1)^(1/2)*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^
(1/2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),
2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^
(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.60.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.45 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\left(ae(d+ex)-\frac{b\left(e+\frac{d}{x}\right)\left(-cex\text{csch}^{-1}(cx)+\frac{\sqrt{2}\sqrt{1+icx}\left(-e^2(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right),\frac{icd+e}{2e}\right)+cd(icd+e)\sqrt{-\frac{e(i+cx)}{cd-ie}}\cdots\right.}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{-\frac{e(i+cx)}{cd-ie}}(cd+cex)\cdots}\right.}{c}\right.}{e^2\sqrt{d+ex}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*(a*e*(d + e*x) - (b*(e + d/x)*(-(c*e*x*ArcCsch[c*x]) + (Sqrt[2]*Sqrt[1
+ I*c*x]*(-(e^2*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin
[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]) + c*d*(I*c*d + e
)*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d
 + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e)
)]], (I*c*d + e)/(2*e)]))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))]*(c*d + c*e*x))))/c))/(e^2*Sqrt[d + e*x])
```

3.60.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 737 vs. 2(284) = 568.

Time = 1.10 (sec) , antiderivative size = 737, normalized size of antiderivative = 2.60, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6844, 1898, 630, 1656, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 6844$$

$$\frac{2b\int \frac{\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{ce}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \; 1898$$

$$\frac{2b\sqrt{\frac{1}{c^2}+x^2}\int \frac{\sqrt{d+ex}}{x\sqrt{x^2+\frac{1}{c^2}}}dx}{cex\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e}$$

$$\Big\downarrow \; 630$$

$$\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{4b\sqrt{\frac{1}{c^2}+x^2}\int -\frac{d+ex}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{cex\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\Big\downarrow \; 1656$$

$$\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-$$

$$\frac{4b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{\sqrt{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\int \frac{1}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}+\frac{d\left(c^2d^2+e^2\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+\cdots}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\cdots}}\cdots}{e^2}\cdots\right.}{cex\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\Big\downarrow \; 1416$$

$$\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-$$

$$\frac{4b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{d\left(c^2d^2+e^2\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}+\frac{\left(c^2d^2+e^2\right)^{3/4}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\cdots}{\cdots}\right.}{cex\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\Big\downarrow \; 2222$$

3.60. $\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

$$\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e}-$$

$$4b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{d(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}{\left(\frac{1}{c^2}+\frac{d^2}{e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticPi}\left(\frac{(cd+\sqrt{c^2}}{4cd\sqrt{c^2}}\right.}{4\sqrt{c}d\sqrt{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}}\right.}{e^2}\right.$$

input
```
Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e - (4*b*Sqrt[c^(-2) + x^2]*(((c^2*
d^2 + e^2)^(3/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d
^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2)
/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticF
[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[
c^2*d^2 + e^2])/2])/(2*Sqrt[c]*e^2*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))
/e^2 + (d + e*x)^2/e^2]) + (d*(c^2*d^2 + e^2)*(1 - (c*d)/Sqrt[c^2*d^2 + e^
2])*((c*(1 + (c*d)/Sqrt[c^2*d^2 + e^2])*ArcTanh[Sqrt[d + e*x]/(c*Sqrt[d]*S
qrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])])/(2*Sqrt[d
]) + ((c^2*d^2 + e^2)^(1/4)*(1 - (c*d)/Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e
*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (
d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]
)^2)]*EllipticPi[(c*d + Sqrt[c^2*d^2 + e^2])^2/(4*c*d*Sqrt[c^2*d^2 + e^2])
, 2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], 1/2 + (d*Sqrt[c
^2*d^2 + e^2])/(2*c*(c^(-2) + d^2/e^2)*e^2)])/(4*Sqrt[c]*d*Sqrt[c^(-2) + d
^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])))/e^2))/(c*e*Sqrt[1 + 1/(
c^2*x^2)]*x)
```

3.60. $\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 630
```
Int[Sqrt[(c_) + (d_.)*(x_)]/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
Simp[-2   Subst[Int[x^2/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1656
```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(-a)*((e + d*q)/(c*d^2 - a*e^2))
   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Simp[a*d*((e + d*q)/(c*d^2 - a*e
^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 -
a*e^2, 0]
```

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

3.60. $\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.60.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\text{EllipticF}\left(\sqrt{\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c}{c}}}\right)}{e}$
default	$\frac{2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\text{EllipticF}\left(\sqrt{\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c}{c}}}\right)}{e}$
parts	$\frac{2a\sqrt{ex+d}}{e}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\text{EllipticF}\left(\right.\right.}{c\sqrt{\frac{c^2(ex+d)^2-}{}}}\right)}{e}$

input
```
int((a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arccsch(c*x)+2/c*(-(I*c*e*(e*x+d)+c^
2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d
)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-Ellip
ticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d
^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(
1/2)))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)))
```

3.60.5 Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/sqrt(e*x + d), x)
```

3.60.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{\sqrt{d+ex}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/sqrt(d + e*x), x)
```

3.60.7 Maxima [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex+d}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
b*(2*sqrt(e*x + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate(2*(c^2*e*x^2 +
 c^2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e*x^2 +
e)*sqrt(e*x + d)), x) - integrate(((e*log(c) + 2*e)*c^2*x^2 + 2*c^2*d*x +
e*log(c) + (c^2*e*x^2 + e)*log(x))/((c^2*e*x^2 + e)*sqrt(e*x + d)), x)) +
2*sqrt(e*x + d)*a/e
```

3.60.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{\sqrt{d + ex}}\, dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{ex + d}}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/sqrt(e*x + d), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{\sqrt{d + ex}}\, dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{\sqrt{d + e\,x}}\, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.61 $\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

3.61.2 Mathematica [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

3.61.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
```

output
```
$Aborted
```

3.61.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.61.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\text{arccsch}\left(cx\right)}{x\sqrt{ex+d}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

3.61.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d + ex}} \, dx = \int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex + d}x} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e*x^2 + d*x), x)
```

3.61.6 Sympy [N/A]

Not integrable

Time = 8.54 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d + ex}} \, dx = \int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{d + ex}} \, dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x)), x)
```

3.61.7 Maxima [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.52

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d + ex}} \, dx = \int \frac{b \operatorname{arcsch}(cx) + a}{\sqrt{ex + d}x} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
-(log(c)*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/sqrt(d)
+ integrate(log(x)/(sqrt(e*x + d)*x), x) - integrate(log(sqrt(c^2*x^2 + 1)
 + 1)/(sqrt(e*x + d)*x), x))*b + a*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x
 + d) + sqrt(d)))/sqrt(d)
```

3.61.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex+d}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x), x)
```

3.61.9 Mupad [N/A]

Not integrable

Time = 4.87 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

3.62 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

3.62.2 Mathematica [N/A]

Not integrable

Time = 8.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

3.62.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
```

output
```
$Aborted
```

3.62.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.62.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\text{arccsch}\left(cx\right)}{x^2\sqrt{ex+d}}dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

3.62.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.48

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\sqrt{d + ex}}\, dx = \int \frac{b \,\text{arcsch}\,(cx) + a}{\sqrt{ex + d}x^2}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e*x^3 + d*x^2), x)
```

3.62.6 Sympy [N/A]

Not integrable

Time = 17.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\sqrt{d + ex}}\, dx = \int \frac{a + b \,\text{acsch}\,(cx)}{x^2\sqrt{d + ex}}\, dx$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**2*sqrt(d + e*x)), x)
```

3.62.7 Maxima [N/A]

Not integrable

Time = 2.38 (sec) , antiderivative size = 175, normalized size of antiderivative = 8.33

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\sqrt{d + ex}}\, dx = \int \frac{b \,\text{arcsch}\,(cx) + a}{\sqrt{ex + d}x^2}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
1/2*((2*sqrt(e*x + d)*e/((e*x + d)*d - d^2) + e*log((sqrt(e*x + d) - sqrt(
d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2))*log(c) - 2*integrate(log(x)/(sqrt(
e*x + d)*x^2), x) + 2*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*
x^2), x))*b - 1/2*a*(2*sqrt(e*x + d)*e/((e*x + d)*d - d^2) + e*log((sqrt(e
*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2))
```

3.62.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex+dx^2}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

3.62.9 Mupad [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{d+e\,x}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

3.63 $\int \frac{x^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 731

$$\begin{aligned}
\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx &= \frac{4b\sqrt{d+ex}(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}x} \\
&+ \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} \\
&- \frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4} \\
&- \frac{32bcd\sqrt{d+ex}\sqrt{1+c^2x^2}E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{8bcd^2\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{4bc(2c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}e^3\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} \\
&- \frac{64bd^3\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce^4\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}
\end{aligned}$$

output
```
-2*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsch(c
*x))/e^4+2*d^3*(a+b*arccsch(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsch(c*x
))*(e*x+d)^(1/2)/e^4+4/15*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1+1/c^2/x
^2)^(1/2)-64/5*b*d^3*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(
1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)
/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/15
*b*c*d*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(
c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)
/e^3/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+8*b*
c*d^2*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c
^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2
)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*
b*c*(2*c^2*d^2-e^2)*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(
-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/
(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+
d)^(1/2)
```

3.63.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 35.09 (sec) , antiderivative size = 1042, normalized size of antiderivative = 1.43

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(4,-\frac{1}{2}\right)}{e^4(d+ex)^{3/2}}$$

$$+ \frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(\frac{32cd\sqrt{1+\frac{1}{c^2x^2}}}{15e^3}-\frac{32c^2d^2\operatorname{csch}^{-1}(cx)}{5e^4}+\frac{2c^2d^2\operatorname{csch}^{-1}(cx)}{e^3\left(e+\frac{d}{x}\right)}-\frac{2c^2x^2\operatorname{csch}^{-1}(cx)}{5e^2}-\frac{2cx\left(2e\sqrt{1+\frac{1}{c^2x^2}}-9cd\operatorname{csch}^{-1}(cx)\right)}{15e^3}\right)}{(d+ex)^{3/2}} - \frac{2(e \cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2)
) + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 + 1/(c^2*x^2)])/(15*e^3) -
(32*c^2*d^2*ArcCsch[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsch[c*x])/(e^3*(e + d/x
)) - (2*c^2*x^2*ArcCsch[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 + 1/(c^2*x^2)]
- 9*c*d*ArcCsch[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c
*x)^(3/2)*(-((Sqrt[2]*(32*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[
(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*
e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2
)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(48*c^3*d^3
 - 8*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)
^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) +
(16*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d
*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[A
rcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((
e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*
d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c
*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d -
 I*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))
/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Elliptic...
```

3.63.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1488 vs. $2(731) = 1462$.

Time = 3.77 (sec) , antiderivative size = 1488, normalized size of antiderivative = 2.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {6864, 27, 7272, 2351, 631, 1540, 1416, 2185, 27, 599, 25, 27, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{6864}$$

$$\frac{b\int \frac{2\left(16d^3+8exd^2-2e^2x^2d+e^3x^3\right)}{5e^4\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{c}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{16d^3+8exd^2-2e^2x^2d+e^3x^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{5ce^4}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{ 7272}$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{16d^3+8exd^2-2e^2x^2d+e^3x^3}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{5ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{ 2351}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(16d^3\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx+\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{5ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{ 631}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx-32d^3\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{5ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{ 1540}$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+e\right.\right.}{5ce^4x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{ 1416}$$

3.63. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}dx$

$$2b\sqrt{c^2x^2+1}\left(\int \frac{x^2e^3-2dxe^2+8d^2e}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+e\right.\right.$$

$5c$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\big\downarrow$ 2185

$$2b\sqrt{c^2x^2+1}\left(-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(c\right.}{}\right.\right.$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\big\downarrow$ 27

$$2b\sqrt{c^2x^2+1}\left(-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(c\right.}{}\right.\right.$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$\big\downarrow$ 599

3.63. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}dx$

$$2b\sqrt{c^2x^2+1}\left(-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}(c\right.\right.$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\downarrow 25$$

$$2b\sqrt{c^2x^2+1}\left(-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}(c\right.\right.$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\downarrow 27$$

$$2b\sqrt{c^2x^2+1}\left(-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}(c\right.\right.$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\downarrow 1511$$

3.63. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$2b\sqrt{c^2x^2+1}\left(-32d^3\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(c\right.}{}\right.\right.$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^3}{e^4\sqrt{d+ex}}+\frac{6\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4}-\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}+$$

$$2b\sqrt{c^2x^2+1}\left(-32\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\right.}{}\right.\right.$$

$$\Big\downarrow \text{1509}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^3}{e^4\sqrt{d+ex}}+\frac{6\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4}-\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^4}+$$

$$2b\sqrt{c^2x^2+1}\left(-32\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\right.}{}\right.\right.$$

3.63. $\displaystyle\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$\downarrow$ 2222

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^3}{e^4\sqrt{d+ex}}+\frac{6\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{e^4}-\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^4}+$$

$$2b\sqrt{c^2x^2+1}\left(-32\left(\left(\frac{c^2d^2}{e^2}+1\right)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right)}{2\sqrt{d}}+\frac{\sqrt[4]{c^2d^2 \cdots}}{\cdots}\right.\right.\right.$$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(2*d^3*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a
 + b*ArcCsch[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/e^4 +
 (2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) + (2*b*Sqrt[1 + c^2*x^2]
*((2*e^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2])/(3*c^2) + (2*(8*c*d*Sqrt[c^2*d^2
 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2
 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^
2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 +
 e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^
2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*E
llipticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*
d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d
+ e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])) + ((c^2*d^2 + e^2)^(1/4)*(32*c^2*d^
2 - e^2 - 8*c*d*Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2
])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e
^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*Ellip
ticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/S
qrt[c^2*d^2 + e^2])/2])/(2*Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d +
e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/(3*c^2) - 32*d^3*(-1/2*(Sqrt[c]*(c^2
*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*
d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d...
```

3.63. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

3.63. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.63.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.98 (sec) , antiderivative size = 2021, normalized size of antiderivative = 2.76

method	result	size
derivativedivides	Expression too large to display	2021
default	Expression too large to display	2021
parts	Expression too large to display	2022

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^4*(-a*(-1/5*(e*x+d)^(5/2)+(e*x+d)^(3/2)*d-3*d^2*(e*x+d)^(1/2)-d^3/(e*x
+d)^(1/2))-b*(-1/5*arccsch(c*x)*(e*x+d)^(5/2)+arccsch(c*x)*(e*x+d)^(3/2)*d
-3*arccsch(c*x)*d^2*(e*x+d)^(1/2)-arccsch(c*x)*d^3/(e*x+d)^(1/2)-2/15/c^3*
(-2*I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*(e*x+d)^(3/2)*c^2*d*e-((c*d+I*e)*c
/(c^2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2)*(e*x+d)^(1/2)*e^3-I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d
^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*
e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/
2)*(e*x+d)^(1/2)*c^2*d^2*e+2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*
x+d)^(3/2)-8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1
/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipt
icE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+48*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/
2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+
I*e)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3-24*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2)...
```

3.63.5 Fricas [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex)^{3/2}} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsch(c*x) + a*x^3)*sqrt(e*x + d)/(e^2*x^2 + 2*d*e*x + d
^2), x)
```

3.63.6 Sympy [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex)^{3/2}} \, dx = \int \frac{x^3(a + b \operatorname{acsch}(cx))}{(d + ex)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

3.63.7 Maxima [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex)^{3/2}} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex + d)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/5*a*((e*x + d)^(5/2)/e^4 - 5*(e*x + d)^(3/2)*d/e^4 + 15*sqrt(e*x + d)*d^
2/e^4 + 5*d^3/(sqrt(e*x + d)*e^4)) + 1/5*b*(2*(e^3*x^3 - 2*d*e^2*x^2 + 8*d
^2*e*x + 16*d^3)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^4) + 5*integr
ate(2/5*(c^2*e^3*x^4 - 2*c^2*d*e^2*x^3 + 8*c^2*d^2*e*x^2 + 16*c^2*d^3*x)/(
(c^2*e^4*x^2 + e^4)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^4*x^2 + e^4)*
sqrt(e*x + d)), x) - 5*integrate(-1/5*(2*c^2*d*e^3*x^4 - 48*c^2*d^3*e*x^2
- (5*e^4*log(c) + 2*e^4)*c^2*x^5 - 32*c^2*d^4*x - (12*c^2*d^2*e^2 + 5*e^4*
log(c))*x^3 - 5*(c^2*e^4*x^5 + e^4*x^3)*log(x))/((c^2*e^5*x^3 + c^2*d*e^4*
x^2 + e^5*x + d*e^4)*sqrt(e*x + d)), x))
```

3.63.8 Giac [F]

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex)^{3/2}}\,dx=\int \frac{x^3\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.64 $\int \frac{x^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.64.1 Optimal result

Integrand size = 21, antiderivative size = 499

$$\begin{aligned}
\int \frac{x^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = &-\frac{2d^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} \\
&-\frac{4d\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^3} \\
&+\frac{4bc\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&-\frac{20bcd\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&+\frac{32bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}$$

output
```
2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2*d^2*(a+b*arccsch(c*x))/e^3/(e*x
+d)^(1/2)-4*d*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+32/3*b*d^2*EllipticPi(1
/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2)
)*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/
x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*EllipticE(1/2*(1-x*(-c^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)
^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+
d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-20/3*b*c*d*EllipticF(1/2*(1-x*(-c^2)^(1/2
))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^
2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(
1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 35.03 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.96

$$\int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex)^{3/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(3,-\frac{1}{2}\right)}{e^3(d+ex)^{3/2}}$$

$$+ \frac{b\left(-\frac{c^2\left(e+\frac{d}{x}\right)^2x^2\left(-\frac{4\sqrt{1+\frac{1}{c^2x^2}}}{3e^2}+\frac{16cd\operatorname{csch}^{-1}(cx)}{3e^3}-\frac{2cd\operatorname{csch}^{-1}(cx)}{e^2\left(e+\frac{d}{x}\right)}-\frac{2cx\operatorname{csch}^{-1}(cx)}{3e^2}\right)}{(d+ex)^{3/2}}+\frac{2\left(e+\frac{d}{x}\right)^{3/2}(cx)^{3/2}\left(-\frac{5\sqrt{2}cde\sqrt{1+icx}(i+cx)\sqrt{\frac{cd+\cdots}{cd-\cdots}}}{\sqrt{1+\frac{1}{c^2x^2}}\cdots}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

output
```
-((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/
2))) + (b*(-((c^2*(e + d/x)^2*x^2*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2) + (1
6*c*d*ArcCsch[c*x])/(3*e^3) - (2*c*d*ArcCsch[c*x])/(e^2*(e + d/x)) - (2*c*
x*ArcCsch[c*x])/(3*e^2)))/(d + e*x)^(3/2)) + (2*(e + d/x)^(3/2)*(c*x)^(3/2
)*((-5*Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I
*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*
e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))
/(I*c*d + e)]) + (I*Sqrt[2]*(c*d - I*e)*(8*c^2*d^2 - e^2)*Sqrt[1 + I*c*x]*
Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e,
ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1
+ 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e*Cosh[2*ArcCsch[c*x]]*(-((
c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt
[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I
*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*
x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d
 + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[S
qrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d + e)*S
qrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c
*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I
+ c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c...
```

3.64.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. $2(499) = 998$.

Time = 3.01 (sec) , antiderivative size = 1415, normalized size of antiderivative = 2.84, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6864, 27, 7272, 2351, 599, 25, 27, 631, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{6864}$$

$$\frac{b\int -\frac{2\left(8d^2+4exd-e^2x^2\right)}{3e^3\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{c}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\int \frac{8d^2+4exd-e^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{3ce^3}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 7272$$

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{8d^2+4exd-e^2x^2}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 2351$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx+\int \frac{4de-e^2x}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 599$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx-\frac{2\int -\frac{e^2(4d-ex)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e^2}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\frac{2\int \frac{e^2(4d-ex)}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e^2}+8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 27$$

3.64. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\int \frac{4d-ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+8d^2\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 631

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\int \frac{4d-ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-16d^2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 1511

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\left(\frac{\left(5cd-\sqrt{c^2d^2+e^2}\right)\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}+\frac{\sqrt{c^2d^2+e^2}\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d \cdots}}{c}\cdots\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 1416

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\left(\frac{\sqrt{c^2d^2+e^2}\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}+\frac{\sqrt[4]{c^2d^2+e^2}\left(5cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\cdots}{\left(\frac{c^2}{e}\cdots\right.}}}{2c^{3/2}\sqrt{\cdots}}\cdots\right.\right.}{3c\cdots}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 1509

3.64. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2\left(\frac{\sqrt{c^2d^2+e^2}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right)|\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{\sqrt{c}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}\cdots}{c}\cdots\right.\right.}{\cdots}$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\Big\downarrow$ 1540

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^3\sqrt{d+ex}}-\frac{4\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-$$
$$\frac{2b\sqrt{c^2x^2+1}\left(2\left(\frac{\sqrt{c^2d^2+e^2}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right)|\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}\cdots\right.\right.}{\sqrt{c}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots}{c}\cdots\right.\right.}{\cdots}$$

$\Big\downarrow$ 1416

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^3\sqrt{d+ex}}-\frac{4\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-$$
$$\frac{2b\sqrt{c^2x^2+1}\left(2\left(\frac{\sqrt{c^2d^2+e^2}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right)|\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}\cdots\right.\right.}{\sqrt{c}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots}{c}\cdots\right.\right.}{\cdots}$$

3.64. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

$$\Big\downarrow \text{2222}$$

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^3\sqrt{d+ex}}-\frac{4\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(2\left(\frac{\sqrt{c^2d^2+e^2}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right)|\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}\right.\right.}{\sqrt{c}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}}{c}\right.\right.}{}$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(-2*d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a
+ b*ArcCsch[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3)
- (2*b*Sqrt[1 + c^2*x^2]*(2*((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1
 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 +
 (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^
2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 -
 (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1
+ (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[
d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqr
t[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/
e^2])))/c + ((c^2*d^2 + e^2)^(1/4)*(5*c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x
))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sq
rt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2
 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[1 + (c
^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])) - 16*d^2*
(-1/2*(Sqrt[c]*(c^2*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x
))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sq
rt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*...
```

3.64. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.64.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.64. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.64.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.03 (sec) , antiderivative size = 896, normalized size of antiderivative = 1.80

method	result
derivativedivides	$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2}{c^2}}}{}\right.$
default	$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2}{c^2}}}{}\right.$
parts	$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)}{e^3}+\frac{2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)}{}}}{}\right.}{}$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e
*x+d)^(3/2)*arccsch(c*x)-2*arccsch(c*x)*d*(e*x+d)^(1/2)-arccsch(c*x)*d^2/(
e*x+d)^(1/2)-2/3/c^2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+
e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2
)*(5*I*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-8*I*EllipticPi((e*x+d)^(1/2)*
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d
)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c*d*e-4*Ellipt
icF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+
8*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c
*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2))*c^2*d^2+EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(
1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE((e*x+d)
^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^
2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x
^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

3.64.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.64.6 Sympy [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx=\int \frac{x^2(a+b\operatorname{acsch}(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

3.64.7 Maxima [F]

$$\int \frac{x^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/3*a*((e*x + d)^(3/2)/e^3 - 6*sqrt(e*x + d)*d/e^3 - 3*d^2/(sqrt(e*x + d)*
e^3)) + 1/3*b*(2*(e^2*x^2 - 4*d*e*x - 8*d^2)*log(sqrt(c^2*x^2 + 1) + 1)/(s
qrt(e*x + d)*e^3) + 3*integrate(2/3*(c^2*e^2*x^3 - 4*c^2*d*e*x^2 - 8*c^2*d
^2*x)/((c^2*e^3*x^2 + e^3)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^2
+ e^3)*sqrt(e*x + d)), x) - 3*integrate(-1/3*(6*c^2*d*e^2*x^3 - (3*e^3*log
(c) + 2*e^3)*c^2*x^4 + 16*c^2*d^3*x + 3*(8*c^2*d^2*e - e^3*log(c))*x^2 - 3
*(c^2*e^3*x^4 + e^3*x^2)*log(x))/((c^2*e^4*x^3 + c^2*d*e^3*x^2 + e^4*x + d
*e^3)*sqrt(e*x + d)), x))
```

3.64.8 Giac [F]

$$\int \frac{x^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^2}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \int \frac{x^2\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.65 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 318

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{4bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}\,e\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}} - \frac{8bd\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
2*d*(a+b*arccsch(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsch(c*x))*(e*x+d)^(1/2
)/e^2-8*b*d*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(
d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2
)^(1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4*b*c*Elliptic
F(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^
(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2
)/(-c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.03 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.83

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}}+\frac{b(2d+ex)\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}-\frac{2ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(\text{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd-ie}}\right.\right.\right.}{c\sqrt{}}\right.}{e^2}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsch[c*x])/Sqrt[d +
e*x] - ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(
c*d - I*e))]*(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (
c*d - I*e)/(c*d + I*e)] - 2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c
/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c*Sqrt[-(c/(c*d
 - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/e^2
```

3.65.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1009 vs. 2(318) = 636.

Time = 2.37 (sec) , antiderivative size = 1009, normalized size of antiderivative = 3.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {6864, 27, 7272, 2351, 27, 510, 631, 1416, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{6864}$$

$$\frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{c}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{2d+ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{ce^2}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \ 7272$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{2d+ex}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \ 2351$$

$$\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{e}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx+2d\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\sqrt{c^2x^2+1}\left(e\int \frac{1}{\sqrt{d+ex}\sqrt{c^2x^2+1}}dx+2d\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \ 510$$

$$\frac{2b\sqrt{c^2x^2+1}\left(2\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}+2d\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \ 631$$

$$\frac{2b\sqrt{c^2x^2+1}\left(2\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}-4d\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}$$

$$\Big\downarrow \ 1416$$

3.65. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}dx$

$$2b\sqrt{c^2x^2+1}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right.}{\sqrt{c}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}$$

$$\overline{\qquad ce^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{2d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}$$

$\downarrow$ 1540

$$2b\sqrt{c^2x^2+1}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right.}{\sqrt{c}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}$$

$$\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{2d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}$$

$\downarrow$ 1416

$$2b\sqrt{c^2x^2+1}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right.}{\sqrt{c}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}$$

$$\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{2d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}$$

$\downarrow$ 2222

$$\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{2d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}+$$

$$2b\sqrt{c^2x^2+1}\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right.}{\sqrt{c}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}$$

3.65. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

output
```
(2*d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*A
rcCsch[c*x]))/e^2 + (2*b*Sqrt[1 + c^2*x^2]*(((c^2*d^2 + e^2)^(1/4)*(1 + (c
*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e
*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/
Sqrt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d
^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c
^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2]) - 4*d*(-1/
2*(Sqrt[c]*(c^2*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d +
e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e
^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c
^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e
^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(e^2*Sqrt[1 + (c^2*d^2)/e^
2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2]) + (1 + (c^2*d^2)/e^2
)*(1 - (c*d)/Sqrt[c^2*d^2 + e^2])*(((1 + (c*d)/Sqrt[c^2*d^2 + e^2])*ArcTan
h[Sqrt[d + e*x]/(Sqrt[d]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2
+ (c^2*(d + e*x)^2)/e^2])])/(2*Sqrt[d]) + ((c^2*d^2 + e^2)^(1/4)*(1 - (c*d
)/Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (
c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2
*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticPi[(c*d + S
qrt[c^2*d^2 + e^2])^2/(4*c*d*Sqrt[c^2*d^2 + e^2]), 2*ArcTan[(Sqrt[c]*Sq...
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.65. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

3.65.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.85 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.32

method	result
parts	$\frac{2a\left(\sqrt{ex+d}+\frac{d}{\sqrt{ex+d}}\right)}{e^2}+\frac{2b\left(\sqrt{ex+d}\,\operatorname{arccsch}(cx)+\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\,\sqrt{\frac{ice(ex+d)-c^2d(e}{c^2d^2}}}{}\right.}{}$
derivativedivides	$\frac{-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsch}(cx)-\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\,\sqrt{\frac{ice(ex+d)-c^2}{c^2}}}{}\right.}{}$
default	$\frac{-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsch}(cx)-\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\,\sqrt{\frac{ice(ex+d)-c^2}{c^2}}}{}\right.}{}$

input
```
int(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*a/e^2*((e*x+d)^(1/2)+d/(e*x+d)^(1/2))+2*b/e^2*((e*x+d)^(1/2)*arccsch(c*x
)+arccsch(c*x)*d/(e*x+d)^(1/2)+2/c*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)
,(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-2*EllipticPi((e*x+d)^(1/2
)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c
*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)))/((c^2*(e*x+
d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((c*d+I*e)*c/(c^2*d
^2+e^2))^(1/2))
```

3.65.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.65.6 Sympy [F]

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx=\int \frac{x(a+b\,\text{acsch}\,(cx))}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

3.65.7 Maxima [F]

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
b*(2*(e*x + 2*d)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2) + integrat
e(2*(c^2*e*x^2 + 2*c^2*d*x)/((c^2*e^2*x^2 + e^2)*sqrt(c^2*x^2 + 1)*sqrt(e*
x + d) + (c^2*e^2*x^2 + e^2)*sqrt(e*x + d)), x) - integrate((6*c^2*d*e*x^2
 + (e^2*log(c) + 2*e^2)*c^2*x^3 + (4*c^2*d^2 + e^2*log(c))*x + (c^2*e^2*x^
3 + e^2*x)*log(x))/((c^2*e^3*x^3 + c^2*d*e^2*x^2 + e^3*x + d*e^2)*sqrt(e*x
 + d)), x)) + 2*a*(sqrt(e*x + d)/e^2 + d/(sqrt(e*x + d)*e^2))
```

3.65.8 Giac [F]

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(3/2), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{(d+ex)^{3/2}}\,dx = \int \frac{x\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.66 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

3.66.1 Optimal result

Integrand size = 18, antiderivative size = 149

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{4b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}$$

output
```
-2*(a+b*arccsch(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(1-x*(-c^2)^(1/2)
)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*
((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\frac{-2e(1+c^2x^2)\left(a+b\text{csch}^{-1}(cx)\right)+2bc(icd+e)\sqrt{2+\frac{2}{c^2x^2}}x\sqrt{1+icx}\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}}{e^2\sqrt{d+ex}\,(1+c^2x^2)}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

output
```
(-2*e*(1 + c^2*x^2)*(a + b*ArcCsch[c*x]) + 2*b*c*(I*c*d + e)*Sqrt[2 + 2/(c
^2*x^2)]*x*Sqrt[1 + I*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*E
llipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*
d + e)/(2*e)])/(e^2*Sqrt[d + e*x]*(1 + c^2*x^2))
```

3.66.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 736 vs. $2(149) = 298$.

Time = 1.09 (sec) , antiderivative size = 736, normalized size of antiderivative = 4.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6844, 1898, 631, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6844}$$

$$-\frac{2b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2\sqrt{d+ex}}dx}{ce}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1898}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 631}$$

$$\frac{4b\sqrt{\frac{1}{c^2}+x^2}\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 1540}$$

$$\frac{4b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{c\left(cd-\sqrt{c^2d^2+e^2}\right)\int \frac{1}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}\cdots}\cdots}{e^2}\cdots\right.}{cex\sqrt{\frac{1}{c^2x^2}+1}}\cdots$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$\downarrow$ 1416

$$\frac{4b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\int -\frac{\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2}-\frac{\sqrt{c}\sqrt[4]{c^2d^2+e^2}\left(cd-\sqrt{c^2d^2+e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\cdots\right.}{\cdots}\right.}{cex\sqrt{\frac{1}{c^2x^2}+1}}\cdots$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

$\downarrow$ 2222

$$\frac{4b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{(c^2d^2+e^2)\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(1-\frac{cd}{\sqrt{c^2d^2+e^2}}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}{\left(\frac{1}{c^2}+\frac{d^2}{e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticPi}\left(\frac{\left(cd+\sqrt{c^2\cdots}\right.}{4cd\sqrt{c^2\cdots}}\cdots\right.}{4\sqrt{c}d\sqrt{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}}\cdots\right.}{e^2}\cdots\right.}{\cdots}$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

3.66. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

output
```
(-2*(a + b*ArcCsch[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[c^(-2) + x^2]*(-1/
2*(Sqrt[c]*(c^2*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d +
e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 +
(d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2
])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)],
(1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(e^2*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d +
 e*x))/e^2 + (d + e*x)^2/e^2]) + ((c^2*d^2 + e^2)*(1 - (c*d)/Sqrt[c^2*d^2
+ e^2])*((c*(1 + (c*d)/Sqrt[c^2*d^2 + e^2])*ArcTanh[Sqrt[d + e*x]/(c*Sqrt[
d]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])])/(2*Sq
rt[d]) + ((c^2*d^2 + e^2)^(1/4)*(1 - (c*d)/Sqrt[c^2*d^2 + e^2])*(1 + (c*(d
 + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2
 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 +
e^2])^2)]*EllipticPi[(c*d + Sqrt[c^2*d^2 + e^2])^2/(4*c*d*Sqrt[c^2*d^2 + e
^2]), 2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], 1/2 + (d*Sq
rt[c^2*d^2 + e^2])/(2*c*(c^(-2) + d^2/e^2)*e^2)])/(4*Sqrt[c]*d*Sqrt[c^(-2)
 + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])))/e^2))/(c*e*Sqrt[1 +
 1/(c^2*x^2)]*x)
```

3.66.3.1 Defintions of rubi rules used

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.66. $\int \frac{a+b\,\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.66.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\,\text{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(c}{c^2}} \cdots\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2+e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}}\right.}{e}$
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\,\text{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(c}{c^2}} \cdots\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2+e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{(cd+ie)c}{c^2d^2+e^2}}}\right.}{e}$
parts	$-\frac{2a}{\sqrt{ex+d}\,e}+\frac{2b\left(-\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ice(ex+d)+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ice(ex+d)-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\,\text{EllipticPi}\left(\sqrt{ex+} \cdots\right.}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2+e^2}{c^2e^2x^2}}\,xd\sqrt{\frac{(cd+ie)}{c^2d^2+e} \cdots}}\right.}{e}$

input
```
int((a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsch(c*x)+2/c/((c^2*(e*x+d)^2
-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/d/((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2)*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1
/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipt
icPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^
2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2))))
```

3.66.5 Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

3.66.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{a+b\operatorname{acsch}(cx)}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

3.66.7 Maxima [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
-(2*c^2*integrate(x/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^
2*e*x^2 + e)*sqrt(e*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x +
 d)*e) + integrate(((e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + e*log(c) + (c^2
*e*x^2 + e)*log(x))/((c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqrt(e*x +
d)), x))*b - 2*a/(sqrt(e*x + d)*e)
```

3.66.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}(cx)+a}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x + d)^(3/2), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

3.67.2 Mathematica [N/A]

Not integrable

Time = 14.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

3.67.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}} \, dx$$
$$\Big\downarrow \text{6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
```

output
```
$Aborted
```

3.67.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.67.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\text{arccsch}\left(cx\right)}{x\left(ex+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

3.67.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^3 + 2*d*e*x^2 + d^2*x),
 x)
```

3.67.6 Sympy [N/A]

Not integrable

Time = 32.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*(d + e*x)**(3/2)), x)
```

3.67.7 Maxima [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 8.52

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
-b*((e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(3/2) +
2*e/(sqrt(e*x + d)*d))*log(c)/e + integrate(log(x)/(sqrt(e*x + d)*e*x^2 +
sqrt(e*x + d)*d*x), x) - integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x +
d)*e*x^2 + sqrt(e*x + d)*d*x), x)) + a*(log((sqrt(e*x + d) - sqrt(d))/(sqr
t(e*x + d) + sqrt(d)))/d^(3/2) + 2/(sqrt(e*x + d)*d))
```

3.67.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

3.67.9 Mupad [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

3.68 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

3.68.2 Mathematica [N/A]

Not integrable

Time = 19.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

3.68.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}} \, dx$$
$$\Big\downarrow \text{6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
```

output
```
$Aborted
```

3.68.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.68.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\operatorname{arccsch}(cx)}{x^2\,(ex+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

3.68.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2
), x)
```

3.68.6 Sympy [N/A]

Not integrable

Time = 59.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{x^2\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

3.68.7 Maxima [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 252, normalized size of antiderivative = 12.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
1/2*b*((3*e^2*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(
5/2) + 2*(3*(e*x + d)*e^2 - 2*d*e^2)/((e*x + d)^(3/2)*d^2 - sqrt(e*x + d)*
d^3))*log(c)/e - 2*integrate(log(x)/(sqrt(e*x + d)*e*x^3 + sqrt(e*x + d)*d
*x^2), x) + 2*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e*x^3 +
sqrt(e*x + d)*d*x^2), x)) - 1/2*a*(2*(3*(e*x + d)*e - 2*d*e)/((e*x + d)^(3
/2)*d^2 - sqrt(e*x + d)*d^3) + 3*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x
 + d) + sqrt(d)))/d^(5/2))
```

3.68.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

3.68.9 Mupad [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

3.69 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 777

$$
\begin{aligned}
\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx &= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&+ \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} \\
&- \frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4} \\
&- \frac{8b\sqrt{-c^2}d^2\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&+ \frac{4bc(2c^2d^2+e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} \\
&- \frac{32bcd\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\
&+ \frac{64bd^2\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^4\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}
\end{aligned}
$$

output
```
2/3*d^3*(a+b*arccsch(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsc
h(c*x))/e^4-6*d^2*(a+b*arccsch(c*x))/e^4/(e*x+d)^(1/2)+4/3*b*d^2*(c^2*x^2+
1)/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsc
h(c*x))*(e*x+d)^(1/2)/e^4+64/3*b*d^2*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/
2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x
+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)+4/3*b*c*(2*c^2*d^2+e^2)*EllipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*
2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c
^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c^2*
(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/3*b*d^2*EllipticE(1/2*(1-x*(-c^2)^
(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c
^2)^(1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x
^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-32/3*b*c*d*EllipticF(
1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/
(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.69.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 34.88 (sec) , antiderivative size = 1108, normalized size of antiderivative = 1.43

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \frac{ad^4\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(4,-\frac{3}{2}\right)}{e^4(d+ex)^{5/2}}$$

$$+ b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{4\sqrt{1+\frac{1}{c^2x^2}}}{3e(c^2d^2+e^2)}+\frac{32cd\,\text{csch}^{-1}(cx)}{3e^4}-\frac{2cd\,\text{csch}^{-1}(cx)}{3e^2\left(e+\frac{d}{x}\right)^2}-\frac{2cx\,\text{csch}^{-1}(cx)}{3e^3}-\frac{2\left(2c^2d^2e\sqrt{1+\frac{1}{c^2x^2}}+7c^3d^3\text{csch}^{-1}(cx)+7cde^2\text{csch}^{-1}\cdots\right.}{3e^3(c^2d^2+e^2)\left(e+\frac{d}{x}\right)}\cdots\right)}{(d+ex)^{5/2}}\cdots\right)$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2)
) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e*(c^2*d^2 +
 e^2)) + (32*c*d*ArcCsch[c*x])/(3*e^4) - (2*c*d*ArcCsch[c*x])/(3*e^2*(e +
d/x)^2) - (2*c*x*ArcCsch[c*x])/(3*e^3) - (2*(2*c^2*d^2*e*Sqrt[1 + 1/(c^2*x
^2)] + 7*c^3*d^3*ArcCsch[c*x] + 7*c*d*e^2*ArcCsch[c*x]))/(3*e^3*(c^2*d^2 +
 e^2)*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*(-((S
qrt[2]*(8*c^3*d^3*e + 8*c*d*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e
*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*
c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*
(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(16*c^4*d^4 + 16*c^2*
d^2*e^2 - e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
 (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) +
 (2*e^3*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*S
qrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*
(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d
+ I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d
 + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I
*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x)...
```

3.69.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2180 vs. 2(777) = 1554.

Time = 5.25 (sec) , antiderivative size = 2180, normalized size of antiderivative = 2.81, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.190, Rules used = {6864, 27, 7272, 2351, 635, 25, 27, 498, 27, 507, 631, 1459, 1416, 1509, 1540, 1416, 2182, 27, 599, 25, 27, 1511, 1416, 1509, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6864}$$

$$\frac{b\int -\frac{2\left(16d^3+24exd^2+6e^2x^2d-e^3x^3\right)}{3e^4\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{c}+\frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\int \frac{16d^3+24exd^2+6e^2x^2d-e^3x^3}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce^4}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 7272$$

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{16d^3+24exd^2+6e^2x^2d-e^3x^3}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 2351$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(16d^3\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 635$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(16d^3\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$$\Big\downarrow \; 27$$

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx}{d}\right)+\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 498

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}\right)\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}\right)\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 507

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+16d^3\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e}{\left(c^2d^2\right.}\right.}{d}\right.\right.}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\big\downarrow$ 631

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+16d^3\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}-\frac{2\int -}{}\right.\right.}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\big\downarrow$ 1459

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{-x^2e^3+6dxe^2+24d^2e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+16d^3\left(-\frac{e\left(\frac{2c^2\left(\frac{\sqrt{c^2d^2+e^2}\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\int \frac{}{\sqrt{(d\cdots}}}{}\cdots\right.}{e\left(c^2d^2+e^2\right)}\cdots\right.}{d}\cdots\right.\right.}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}\cdots}$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\downarrow$ 1416

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(16d^3\left(-\frac{e\left(\frac{2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\cdots\right.\right.}{2c^{3/2}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}\cdots\right.}{e\left(c^2d^2+e^2\right)}\cdots\right.}{d}\cdots\right.\right.}{\cdots}$$

$$\frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}$$

$\Big\downarrow$ 1509

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{\cdots}\right.\right.$$

$\downarrow$ 1540

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^4}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(16-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

↓ 1416

3.69. $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}\cdots}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{\cdots}\right.\right.$$

↓ 2182

3.69. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{}\right.\right.$$

↓ 27

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.}{}\right.\right.\right.$$

↓ 599

3.69. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{\quad}\right.\right.$$

↓ 25

3.69. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\cdots\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.\right.}{\cdots}$$

↓ 27

3.69. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{}\right.\right.$$

↓ 1511

3.69. $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\cdots\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.\right.}{\cdots}\cdots\right.\right.}{\cdots}$$

↓ 1416

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2}}\cdots\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.\right.}{\cdots}\cdots\right.\right.$$

↓ 1509

3.69. $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\text{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{\cdots}\right.\right.}{\cdots}$$

$\downarrow$ 2222

3.69. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^3}{3e^4(d+ex)^{3/2}}-\frac{6\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{e^4\sqrt{d+ex}}-\frac{6\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}-$$

$$2b\sqrt{c^2x^2+1}\left(16\left(-\frac{e\left(\frac{2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\Big(\cdots\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots}{\cdots}\cdots}{\cdots}\cdots\right.\right.$$

3.69. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(2*d^3*(a + b*ArcCsch[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^2*(a + b*ArcCs
ch[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e
^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^4) - (2*b*Sqrt[1 + c^2*
x^2]*((-34*d^2*e^2*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*Sqrt[d + e*x]) + (2
*(-(((16*c^2*d^2 - e^2)*Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^
2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*
d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/
4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^
2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*
x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*S
qrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2]))
)/c) + ((c^2*d^2 + e^2)^(3/4)*(16*c^2*d^2 - e^2 + 8*c*d*Sqrt[c^2*d^2 + e^2
])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^
2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(
d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*
x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)
*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2]
)))/(c^2*d^2 + e^2) + 16*d^3*(-((e*((-2*e*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e
^2)*Sqrt[d + e*x]) + (2*c^2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*S...
```

3.69.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

3.69. $\int \frac{x^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.69. $\int \frac{x^3 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}} \, dx$

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.69.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.29 (sec) , antiderivative size = 2728, normalized size of antiderivative = 3.51

method	result	size
derivativedivides	Expression too large to display	2728
default	Expression too large to display	2728
parts	Expression too large to display	2729

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^4*(-a*(-1/3*(e*x+d)^(3/2)+3*d*(e*x+d)^(1/2)-1/3*d^3/(e*x+d)^(3/2)+3*d^
2/(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsch(c*x)+3*arccsch(c*x)*d*(e*x+
d)^(1/2)-1/3*arccsch(c*x)*d^3/(e*x+d)^(3/2)+3*arccsch(c*x)*d^2/(e*x+d)^(1/
2)-2/3/c^2*(-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2)
)^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*El
lipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3*(e*x+d)^(1/2)+I*((c*d+I*e)*c/(c^2*d^
2+e^2))^(1/2)*c^3*d^4*e-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*
c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e*(e*x+d)^(1/2)-((c*d+I*e
)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*(e*x+d)^2+8*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d
)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^
2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4*(e*x+d)^
(1/2)-16*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*
((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi
((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))
*c^4*d^4*(e*x+d)^(1/2)+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*e*(e...
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.69.6 Sympy [F]

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x^3(a+b\,\text{acsch}\,(cx))}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

3.69.7 Maxima [F]

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*b*(2*(e^3*x^3 - 6*d*e^2*x^2 - 24*d^2*e*x - 16*d^3)*log(sqrt(c^2*x^2 +
1) + 1)/((e^5*x + d*e^4)*sqrt(e*x + d)) + 3*integrate(2/3*(c^2*e^3*x^4 - 6
*c^2*d*e^2*x^3 - 24*c^2*d^2*e*x^2 - 16*c^2*d^3*x)/((c^2*e^5*x^3 + c^2*d*e^
4*x^2 + e^5*x + d*e^4)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^5*x^3 + c^
2*d*e^4*x^2 + e^5*x + d*e^4)*sqrt(e*x + d)), x) - 3*integrate(-1/3*(10*c^2
*d*e^3*x^4 + 80*c^2*d^3*e*x^2 - (3*e^4*log(c) + 2*e^4)*c^2*x^5 + 32*c^2*d^
4*x + 3*(20*c^2*d^2*e^2 - e^4*log(c))*x^3 - 3*(c^2*e^4*x^5 + e^4*x^3)*log(
x))/((c^2*e^6*x^4 + 2*c^2*d*e^5*x^3 + 2*d*e^5*x + d^2*e^4 + (c^2*d^2*e^4 +
 e^6)*x^2)*sqrt(e*x + d)), x)) + 2/3*a*((e*x + d)^(3/2)/e^4 - 9*sqrt(e*x +
 d)*d/e^4 - 9*d^2/(sqrt(e*x + d)*e^4) + d^3/((e*x + d)^(3/2)*e^4))
```

3.69.8 Giac [F]

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{x^3\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d + e\,x)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.70 $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 569

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{4b\sqrt{-c^2}\,d\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}} + \frac{4bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{32bd\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}\,d+e}}\sqrt{1+c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{3ce^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
-2/3*d^2*(a+b*arccsch(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsch(c*x))/e^3/(
e*x+d)^(1/2)-4/3*b*d*(c^2*x^2+1)/c/e/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(
e*x+d)^(1/2)+2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3-32/3*b*d*EllipticPi(1/
2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))
*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x
/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*d*EllipticE(1/2*(1-x*(-c^2)^(1/2)
)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(
1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(
1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+4*b*c*EllipticF(1/2*(1-x*(
-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2
))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/
2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.70.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 35.05 (sec) , antiderivative size = 1076, normalized size of antiderivative = 1.89

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex)^{5/2}}\,dx = -\frac{ad^3\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(3,-\frac{3}{2}\right)}{e^3(d+ex)^{5/2}}$$

$$+\frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{4cd\sqrt{1+\frac{1}{c^2x^2}}}{3e^2(c^2d^2+e^2)}-\frac{16\text{csch}^{-1}(cx)}{3e^3}+\frac{2\text{csch}^{-1}(cx)}{3e\left(e+\frac{d}{x}\right)^2}+\frac{4\left(cde\sqrt{1+\frac{1}{c^2x^2}}+2c^2d^2\text{csch}^{-1}(cx)+2e^2\text{csch}^{-1}(cx)\right)}{3e^2(c^2d^2+e^2)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}}-\frac{2\left(e+\frac{d}{x}\right)^{5/2}(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

output
```
-((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/
2))) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2*(
c^2*d^2 + e^2)) - (16*ArcCsch[c*x])/(3*e^3) + (2*ArcCsch[c*x])/(3*e*(e + d
/x)^2) + (4*(c*d*e*Sqrt[1 + 1/(c^2*x^2)] + 2*c^2*d^2*ArcCsch[c*x] + 2*e^2*
ArcCsch[c*x]))/(3*e^2*(c^2*d^2 + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(
e + d/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]*(3*c^2*d^2*e + 3*e^3)*Sqrt[1 + I*c*
x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I
 + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e
 + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d
- I*e)*(8*c^3*d^3 + 9*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*
e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))
/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x
]*(c*x)^(3/2)) - (2*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x
^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I
*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*
e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(
c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]]
, (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d
- I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt
[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d ...
```

3.70.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2123 vs. $2(569) = 1138$.

Time = 4.48 (sec) , antiderivative size = 2123, normalized size of antiderivative = 3.73, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.190$, Rules used = {6864, 27, 7272, 2351, 635, 25, 27, 498, 27, 507, 631, 688, 27, 599, 25, 27, 1459, 1416, 1509, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \; 6864$$

$$\frac{b\int \frac{2\left(8d^2+12exd+3e^2x^2\right)}{3e^3\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{c}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\int \frac{8d^2+12exd+3e^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce^3}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 7272$$

$$\frac{2b\sqrt{c^2x^2+1}\int \frac{8d^2+12exd+3e^2x^2}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 2351$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 635$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 25$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 27$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

3.70. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$\downarrow$ 498

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 27

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}\right)+\int \frac{3xe^2+12de}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 507

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}\right)+\int \frac{3xe^2+1}{(d+ex)^{3/2}\sqrt{}}\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 631

3.70. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+\cdots}}}{d}\cdots\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 688

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+\cdots}}}{d}\cdots\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 27

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{c^2x^2+1}}{\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+\cdots}}}{d}\cdots\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$\downarrow$ 599

3.70. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+}}}{d}\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(a+b\text{csch}^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\text{csch}^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^3}$$

$$\Big\downarrow 25$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+}}}{d}\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(a+b\text{csch}^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\text{csch}^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^3}$$

$$\Big\downarrow 27$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+}}}{d}\right.\right.}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2d^2(a+b\text{csch}^{-1}(cx))}{3e^3(d+ex)^{3/2}}+\frac{4d(a+b\text{csch}^{-1}(cx))}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\text{csch}^{-1}(cx))}{e^3}$$

$$\Big\downarrow 1459$$

3.70. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\frac{2b\sqrt{c^2x^2+1}\left(8d^2-\frac{e\left(\frac{2c^2\left(\frac{\sqrt{c^2d^2+e^2}\int\frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\int\frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+e}\cdots}{c}\cdots\right)}{e\left(c^2d^2+e^2\right)}\cdots\right)}{d}\cdots\right)}{3ce\cdots}$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 1416$$

3.70. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$2b\sqrt{c^2x^2+1}\left(8d^2\left(-\frac{e\left(\frac{2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{c}{\sqrt{c^2d^2}}\right.\right.}{2c^{3/2}\sqrt{\frac{c^2d^2}{e^2}+\frac{c^2(d+ex)^2}{e^2}-\frac{2c^2d(d+ex)}{e^2}+1}}}{e\left(c^2d^2+e^2\right)}}{d}\right.\right.$$

$$\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \ 1509$$

3.70. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{3e^3(d+ex)^{3/2}}+\frac{4\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.\right.}{\cdots}\cdots\right.\right.}{\cdots}$$

↓ 1511

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{3e^3(d+ex)^{3/2}}+\frac{4\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$$

↓ 1416

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{3e^3(d+ex)^{3/2}}+\frac{4\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.\right)}{\cdots}\cdots\right.\right)$$

↓ 1509

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\,\text{csch}^{-1}(cx)\right)d^2}{3e^3(d+ex)^{3/2}}+\frac{4\left(a+b\,\text{csch}^{-1}(cx)\right)d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(\frac{2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\cdots\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.}{\cdots}\cdots}{\cdots}\cdots\right.\right.}{\cdots}$$

$\downarrow$ 1540

3.70. $\int \frac{x^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{3e^3(d+ex)^{3/2}}+\frac{4\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right.\right)}{\cdots}\right.\right.}{\cdots}$$

↓ 1416

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)d^2}{3e^3(d+ex)^{3/2}}+\frac{4\left(a+b\operatorname{csch}^{-1}(cx)\right)d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}+$$

$$\frac{2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$$

$\downarrow$ 2222

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2(a+b\operatorname{csch}^{-1}(cx))\,d^2}{3e^3(d+ex)^{3/2}}+\frac{4(a+b\operatorname{csch}^{-1}(cx))\,d}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}(a+b\operatorname{csch}^{-1}(cx))}{e^3}+$$

$$2b\sqrt{c^2x^2+1}\left(8\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+\cdots}}\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{\cdots}\right.\right.}{\cdots}\right.\right.$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

output
```
(-2*d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a + b*ArcCsc
h[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3
+ (2*b*Sqrt[1 + c^2*x^2]*((-18*d*e^2*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*S
qrt[d + e*x]) + (6*(-3*c*d*Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 +
(c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c
^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^
(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2
*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (
c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[d +
 e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c
]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2
])) + ((c^2*d^2 + e^2)^(3/4)*(3*c*d + Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*
x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2
 + (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2
*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2
)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*Sqrt[c]*Sqrt[1 + (c^2*d^2
)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/(c^2*d^2 + e^2
) + 8*d^2*(-((e*((-2*e*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*Sqrt[d + e*x])
+ (2*c^2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2
- (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2...
```

3.70.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

3.70. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.70.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.88 (sec) , antiderivative size = 2492, normalized size of antiderivative = 4.38

method	result	size
derivativedivides	Expression too large to display	2492
default	Expression too large to display	2492
parts	Expression too large to display	2500

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^3*(a*((e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2)+2*d/(e*x+d)^(1/2))+b*((e*x+d
)^(1/2)*arccsch(c*x)-1/3*arccsch(c*x)*d^2/(e*x+d)^(3/2)+2*arccsch(c*x)*d/(
e*x+d)^(1/2)+2/3/c*(-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^
(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*
e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*
d^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)-4*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1
/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c
*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+
d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-8*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2
))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/
((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1/2)+8*(-(I*c*e*(e*x+d)+c^
2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d
)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)...
```

3.70.5 Fricas [F]

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^2}{(ex + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2
 + 3*d^2*e*x + d^3), x)
```

3.70.6 Sympy [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx=\int \frac{x^2(a+b\operatorname{acsch}(cx))}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

3.70.7 Maxima [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*b*(2*(3*e^2*x^2 + 12*d*e*x + 8*d^2)*log(sqrt(c^2*x^2 + 1) + 1)/((e^4*x
 + d*e^3)*sqrt(e*x + d)) + 3*integrate(2/3*(3*c^2*e^2*x^3 + 12*c^2*d*e*x^2
 + 8*c^2*d^2*x)/((c^2*e^4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3)*sqrt(c^2*x^
2 + 1)*sqrt(e*x + d) + (c^2*e^4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3)*sqrt(
e*x + d)), x) - 3*integrate(1/3*(30*c^2*d*e^2*x^3 + 3*(e^3*log(c) + 2*e^3)
*c^2*x^4 + 16*c^2*d^3*x + (40*c^2*d^2*e + 3*e^3*log(c))*x^2 + 3*(c^2*e^3*x
^4 + e^3*x^2)*log(x))/((c^2*e^5*x^4 + 2*c^2*d*e^4*x^3 + 2*d*e^4*x + d^2*e^
3 + (c^2*d^2*e^3 + e^5)*x^2)*sqrt(e*x + d)), x)) + 2/3*a*(3*sqrt(e*x + d)/
e^3 + 6*d/(sqrt(e*x + d)*e^3) - d^2/((e*x + d)^(3/2)*e^3))
```

3.70.8 Giac [F]

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex)^{5/2}}\,dx = \int \frac{x^2\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.71 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

3.71.1 Optimal result

Integrand size = 19, antiderivative size = 393

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}} + \frac{8b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
2/3*d*(a+b*arccsch(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsch(c*x))/e^2/(e*x+d
)^(1/2)+4/3*b*(c^2*x^2+1)/c/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1
/2)+8/3*b*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*
(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^
(1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(
1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(-c^2)^(1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e/(c^2*d^2+e^2
)/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.93 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.99

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx=\frac{2}{3}\left(\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}x}{(c^2d^2+e^2)\sqrt{d+ex}}-\frac{a(2d+3ex)}{e^2(d+ex)^{3/2}}-\frac{b(2d+3ex)\operatorname{csch}^{-1}(cx)}{e^2(d+ex)^{3/2}}\right.$$
$$+\frac{2ib\sqrt{-\frac{c}{cd-ie}}\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(cdE\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)-cd\,\mathrm{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\right.\right.\right.}{c^2de^2\sqrt{1+\frac{1}{c^2x^2}}}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

output
```
(2*((2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/((c^2*d^2 + e^2)*Sqrt[d + e*x]) - (a*(
2*d + 3*e*x))/(e^2*(d + e*x)^(3/2)) - (b*(2*d + 3*e*x)*ArcCsch[c*x])/(e^2*
(d + e*x)^(3/2)) + ((2*I)*b*Sqrt[-(c/(c*d - I*e))]*Sqrt[-((e*(-I + c*x))/(
c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(c*d*EllipticE[I*ArcSinh[S
qrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - c*d*Ellip
ticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I
*e)] + 2*(c*d - I*e)*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d -
 I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^2*d*e^2*Sqrt[1 + 1/(
c^2*x^2)]*x)))/3
```

3.71.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2084 vs. 2(393) = 786.

Time = 3.83 (sec) , antiderivative size = 2084, normalized size of antiderivative = 5.30, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.105, Rules used = {6864, 27, 7272, 2351, 27, 498, 27, 507, 635, 25, 27, 498, 27, 507, 631, 1459, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$$
$$\Big\downarrow \; 6864$$

$$\frac{b\int -\frac{2(2d+3ex)}{3e^2\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{c}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 27

$$-\frac{2b\int \frac{2d+3ex}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce^2}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 7272

$$-\frac{2b\sqrt{c^2x^2+1}\int \frac{2d+3ex}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 2351

$$-\frac{2b\sqrt{c^2x^2+1}\left(\int \frac{3e}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\int \frac{1}{(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 498

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

3.71. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}dx$

$$\Big\downarrow \; 507$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\int \frac{1}{x(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 635$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{c^2x^2+1}}dx+\cdots\right)\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}\cdots\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\int \frac{\cdots}{d(d+e\cdots}\right)\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}\cdots\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\int \frac{\cdots}{(d+ex\cdots}}{\cdots}\right)\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}\cdots\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 498$$

3.71. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}\right)+3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+d}}}{e(c^2d^2+e^2)}\right.\right.}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\text{csch}^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\big\downarrow$ 27

$$-\frac{2b\sqrt{c^2x^2+1}\left(2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{c^2x^2+1}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}\right)+3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+}}}{e(c^2d^2+e^2)}\right.\right.}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\text{csch}^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\big\downarrow$ 507

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{c^2x^2+1}}dx}{d}-\frac{e\left(2c^2\int\right.}{}\right.\right.}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\text{csch}^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

$\big\downarrow$ 631

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2e\sqrt{c^2x^2+1}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)+2d\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}}}}{e(c^2d^2+e^2)}\right.}{}\right.\right.}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

$$\frac{2(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex}}+\frac{2d(a+b\text{csch}^{-1}(cx))}{3e^2(d+ex)^{3/2}}$$

3.71. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$\Big\downarrow \; 1459$$

$$-\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\left(\frac{\sqrt{c^2d^2+e^2}\int \frac{1}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}d\sqrt{d+ex}}{c}\right)}{e(c^2d^2+e^2)}\right.\right.}{}$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}$$

$$\Big\downarrow \; 1416$$

3.71. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(3e\left(\frac{2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}}{e\left(c^2d^2+e^2\right)}\right.\right.}{}$$

↓ 1509

3.71. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(3e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.}{}$$

$\downarrow$ 1540

3.71. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\,\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\,\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(3e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\ \text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.}{}$$

↓ 1416

3.71. $\int\frac{x\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-$$

$$\frac{2b\sqrt{c^2x^2+1}\left(3e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.}{}$$

$\downarrow$ 2222

3.71. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-$$

$$2b\sqrt{c^2x^2+1}\left(3e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}{\left(\frac{c^2d^2}{e^2}+1\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+\right.\right.}{2c^{3/2}\sqrt{\frac{(d+ex)^2c^2}{e^2}-\frac{2d(d+ex)c^2}{e^2}+\frac{d^2c^2}{e^2}+1}}\right.\right.\right.$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

3.71. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

output
```
(2*d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCsch[c*x
]))/(e^2*Sqrt[d + e*x]) - (2*b*Sqrt[1 + c^2*x^2]*(3*e*((-2*e*Sqrt[1 + c^2*
x^2])/((c^2*d^2 + e^2)*Sqrt[d + e*x]) + (2*c^2*(-((Sqrt[c^2*d^2 + e^2]*(-(
(Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d
+ e*x)^2)/e^2]))/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2
]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt
[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2)/((1
 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*
ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2
*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2
 + (c^2*(d + e*x)^2)/e^2])))/c) + ((c^2*d^2 + e^2)^(3/4)*(1 + (c*(d + e*x)
)/Sqrt[c^2*d^2 + e^2])*Sqrt[(1 + (c^2*d^2)/e^2 - (2*c^2*d*(d + e*x))/e^2 +
 (c^2*(d + e*x)^2)/e^2)/((1 + (c^2*d^2)/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d
^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^
(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[1 + (c^2*d^2)/
e^2 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])))/(e*(c^2*d^2 + e^
2))) + 2*d*(-((e*((-2*e*Sqrt[1 + c^2*x^2])/((c^2*d^2 + e^2)*Sqrt[d + e*x])
 + (2*c^2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[1 + (c^2*d^2)/e^2
 - (2*c^2*d*(d + e*x))/e^2 + (c^2*(d + e*x)^2)/e^2])/((1 + (c^2*d^2)/e^2)*
(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 +...
```

3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.71. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.71.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.47 (sec) , antiderivative size = 2106, normalized size of antiderivative = 5.36

method	result	size
derivativedivides	Expression too large to display	2106
default	Expression too large to display	2106
parts	Expression too large to display	2110

input
```
int(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e^2*(-a*(1/(e*x+d)^(1/2)-1/3*d/(e*x+d)^(3/2))-b*(1/(e*x+d)^(1/2)*arccsch
(c*x)-1/3*arccsch(c*x)*d/(e*x+d)^(3/2)-2/3/c*(I*((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2)*(e*x+d)^2*c^2*d*e+2*I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/
(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e
^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c
*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/
(c^2*d^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+
d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2
+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)
^(1/2)-(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((
I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e
*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-2*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)
-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^
2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)
/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^
2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^2+I*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^
2*d^3*e+2*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)+I*((c*d+I*e...
```

3.71.5 Fricas [F]

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x*arccsch(c*x) + a*x)*sqrt(e*x + d)/(e^3*x^3 + 3*d*e^2*x^2 + 3
*d^2*e*x + d^3), x)
```

3.71.6 Sympy [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{x(a + b\,\text{acsch}\,(cx))}{(d + ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

3.71.7 Maxima [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{(ex + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*b*(2*(3*e*x + 2*d)*log(sqrt(c^2*x^2 + 1) + 1)/((e^3*x + d*e^2)*sqrt(e
*x + d)) + 3*integrate(2/3*(3*c^2*e*x^2 + 2*c^2*d*x)/((c^2*e^3*x^3 + c^2*d
*e^2*x^2 + e^3*x + d*e^2)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3*x^3 +
 c^2*d*e^2*x^2 + e^3*x + d*e^2)*sqrt(e*x + d)), x) + 3*integrate(-1/3*(10*
c^2*d*e*x^2 - 3*(e^2*log(c) - 2*e^2)*c^2*x^3 + (4*c^2*d^2 - 3*e^2*log(c))*
x - 3*(c^2*e^2*x^3 + e^2*x)*log(x))/((c^2*e^4*x^4 + 2*c^2*d*e^3*x^3 + 2*d*
e^3*x + d^2*e^2 + (c^2*d^2*e^2 + e^4)*x^2)*sqrt(e*x + d)), x)) - 2/3*a*(3/
(sqrt(e*x + d)*e^2) - d/((e*x + d)^(3/2)*e^2))
```

3.71.8 Giac [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{(ex + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(5/2), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex)^{5/2}}\,dx = \int \frac{x\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(d + e\,x)^{5/2}}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.72 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

3.72.1 Optimal result

Integrand size = 18, antiderivative size = 369

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{4be(1+c^2x^2)}{3cd\,(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$
$$+ \frac{4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3cd\,(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}$$
$$+ \frac{4b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{3cde\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

output
```
-2/3*(a+b*arccsch(c*x))/e/(e*x+d)^(3/2)-4/3*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e
^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*EllipticE(1/2*(1-x*(-c^2)^(1
/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2
)^(1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^
(1/2)/((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)+4/3*b*EllipticPi(1/2*(1-x*(-c^2)^
(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(
1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/d/e/x/(1+1/c^2/x^2)
^(1/2)/(e*x+d)^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.43 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.42

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2a}{3e(d+ex)^{3/2}} + \frac{b\left(-\frac{c^3\left(e+\frac{d}{x}\right)^3x^3\left(-\frac{4\sqrt{1+\frac{1}{c^2x^2}}}{3cd\left(c^2d^2+e^2\right)}+\frac{2\operatorname{csch}^{-1}(cx)}{3c^2d^2e}+\frac{2e\operatorname{csch}^{-1}(cx)}{3c^2d^2\left(e+\frac{d}{x}\right)^2}-\frac{4\left(-cde\sqrt{1+\frac{1}{c^2x^2}}+c^2d^2\operatorname{csch}^{-1}(cx)+e^2\operatorname{csch}^{-1}(cx)\right)}{3c^2d^2\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}\right)}{(d+ex)^{5/2}} + \frac{2\left(e+\frac{d}{x}\right)^{5/2}(\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

output
```
(-2*a)/(3*e*(d + e*x)^(3/2)) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/
(c^2*x^2)])/(3*c*d*(c^2*d^2 + e^2)) + (2*ArcCsch[c*x])/(3*c^2*d^2*e) + (2*
e*ArcCsch[c*x])/(3*c^2*d^2*(e + d/x)^2) - (4*(-(c*d*e*Sqrt[1 + 1/(c^2*x^2)
]) + c^2*d^2*ArcCsch[c*x] + e^2*ArcCsch[c*x]))/(3*c^2*d^2*(c^2*d^2 + e^2)*
(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*((I*Sqrt[2]
*c*d*(c*d - I*e)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d +
 e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]
], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2))
 + (2*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*S
qrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*
(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d
+ I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d
 + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I
*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(
c*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1
+ (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)
]))/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sq
rt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2))))/(3*e*(c^2*d^2 + e^2)*(d + e*x)^(5/2
))))/c
```

3.72.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1359 vs. $2(369) = 738$.

Time = 1.88 (sec) , antiderivative size = 1359, normalized size of antiderivative = 3.68, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6844, 1898, 635, 25, 27, 498, 27, 507, 631, 1459, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}} dx$$

$$\Big\downarrow \text{6844}$$

$$-\frac{2b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{3/2}}dx}{3ce}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1898}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\int \frac{1}{x(d+ex)^{3/2}\sqrt{x^2+\frac{1}{c^2}}}dx}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{635}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\int -\frac{e}{d(d+ex)^{3/2}\sqrt{x^2+\frac{1}{c^2}}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}-\int \frac{e}{d(d+ex)^{3/2}\sqrt{x^2+\frac{1}{c^2}}}dx\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}-\frac{e\int \frac{1}{(d+ex)^{3/2}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{498}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}-\frac{e\left(-\frac{2c^2\int -\frac{\sqrt{d+ex}}{2\sqrt{x^2+\frac{1}{c^2}}}dx}{c^2d^2+e^2}-\frac{2c^2e\sqrt{\frac{1}{c^2}+x^2}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{c^2\int \frac{\sqrt{d+ex}}{\sqrt{x^2+\frac{1}{c^2}}}dx}{c^2d^2+e^2}-\frac{2c^2e\sqrt{\frac{1}{c^2}+x^2}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 507$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d}-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2c^2e\sqrt{\frac{1}{c^2}+x^2}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 631$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(\frac{2c^2\int \frac{d+ex}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e(c^2d^2+e^2)}-\frac{2c^2e\sqrt{\frac{1}{c^2}+x^2}}{(c^2d^2+e^2)\sqrt{d+ex}}\right)}{d}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{d}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 1459$$

3.72. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(\frac{2c^2\left(\frac{\sqrt{c^2d^2+e^2}\int \frac{1}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{c}-\frac{\sqrt{c^2d^2+e^2}\int \frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{c}\right)}{e\left(c^2d^2+e^2\right)}-\frac{2c^2e\sqrt{\cdots}}{\left(c^2d^2+e^2\cdots\right.}\cdots\right)}{d}\cdots\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\cdots}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 1416

3.72. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(\frac{2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}{\left(\frac{1}{c^2}+\frac{d^2}{e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right.}{2c^{3/2}\sqrt{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}}}{e\left(c^2d^2+e^2\right)}}{d}\right.}{3cex\sqrt{\frac{1}{c^2x^2}+}}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1509}$$

3.72. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}\,d\sqrt{d+ex}}{d}-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+ex)}{e^2}+\frac{(d+ex)^2}{e^2}}{\left(\frac{1}{c^2}+\frac{d^2}{e^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{Ellip}\ldots}{2c^{3/2}\sqrt{\frac{1}{c^2}+\frac{d^2}{e^2}-\frac{2d(d+e\ldots}{e^2}\ldots}}\ldots\right.\right.}{\ldots}\ldots\right.}{\ldots}$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}$$

$\downarrow$ 1540

3.72. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}-$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right)}{2c^{3/2}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}-\cdots\right.}{\cdots}\right.}{\cdots}$$

↓ 1416

3.72. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}-$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right)}{2c^{3/2}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}-\cdots\right.}{\cdots}}{\cdots}\right.$$

$\downarrow$ 2222

3.72. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

$$-\frac{2(a+b\operatorname{csch}^{-1}(cx))}{3e(d+ex)^{3/2}}-$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(-\frac{e\left(2c^2\left(\frac{\left(c^2d^2+e^2\right)^{3/4}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt[4]{c^2d^2+e^2}}\right),\frac{1}{2}\left(\frac{cd}{\sqrt{c^2d^2+e^2}}+1\right)\right)}{2c^{3/2}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}\right.\right.\right.$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

3.72. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx$

output
```
(-2*(a + b*ArcCsch[c*x]))/(3*e*(d + e*x)^(3/2)) - (2*b*Sqrt[c^(-2) + x^2]*
(-((e*((-2*c^2*e*Sqrt[c^(-2) + x^2])/((c^2*d^2 + e^2)*Sqrt[d + e*x]) + (2*
c^2*(-((Sqrt[c^2*d^2 + e^2]*(-((Sqrt[d + e*x]*Sqrt[c^(-2) + d^2/e^2 - (2*d
*(d + e*x))/e^2 + (d + e*x)^2/e^2])/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))
/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c
^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/
e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*Ellip
ticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d)/S
qrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e
^2 + (d + e*x)^2/e^2]))))/c) + ((c^2*d^2 + e^2)^(3/4)*(1 + (c*(d + e*x))/Sq
rt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x
)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*E
llipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*
d)/Sqrt[c^2*d^2 + e^2])/2])/(2*c^(3/2)*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e
*x))/e^2 + (d + e*x)^2/e^2])))/(e*(c^2*d^2 + e^2))))/d) - (2*(-1/2*(Sqrt[c
]*(c^2*d^2 + e^2)^(1/4)*(c*d - Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqr
t[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)
^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])^2)]*El
lipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d^2 + e^2)^(1/4)], (1 + (c*d
)/Sqrt[c^2*d^2 + e^2])/2])/(e^2*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))...
```

3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 507
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[2/
d   Subst[Int[x^2/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]
, x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1459
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Simp[1/q   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[1/q
 Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.72. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}} \, dx$

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.72.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.24 (sec) , antiderivative size = 2079, normalized size of antiderivative = 5.63

method	result	size
derivativedivides	Expression too large to display	2079
default	Expression too large to display	2079
parts	Expression too large to display	2081

input
```
int((a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsch(c*x)-2/3/c*(I*((c*
d+I*e)*c/(c^2*d^2+e^2))^(1/2)*d*e^3-I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d
^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1
/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d
+I*e)*c/(c^2*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1/2)-((c*d+I*e)*c/(c^2*d^2+e^2)
)^(1/2)*c^3*d^2*(e*x+d)^2+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2
*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-(-(I*c*e*(e*x
+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*
(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((c*d+I*
e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*
c^3*d^3*(e*x+d)^(1/2)+(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/
c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2
+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-I*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d
^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))...
```

3.72.5 Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2
*e*x + d^3), x)
```

3.72.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{a+b\,\text{acsch}\,(cx)}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

3.72.7 Maxima [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(6*c^2*integrate(1/3*x/((c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*sqr
t(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e)*s
qrt(e*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((e^2*x + d*e)*sqrt(e*x +
 d)) + 3*integrate(1/3*((3*e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x + 3*e*log(c
) + 3*(c^2*e*x^2 + e)*log(x))/((c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x
+ d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(e*x + d)), x))*b - 2/3*a/((e*x + d)^
(3/2)*e)
```

3.72.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x + d)^(5/2), x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.73 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

3.73.2 Mathematica [N/A]

Not integrable

Time = 38.79 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

3.73.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
```

output
```
$Aborted
```

3.73.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.73.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\operatorname{arccsch}\left(cx\right)}{x\left(ex+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

3.73.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x(d + ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex + d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2
*e*x^2 + d^3*x), x)
```

3.73.6 Sympy [N/A]

Not integrable

Time = 162.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x(d + ex)^{5/2}}\,dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x\,(d + ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x+d)**(5/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*(d + e*x)**(5/2)), x)
```

3.73.7 Maxima [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 239, normalized size of antiderivative = 11.38

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x(d + ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex + d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*b*((3*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5
/2) + 2*(3*(e*x + d)*e + d*e)/((e*x + d)^(3/2)*d^2))*log(c)/e + 3*integrat
e(log(x)/(sqrt(e*x + d)*e^2*x^3 + 2*sqrt(e*x + d)*d*e*x^2 + sqrt(e*x + d)*
d^2*x), x) - 3*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2*x^3
 + 2*sqrt(e*x + d)*d*e*x^2 + sqrt(e*x + d)*d^2*x), x)) + 1/3*a*(3*log((sqr
t(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(5/2) + 2*(3*e*x + 4*d)
/((e*x + d)^(3/2)*d^2))
```

3.73.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\int \frac{b\operatorname{arcsch}\left(cx\right)+a}{(ex+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

3.73.9 Mupad [N/A]

Not integrable

Time = 5.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

3.74 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

3.74.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

3.74.2 Mathematica [N/A]

Not integrable

Time = 34.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

3.74.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

output
```
$Aborted
```

3.74.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.74.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\operatorname{arccsch}\left(cx\right)}{x^{2}\left(ex+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

3.74.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2(d + ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex + d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(b*arccsch(c*x) + a)/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2
*e*x^3 + d^3*x^2), x)
```

3.74.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2(d + ex)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.74.7 Maxima [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 316, normalized size of antiderivative = 15.05

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2(d + ex)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex + d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
1/6*b*((2*(15*(e*x + d)^2*e^2 - 10*(e*x + d)*d*e^2 - 2*d^2*e^2)/((e*x + d)
^(5/2)*d^3 - (e*x + d)^(3/2)*d^4) + 15*e^2*log((sqrt(e*x + d) - sqrt(d))/(
sqrt(e*x + d) + sqrt(d)))/d^(7/2))*log(c)/e - 6*integrate(log(x)/(sqrt(e*x
 + d)*e^2*x^4 + 2*sqrt(e*x + d)*d*e*x^3 + sqrt(e*x + d)*d^2*x^2), x) + 6*i
ntegrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x + d)*e^2*x^4 + 2*sqrt(e*x +
d)*d*e*x^3 + sqrt(e*x + d)*d^2*x^2), x)) - 1/6*a*(2*(15*(e*x + d)^2*e - 10
*(e*x + d)*d*e - 2*d^2*e)/((e*x + d)^(5/2)*d^3 - (e*x + d)^(3/2)*d^4) + 15
*e*log((sqrt(e*x + d) - sqrt(d))/(sqrt(e*x + d) + sqrt(d)))/d^(7/2))
```

3.74.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex+d)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

3.74.9 Mupad [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

3.75 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

3.75.1 Optimal result

Integrand size = 18, antiderivative size = 648

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx &= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} \\ &- \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ &- \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} \\ &- \frac{4bc(7c^2d^2+3e^2)\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2\sqrt{-c^2}e}{-c^2d+\sqrt{-c^2}e}\right)}{15\sqrt{-c^2}d^2\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}} \\ &- \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} \\ &+ \frac{4b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{1+c^2x^2}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right),\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cd^2e\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}\end{aligned}$$

output
```
-2/5*(a+b*arccsch(c*x))/e/(e*x+d)^(5/2)-4/15*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+
e^2)/x/(e*x+d)^(3/2)/(1+1/c^2/x^2)^(1/2)-16/15*b*c*e*(c^2*x^2+1)/(c^2*d^2+
e^2)^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*e*(c^2*x^2+1)/c/d^2/(c^2*
d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*c*(7*c^2*d^2+3*e^2)*El
lipticE(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2^(1/2)*(e*(-c^2)^(1/2)/(-c^2
*d+e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d^2+e^
2)^2/x/(-c^2)^(1/2)/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)
-4/15*b*EllipticF(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/
(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*((e*x+d)/(d+
e/(-c^2)^(1/2)))^(1/2)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/
2)+4/5*b*EllipticPi(1/2*(1-x*(-c^2)^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(
-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(
1/2)+e))^(1/2)/c/d^2/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

3.75.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 34.91 (sec) , antiderivative size = 1217, normalized size of antiderivative = 1.88

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2a}{5e(d+ex)^{5/2}} + \frac{b\left(-\frac{c^4\left(e+\frac{d}{x}\right)^4x^4\left(-\frac{4\left(7c^2d^2+3e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}{15c^2d^2\left(c^2d^2+e^2\right)^2}+\frac{2\text{csch}^{-1}(cx)}{5c^3d^3e}-\frac{2e^2\text{csch}^{-1}(cx)}{5c^3d^3\left(e+\frac{d}{x}\right)^3}+\frac{2\left(-2cde^2\sqrt{1+\frac{1}{c^2x^2}}+9c^2d^2e\,\text{csch}^{-1}(cx)+9e^3\text{csch}^{-1}(cx)\right)}{15c^3d^3\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)^2}-\cdots\right)}{(d+ex)^{7/2}}\cdots\right)}{\cdots}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
```

output
```
(-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((-4*(7*c^2*d^2
+ 3*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(15*c^2*d^2*(c^2*d^2 + e^2)^2) + (2*ArcCsc
h[c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsch[c*x])/(5*c^3*d^3*(e + d/x)^3) + (2*
(-2*c*d*e^2*Sqrt[1 + 1/(c^2*x^2)] + 9*c^2*d^2*e*ArcCsch[c*x] + 9*e^3*ArcCs
ch[c*x]))/(15*c^3*d^3*(c^2*d^2 + e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqr
t[1 + 1/(c^2*x^2)] - 8*c*d*e^3*Sqrt[1 + 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsch[c
*x] + 18*c^2*d^2*e^2*ArcCsch[c*x] + 9*e^4*ArcCsch[c*x]))/(15*c^3*d^3*(c^2*
d^2 + e^2)^2*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2
)*(-((Sqrt[2]*(c^2*d^2*e + e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*
x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c
*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(
1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(3*c^3*d^3 - c*d*e^2)*
Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi
[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2
*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-7*c^2*d^2
*e - 3*e^3)*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c
*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF
[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-
((e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((
c*d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*...
```

3.75.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1489 vs. $2(648) = 1296$.

Time = 2.19 (sec) , antiderivative size = 1489, normalized size of antiderivative = 2.30, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {6844, 1898, 635, 631, 688, 27, 688, 27, 599, 27, 1511, 1416, 1509, 1540, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 6844}$$

$$-\frac{2b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}x^2(d+ex)^{5/2}}dx}{5ce}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{ 1898}$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\int \frac{1}{x(d+ex)^{5/2}\sqrt{x^2+\frac{1}{c^2}}}dx}{5cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 635

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2+\frac{1}{c^2}}}dx+\frac{\int \frac{1}{x\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{d^2}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 631

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(\int \frac{-\frac{xe^2}{d^2}-\frac{2e}{d}}{(d+ex)^{5/2}\sqrt{x^2+\frac{1}{c^2}}}dx-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{d^2}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 688

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{2\int \frac{e\left(3d\left(\frac{e^2}{c^2d^2}+2\right)-ex\right)}{2d(d+ex)^{3/2}\sqrt{x^2+\frac{1}{c^2}}}dx}{3\left(\frac{e^2}{c^2}+d^2\right)}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{d^2}+\frac{2e^2\sqrt{\frac{1}{c^2}+x^2}}{3d\left(\frac{e^2}{c^2}+d^2\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 27

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\int \frac{3d\left(\frac{e^2}{c^2d^2}+2\right)-ex}{(d+ex)^{3/2}\sqrt{x^2+\frac{1}{c^2}}}dx}{3d\left(\frac{e^2}{c^2}+d^2\right)}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{d^2}+\frac{2e^2\sqrt{\frac{1}{c^2}+x^2}}{3d\left(\frac{e^2}{c^2}+d^2\right)(d+ex)^{3/2}}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 688

3.75. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(-\frac{2c^2\int -\frac{6d^2+e\left(\frac{3e^2}{c^2d^2}+7\right)xd+\frac{2e^2}{c^2}}{2\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{\frac{1}{c^2}+x^2}\left(7c^2d^2+3e^2\right)}{d\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{3d\left(\frac{e^2}{c^2}+d^2\right)}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{d^2}+\frac{\cdots}{3d\cdots}\cdots\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}\cdots$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(\frac{c^2\int \frac{2\left(3d^2+\frac{e^2}{c^2}\right)+de\left(\frac{3e^2}{c^2d^2}+7\right)x}{\sqrt{d+ex}\sqrt{x^2+\frac{1}{c^2}}}dx}{c^2d^2+e^2}-\frac{2e\sqrt{\frac{1}{c^2}+x^2}\left(7c^2d^2+3e^2\right)}{d\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{3d\left(\frac{e^2}{c^2}+d^2\right)}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{d^2}+\frac{\cdots}{3d(\cdots}\cdots\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}\cdots$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 599$$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(-\frac{2c^2\int \frac{e\left(d^2-\left(\frac{3e^2}{c^2d^2}+7\right)(d+ex)d+\frac{e^2}{c^2}\right)}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e^2\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{\frac{1}{c^2}+x^2}\left(7c^2d^2+3e^2\right)}{d\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{3d\left(\frac{e^2}{c^2}+d^2\right)}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d\cdots}}{d^2}\cdots\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}}\cdots$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 27$$

3.75. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(-\frac{2c^2\int \frac{d^2-\left(\frac{3e^2}{c^2d^2}+7\right)(d+ex)d+\frac{e^2}{c^2}}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{e\left(c^2d^2+e^2\right)}-\frac{2e\sqrt{\frac{1}{c^2}+x^2}\left(7c^2d^2+3e^2\right)}{d\left(c^2d^2+e^2\right)\sqrt{d+ex}}\right)}{3d\left(\frac{e^2}{c^2}+d^2\right)}-\frac{2\int -\frac{1}{ex\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d\cdots}}{d^2}\cdots\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}\cdots}\cdots$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 1511

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(-\frac{2c^2\left(\left(-\frac{d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)}{c}+\frac{e^2}{c^2}+d^2\right)\int\frac{1}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}+\frac{d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)\int\frac{1-\cdots}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex\cdots}{e^2}\cdots}}\cdots}{c}\cdots\right)}{e\left(c^2d^2+e^2\right)}\cdots\right)}{3d\left(\frac{e^2}{c^2}+d^2\right)}\cdots\right)}{5cex\sqrt{\frac{1}{c^2x^2}\cdots}}\cdots$$

$$\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$\downarrow$ 1416

3.75. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2b\sqrt{\frac{1}{c^2}+x^2}\left(-\frac{e\left(-\frac{2c^2\left(\frac{d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)\int\frac{1-\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}}{\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}}d\sqrt{d+ex}}{c}+\frac{\sqrt[4]{c^2d^2+e^2}\left(-\frac{d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)}{c}+\frac{e^2}{c^2}+d^2\right.}{\cdots}\right)}{e\left(c^2d^2+e^2\right)\cdots}\right)}{3\cdots}\right.}{\cdots}$$

$$\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 1509$$

3.75. $\int\frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(\frac{2\sqrt{x^2+\frac{1}{c^2}}\,e^2}{3d\left(d^2+\frac{e^2}{c^2}\right)(d+ex)^{3/2}}-\left(-\frac{2\left(\frac{d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,E\left(2\operatorname{arcta}\right.}{\sqrt{c}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c}}}\right.}{c}\right.}{}\right.}{}\right.$$

↓ 1540

3.75. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(\frac{2\sqrt{x^2+\frac{1}{c^2}}\,e^2}{3d\left(d^2+\frac{e^2}{c^2}\right)(d+ex)^{3/2}}-\frac{-\frac{2\left(d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}\,E\left(2\,\text{arcta}\cdots\right.}{\sqrt{c}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\cdots}}\cdots\right)}{c\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots\right)$$

↓ 1416

3.75. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-$$

$$2b\sqrt{x^2+\frac{1}{c^2}}\left(\frac{2\sqrt{x^2+\frac{1}{c^2}}e^2}{3d\left(d^2+\frac{e^2}{c^2}\right)(d+ex)^{3/2}}-\frac{\left(-\frac{2\left(d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}E\left(2\operatorname{arcta}\right.}{\sqrt{c}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c}}}\right.}{c}\right.}{}\right.}{}\right)$$

↓ 2222

3.75. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-\frac{2b\sqrt{x^2+\frac{1}{c^2}}\left(\frac{2\sqrt{x^2+\frac{1}{c^2}}e^2}{3d\left(d^2+\frac{e^2}{c^2}\right)(d+ex)^{3/2}}-\frac{-\frac{2\left(\frac{d\sqrt{c^2d^2+e^2}\left(\frac{3e^2}{c^2d^2}+7\right)\left(\frac{\sqrt[4]{c^2d^2+e^2}\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)\sqrt{\frac{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\frac{1}{c^2}}{\left(\frac{d^2}{e^2}+\frac{1}{c^2}\right)\left(\frac{c(d+ex)}{\sqrt{c^2d^2+e^2}}+1\right)^2}}E\left(2\operatorname{arcta}\ldots\right.}{\sqrt{c}\sqrt{\frac{d^2}{e^2}-\frac{2(d+ex)d}{e^2}+\frac{(d+ex)^2}{e^2}+\ldots}}\right.}{c}\ldots\right)}{\ldots}}{\ldots}\right.}{\ldots}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
```

3.75. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

output
```
(-2*(a + b*ArcCsch[c*x]))/(5*e*(d + e*x)^(5/2)) - (2*b*Sqrt[c^(-2) + x^2]*
((2*e^2*Sqrt[c^(-2) + x^2])/(3*d*(d^2 + e^2/c^2)*(d + e*x)^(3/2)) - (e*((-
2*e*(7*c^2*d^2 + 3*e^2)*Sqrt[c^(-2) + x^2])/(d*(c^2*d^2 + e^2)*Sqrt[d + e*
x]) - (2*c^2*((d*Sqrt[c^2*d^2 + e^2]*(7 + (3*e^2)/(c^2*d^2))*(-((Sqrt[d +
e*x]*Sqrt[c^(-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])/((c^(
-2) + d^2/e^2)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2]))) + ((c^2*d^2 + e^2
)^(1/4)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (
2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x
))/Sqrt[c^2*d^2 + e^2])^2)]*EllipticE[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^
2*d^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(Sqrt[c]*Sqrt[c^(
-2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])))/c + ((c^2*d^2 +
e^2)^(1/4)*(d^2 + e^2/c^2 - (d*Sqrt[c^2*d^2 + e^2]*(7 + (3*e^2)/(c^2*d^2))
)/c)*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2) + d^2/e^2 - (2*d
*(d + e*x))/e^2 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2)*(1 + (c*(d + e*x))/
Sqrt[c^2*d^2 + e^2])^2)]*EllipticF[2*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/(c^2*d
^2 + e^2)^(1/4)], (1 + (c*d)/Sqrt[c^2*d^2 + e^2])/2])/(2*Sqrt[c]*Sqrt[c^(-
2) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2])))/(e*(c^2*d^2 + e^2
))))/(3*d*(d^2 + e^2/c^2)) - (2*(-1/2*(Sqrt[c]*(c^2*d^2 + e^2)^(1/4)*(c*d
- Sqrt[c^2*d^2 + e^2])*(1 + (c*(d + e*x))/Sqrt[c^2*d^2 + e^2])*Sqrt[(c^(-2
) + d^2/e^2 - (2*d*(d + e*x))/e^2 + (d + e*x)^2/e^2)/((c^(-2) + d^2/e^2...
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 631
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[-2   Subst[Int[1/((c - x^2)*Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^
2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[b/a]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

3.75. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx$

rule 1898
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Simp[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p])   Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n
))^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !I
ntegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 6844
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Simp[
b/(c*e*(m + 1))   Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
 /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

3.75.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.51 (sec) , antiderivative size = 3782, normalized size of antiderivative = 5.84

method	result	size
derivativedivides	Expression too large to display	3782
default	Expression too large to display	3782
parts	Expression too large to display	3784

input
```
int((a+b*arccsch(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsch(c*x)-2/15/c*(7*I*(
(c*d+I*e)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*e*(e*x+d)^3+3*EllipticPi((e*x+d)^
(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c*(c^2*d^2+e^2)/d,(-(I
*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*e
*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-c
^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c*d*e^4*(e*x+d)^(3/2)+3*(-(
I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x
+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2
)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2))*c*d*e^4*(e*x+d)^(3/2)-10*EllipticE((e*x+d)^(1/2)*((c*d+I*e)*c/(c
^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*(-(I*c*
e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*e*(e*x+d)-
c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e^2*(e*x+d)^(3/2)+
6*EllipticPi((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),1/(c*d+I*e)/c
*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((c*d+I*e)*c/(c^2*d^2+
e^2))^(1/2))*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1
/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^3*d^
3*e^2*(e*x+d)^(3/2)+9*(-(I*c*e*(e*x+d)+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2
+e^2))^(1/2)*((I*c*e*(e*x+d)-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2)*EllipticF((e*x+d)^(1/2)*((c*d+I*e)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d...
```

3.75.5 Fricas [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^{7/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.75.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/(e*x+d)**(7/2),x)
```

output
```
Timed out
```

3.75.7 Maxima [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*(10*c^2*integrate(1/5*x/((c^2*e^3*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x +
 d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(c^2*x^2 + 1)*sqrt(e*x + d) + (c^2*e^3
*x^4 + 2*c^2*d*e^2*x^3 + 2*d*e^2*x + d^2*e + (c^2*d^2*e + e^3)*x^2)*sqrt(e
*x + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((e^3*x^2 + 2*d*e^2*x + d^2*e)
*sqrt(e*x + d)) + 5*integrate(1/5*((5*e*log(c) - 2*e)*c^2*x^2 - 2*c^2*d*x
+ 5*e*log(c) + 5*(c^2*e*x^2 + e)*log(x))/((c^2*e^4*x^5 + 3*c^2*d*e^3*x^4 +
 3*d^2*e^2*x + d^3*e + (3*c^2*d^2*e^2 + e^4)*x^3 + (c^2*d^3*e + 3*d*e^3)*x
^2)*sqrt(e*x + d)), x))*b - 2/5*a/((e*x + d)^(5/2)*e)
```

3.75.8 Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = \int \frac{b\operatorname{arcsch}(cx)+a}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x + d)^(7/2), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex)^{7/2}}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{7/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(7/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.76 $\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.76.1 Optimal result

Integrand size = 19, antiderivative size = 214

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b(42c^2d-25e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{560c^6\sqrt{-c^2x^2}}$$

output
```
1/5*d*x^5*(a+b*arccsch(c*x))+1/7*e*x^7*(a+b*arccsch(c*x))-1/560*b*(42*c^2*
d-25*e)*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^6/(-c^2*x^2)^(1/2)-1/560*b*(42*
c^2*d-25*e)*x^2*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)+1/840*b*(42*c^2*d-
25*e)*x^4*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/42*b*e*x^6*(-c^2*x^2-1
)^(1/2)/c/(-c^2*x^2)^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.64

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{48ac^7x^5(7d+5ex^2)+bc^2\sqrt{1+\frac{1}{c^2x^2}}x^2(75e-2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4))+48bc^7x^5(7d+5ex}{1680c^7}$$

input
```
Integrate[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2*(75*e - 2*
c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5
*e*x^2)*ArcCsch[c*x] + 3*b*(42*c^2*d - 25*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)
])*x])/(1680*c^7)
```

3.76.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {6856, 27, 363, 262, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \ 6856$$
$$-\frac{bcx\int\frac{x^4(5ex^2+7d)}{35\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)$$
$$\Big\downarrow \ 27$$
$$-\frac{bcx\int\frac{x^4(5ex^2+7d)}{\sqrt{-c^2x^2-1}}dx}{35\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)$$
$$\Big\downarrow \ 363$$

$$-\frac{bcx\left(\frac{1}{6}\left(42d-\frac{25e}{c^2}\right)\int \frac{x^4}{\sqrt{-c^2x^2-1}}dx-\frac{5ex^5\sqrt{-c^2x^2-1}}{6c^2}\right)}{35\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 262$$

$$-\frac{bcx\left(\frac{1}{6}\left(42d-\frac{25e}{c^2}\right)\left(-\frac{3\int \frac{x^2}{\sqrt{-c^2x^2-1}}dx}{4c^2}-\frac{x^3\sqrt{-c^2x^2-1}}{4c^2}\right)-\frac{5ex^5\sqrt{-c^2x^2-1}}{6c^2}\right)}{35\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 262$$

$$-\frac{bcx\left(\frac{1}{6}\left(42d-\frac{25e}{c^2}\right)\left(-\frac{3\left(-\frac{\int \frac{1}{\sqrt{-c^2x^2-1}}dx}{2c^2}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{-c^2x^2-1}}{4c^2}\right)-\frac{5ex^5\sqrt{-c^2x^2-1}}{6c^2}\right)}{35\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 224$$

$$-\frac{bcx\left(\frac{1}{6}\left(42d-\frac{25e}{c^2}\right)\left(-\frac{3\left(-\frac{\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}}{2c^2}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{-c^2x^2-1}}{4c^2}\right)-\frac{5ex^5\sqrt{-c^2x^2-1}}{6c^2}\right)}{35\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{5}dx^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{bcx\left(\frac{1}{6}\left(-\frac{3\left(-\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{2c^3}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)}{4c^2}-\frac{x^3\sqrt{-c^2x^2-1}}{4c^2}\right)\left(42d-\frac{25e}{c^2}\right)-\frac{5ex^5\sqrt{-c^2x^2-1}}{6c^2}\right)}{35\sqrt{-c^2x^2}}$$

input
```
Int[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

3.76. $\int x^4(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

output
```
(d*x^5*(a + b*ArcCsch[c*x]))/5 + (e*x^7*(a + b*ArcCsch[c*x]))/7 - (b*c*x*(
(-5*e*x^5*Sqrt[-1 - c^2*x^2])/(6*c^2) + ((42*d - (25*e)/c^2)*(-1/4*(x^3*Sq
rt[-1 - c^2*x^2])/c^2 - (3*(-1/2*(x*Sqrt[-1 - c^2*x^2])/c^2 - ArcTan[(c*x)
/Sqrt[-1 - c^2*x^2]]/(2*c^3)))/(4*c^2)))/6))/(35*Sqrt[-(c^2*x^2)])
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.76.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93

method	result
parts	$a\left(\frac{1}{7}e\,x^{7}+\frac{1}{5}d\,x^{5}\right)+\frac{b\left(\frac{c^{5}\operatorname{arccsch}(cx)e\,x^{7}}{7}+\frac{\operatorname{arccsch}(cx)x^{5}c^{5}d}{5}+\frac{\sqrt{c^{2}x^{2}+1}\left(84d\,c^{5}x^{3}\sqrt{c^{2}x^{2}+1}+40e\,c^{5}x^{5}\sqrt{c^{2}x^{2}+1}-126d\,c\right.}{}\right.}{c^{5}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^{7}x^{5}}{5}+\frac{\operatorname{arccsch}(cx)e\,c^{7}x^{7}}{7}+\frac{\sqrt{c^{2}x^{2}+1}\left(84d\,c^{5}x^{3}\sqrt{c^{2}x^{2}+1}+40e\,c^{5}x^{5}\sqrt{c^{2}x^{2}+1}-126d\,c^{3}x\sqrt{c\right.}{}\right.}{c^{2}}}{c^{5}}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^{7}x^{5}+\frac{1}{7}e\,c^{7}x^{7}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^{7}x^{5}}{5}+\frac{\operatorname{arccsch}(cx)e\,c^{7}x^{7}}{7}+\frac{\sqrt{c^{2}x^{2}+1}\left(84d\,c^{5}x^{3}\sqrt{c^{2}x^{2}+1}+40e\,c^{5}x^{5}\sqrt{c^{2}x^{2}+1}-126d\,c^{3}x\sqrt{c\right.}{}\right.}{c^{2}}}{c^{5}}$

input
```
int(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e*x^7+1/5*d*x^5)+b/c^5*(1/7*c^5*arccsch(c*x)*e*x^7+1/5*arccsch(c*x)
*x^5*c^5*d+1/1680/c^3*(c^2*x^2+1)^(1/2)*(84*d*c^5*x^3*(c^2*x^2+1)^(1/2)+40
*e*c^5*x^5*(c^2*x^2+1)^(1/2)-126*d*c^3*x*(c^2*x^2+1)^(1/2)-50*e*c^3*x^3*(c
^2*x^2+1)^(1/2)+126*d*c^2*arcsinh(c*x)+75*e*c*x*(c^2*x^2+1)^(1/2)-75*e*arc
sinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

$$\int x^{4}\left(d+ex^{2}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{240\,ac^{7}ex^{7}+336\,ac^{7}dx^{5}+48\left(7\,bc^{7}d+5\,bc^{7}e\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx+1\right)-3\left(42\,bc^{2}d-25\,be\right)\log\left(cx\sqrt{\right.}{}$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e*x^7 + 336*a*c^7*d*x^5 + 48*(7*b*c^7*d + 5*b*c^7*e)*log
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 3*(42*b*c^2*d - 25*b*e)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 48*(7*b*c^7*d + 5*b*c^7*e)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 48*(5*b*c^7*e*x^7 + 7*b*c
^7*d*x^5 - 7*b*c^7*d - 5*b*c^7*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + (40*b*c^6*e*x^6 + 2*(42*b*c^6*d - 25*b*c^4*e)*x^4 - 3*(42*b*c
^4*d - 25*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

3.76.6 Sympy [F]

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^4(a+b\operatorname{acsch}(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**4*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**4*(a + b*acsch(c*x))*(d + e*x**2), x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.35

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{7}aex^7+\frac{1}{5}adx^5$$
$$+\frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd$$
$$+\frac{1}{672}\left(96\,x^7\operatorname{arcsch}(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3\,c^6\left(\frac{1}{c^2x^2}+1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right.}{c^6}}{c}\right.$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2)
 + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1
/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt
(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(
1/(c^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2)
+ 1))/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2
*x^2) + 1) - c^6) - 15*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/
(c^2*x^2) + 1) - 1)/c^6)/c)*b*e
```

3.76.8 Giac [F]

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int(ex^2+d)(b\,\text{arcsch}\,(cx)+a)x^4\,dx$$

input
```
integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^4, x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^4\left(e\,x^2+d\right)\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^4*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^4*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.77 $\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.77.1 Optimal result

Integrand size = 19, antiderivative size = 167

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{b(20c^2d-9e)\,x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b(20c^2d-9e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{120c^4\sqrt{-c^2x^2}}$$

output
```
1/3*d*x^3*(a+b*arccsch(c*x))+1/5*e*x^5*(a+b*arccsch(c*x))+1/120*b*(20*c^2*
d-9*e)*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^4/(-c^2*x^2)^(1/2)+1/120*b*(20*c
^2*d-9*e)*x^2*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/20*b*e*x^4*(-c^2*x
^2-1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.71

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{c^2x^2\left(8ac^3x(5d+3ex^2)+b\sqrt{1+\frac{1}{c^2x^2}}(-9e+c^2(20d+6ex^2))\right)+8bc^5x^3(5d+3ex^2)\,\text{csch}^{-1}(cx)+b(-20c}{120c^5}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-9*e + c^2*
(20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsch[c*x] + b*(-20*c^2
*d + 9*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(120*c^5)
```

3.77.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6856, 27, 363, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+e x^2\right) \left(a+b \text{csch}^{-1}(c x)\right) \, dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{b c x \int \frac{x^2 \left(3 e x^2+5 d\right)}{15 \sqrt{-c^2 x^2-1}}dx}{\sqrt{-c^2 x^2}}+\frac{1}{3} d x^3 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{5} e x^5 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{b c x \int \frac{x^2 \left(3 e x^2+5 d\right)}{\sqrt{-c^2 x^2-1}}dx}{15 \sqrt{-c^2 x^2}}+\frac{1}{3} d x^3 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{5} e x^5 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{363}$$

$$-\frac{b c x \left(\frac{1}{4} \left(20 d-\frac{9 e}{c^2}\right) \int \frac{x^2}{\sqrt{-c^2 x^2-1}}dx-\frac{3 e x^3 \sqrt{-c^2 x^2-1}}{4 c^2}\right)}{15 \sqrt{-c^2 x^2}}+\frac{1}{3} d x^3 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{5} e x^5 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{262}$$

$$-\frac{b c x \left(\frac{1}{4} \left(20 d-\frac{9 e}{c^2}\right) \left(-\frac{\int \frac{1}{\sqrt{-c^2 x^2-1}}dx}{2 c^2}-\frac{x \sqrt{-c^2 x^2-1}}{2 c^2}\right)-\frac{3 e x^3 \sqrt{-c^2 x^2-1}}{4 c^2}\right)}{15 \sqrt{-c^2 x^2}}+\frac{1}{3} d x^3 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{5} e x^5 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{224}$$

$$-\frac{bcx\left(\frac{1}{4}\left(20d-\frac{9e}{c^2}\right)\left(-\frac{\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}}{2c^2}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)-\frac{3ex^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bcx\left(\frac{1}{4}\left(-\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{2c^3}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)\left(20d-\frac{9e}{c^2}\right)-\frac{3ex^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcCsch[c*x]))/3 + (e*x^5*(a + b*ArcCsch[c*x]))/5 - (b*c*x*(
(-3*e*x^3*Sqrt[-1 - c^2*x^2])/(4*c^2) + ((20*d - (9*e)/c^2)*(-1/2*(x*Sqrt[
-1 - c^2*x^2])/c^2 - ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]]/(2*c^3)))/4))/(15*Sq
rt[-(c^2*x^2)])
```

3.77.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.77.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

method	result
parts	$a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\left(\frac{c^3\,\mathrm{arccsch}(cx)e\,x^5}{5}+\frac{\mathrm{arccsch}(cx)x^3c^3d}{3}-\frac{\sqrt{c^2x^2+1}\,\left(-6e\,c^3x^3\sqrt{c^2x^2+1}-20d\,c^3x\sqrt{c^2x^2+1}+20d\,c^2 \cdots\right.}{120c^3\sqrt{\frac{c^2x^2+1}{c^2x^2}}\cdots}\right.}{c^3}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsch}(cx)d\,c^5x^3}{3}+\frac{\mathrm{arccsch}(cx)e\,c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\,\left(20d\,c^3x\sqrt{c^2x^2+1}+6e\,c^3x^3\sqrt{c^2x^2+1}-20d\,c^2\,\mathrm{arcsinh}\cdots\right.}{120\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^3}$
default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\mathrm{arccsch}(cx)d\,c^5x^3}{3}+\frac{\mathrm{arccsch}(cx)e\,c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\,\left(20d\,c^3x\sqrt{c^2x^2+1}+6e\,c^3x^3\sqrt{c^2x^2+1}-20d\,c^2\,\mathrm{arcsinh}\cdots\right.}{120\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^3}$

input
```
int(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arccsch(c*x)*e*x^5+1/3*arccsch(c*x)
*x^3*c^3*d-1/120/c^3*(c^2*x^2+1)^(1/2)*(-6*e*c^3*x^3*(c^2*x^2+1)^(1/2)-20*
d*c^3*x*(c^2*x^2+1)^(1/2)+20*d*c^2*arcsinh(c*x)+9*e*c*x*(c^2*x^2+1)^(1/2)-
9*e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.63

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{24\,ac^5ex^5+40\,ac^5dx^3+8\,(5\,bc^5d+3\,bc^5e)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)+(20\,bc^2d-9\,be)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right.}{}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e*x^5 + 40*a*c^5*d*x^3 + 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (20*b*c^2*d - 9*b*e)*log(c*x*s
qrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(5*b*c^5*d + 3*b*c^5*e)*log(c*x*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3
- 5*b*c^5*d - 3*b*c^5*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) + (6*b*c^4*e*x^4 + (20*b*c^4*d - 9*b*c^2*e)*x^2)*sqrt((c^2*x^2 + 1)/(c^2
*x^2)))/c^5
```

3.77.6 Sympy [F]

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2), x)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.36

$$\int x^2(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}\,aex^5 + \frac{1}{3}\,adx^3 + \frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd + \frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx) - \frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4} - \frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4} + \frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2)
 + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 +
 log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsch(c*x) -
(2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2)
 + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1
)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

3.77.8 Giac [F]

$$\int x^2(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int (ex^2+d)(b\operatorname{arcsch}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.78 $\int (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx$

3.78.1 Optimal result

Integrand size = 16, antiderivative size = 115

$$\int (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} + dx\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{b(6c^2d-e)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6c^2\sqrt{-c^2x^2}}$$

output
```
d*x*(a+b*arccsch(c*x))+1/3*e*x^3*(a+b*arccsch(c*x))-1/6*b*(6*c^2*d-e)*x*ar
ctan(c*x/(-c^2*x^2-1)^(1/2))/c^2/(-c^2*x^2)^(1/2)+1/6*b*e*x^2*(-c^2*x^2-1)
^(1/2)/c/(-c^2*x^2)^(1/2)
```

3.78.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.35

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = adx+\frac{1}{3}aex^3+\frac{bex^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{6c}+bdx\text{csch}^{-1}(cx)+\frac{1}{3}bex^3\text{csch}^{-1}(cx)+\frac{2bd\sqrt{1+\frac{1}{c^2x^2}}x\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}-\frac{be\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*
x*ArcCsch[c*x] + (b*e*x^3*ArcCsch[c*x])/3 + (2*b*d*Sqrt[1 + 1/(c^2*x^2)]*x
*ArcTanh[(-1 + Sqrt[1 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2] - (b*e*Log[x*(
1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

3.78.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6846, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6846}$$

$$-\frac{bcx\int \frac{ex^2+3d}{3\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{ex^2+3d}{\sqrt{-c^2x^2-1}}dx}{3\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 299$$

$$-\frac{bcx\left(\frac{1}{2}\left(6d-\frac{e}{c^2}\right)\int \frac{1}{\sqrt{-c^2x^2-1}}dx-\frac{ex\sqrt{-c^2x^2-1}}{2c^2}\right)}{3\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 224$$

$$-\frac{bcx\left(\frac{1}{2}\left(6d-\frac{e}{c^2}\right)\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}-\frac{ex\sqrt{-c^2x^2-1}}{2c^2}\right)}{3\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \; 216$$

$$dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bcx\left(\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)\left(6d-\frac{e}{c^2}\right)}{2c}-\frac{ex\sqrt{-c^2x^2-1}}{2c^2}\right)}{3\sqrt{-c^2x^2}}$$

input
```
Int[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
d*x*(a + b*ArcCsch[c*x]) + (e*x^3*(a + b*ArcCsch[c*x]))/3 - (b*c*x*(-1/2*(
e*x*Sqrt[-1 - c^2*x^2])/c^2 + ((6*d - e/c^2)*ArcTan[(c*x)/Sqrt[-1 - c^2*x^
2]])/(2*c)))/(3*Sqrt[-(c^2*x^2)])
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 6846
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsch[c*x])   u
, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[-1 -
 c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ
[p + 1/2, 0])
```

3.78.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

method	result
parts	$a\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b\left(\frac{c\ \operatorname{arccsch}(cx)e\,x^3}{3}+\operatorname{arccsch}(cx)dxc+\frac{\sqrt{c^2x^2+1}\left(6d\,c^2\ \operatorname{arcsinh}(cx)+ecx\sqrt{c^2x^2+1}-e\ \operatorname{arcsinh}(cx)\right)}{6c^3x\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c}$
derivativedivides	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^3x+\frac{\operatorname{arccsch}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(6d\,c^2\ \operatorname{arcsinh}(cx)+ecx\sqrt{c^2x^2+1}-e\ \operatorname{arcsinh}(cx)\right)}{6cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}}{c}$
default	$\frac{\frac{a\left(d\,c^3x+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^3x+\frac{\operatorname{arccsch}(cx)e\,c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(6d\,c^2\ \operatorname{arcsinh}(cx)+ecx\sqrt{c^2x^2+1}-e\ \operatorname{arcsinh}(cx)\right)}{6cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}\right)}{c^2}}{c}$

input
```
int((e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arccsch(c*x)*e*x^3+arccsch(c*x)*d*x*c+1/6/c^3
*(c^2*x^2+1)^(1/2)*(6*d*c^2*arcsinh(c*x)+e*c*x*(c^2*x^2+1)^(1/2)-e*arcsinh
(c*x))/x/((c^2*x^2+1)/c^2/x^2)^(1/2))
```

3.78.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(101) = 202$.

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.13

$$\int \left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$= \frac{2\,ac^3ex^3+bc^2ex^2\sqrt{\frac{c^2x^2+1}{c^2x^2}}+6\,ac^3dx+2\,(3\,bc^3d+bc^3e)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-(6\,bc^2d-be)\log\left(c\right.}{}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 + b*c^2*e*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 6*a*c^3*d
*x + 2*(3*b*c^3*d + b*c^3*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x +
 1) - (6*b*c^2*d - b*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(
3*b*c^3*d + b*c^3*e)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*
(b*c^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)))/c^3
```

3.78.6 Sympy [F]

$$\int \left(d+ex^2\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(a+b\operatorname{acsch}\left(cx\right)\right)\left(d+ex^2\right)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2), x)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{3}aex^3+\frac{1}{12}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be$$
$$+adx+\frac{\left(2cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd}{2c}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/
(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c
^2*x^2) + 1) - 1)/c^2)/c)*b*e + a*d*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt
(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d/c
```

3.78.8 Giac [F]

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (ex^2+d)(b\operatorname{arcsch}(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (e\,x^2+d) \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int((d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bex\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

output
```
-d*(a+b*arccsch(c*x))/x+e*x*(a+b*arccsch(c*x))-b*e*x*arctan(c*x/(-c^2*x^2-
1)^(1/2))/(-c^2*x^2)^(1/2)+b*c*d*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)
```

3.79.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=-\frac{ad}{x}+aex+bcd\sqrt{\frac{1+c^2x^2}{c^2x^2}}-\frac{bd\text{csch}^{-1}(cx)}{x}+bex\text{csch}^{-1}(cx)+\frac{2be\sqrt{1+\frac{1}{c^2x^2}}\,x\text{arctanh}\left(\frac{-1+\sqrt{1+c^2x^2}}{cx}\right)}{\sqrt{1+c^2x^2}}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x + b*c*d*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsch[c*
x])/x + b*e*x*ArcCsch[c*x] + (2*b*e*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(-1 +
Sqrt[1 + c^2*x^2])/(c*x)])/Sqrt[1 + c^2*x^2]
```

3.79.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6856, 25, 358, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\operatorname{csch}^{-1}(cx))}{x^2}dx$$

$$\Big\downarrow \; 6856$$

$$-\frac{bcx\int -\frac{d-ex^2}{x^2\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d(a+b\operatorname{csch}^{-1}(cx))}{x}+ex(a+b\operatorname{csch}^{-1}(cx))$$

$$\Big\downarrow \; 25$$

$$\frac{bcx\int \frac{d-ex^2}{x^2\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d(a+b\operatorname{csch}^{-1}(cx))}{x}+ex(a+b\operatorname{csch}^{-1}(cx))$$

$$\Big\downarrow \; 358$$

$$\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{x}-e\int \frac{1}{\sqrt{-c^2x^2-1}}dx\right)}{\sqrt{-c^2x^2}}-\frac{d(a+b\operatorname{csch}^{-1}(cx))}{x}+ex(a+b\operatorname{csch}^{-1}(cx))$$

$$\Big\downarrow \; 224$$

$$\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{x}-e\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}-\frac{d(a+b\operatorname{csch}^{-1}(cx))}{x}+ex(a+b\operatorname{csch}^{-1}(cx))$$

$$\Big\downarrow \; 216$$

$$-\frac{d(a+b\operatorname{csch}^{-1}(cx))}{x}+ex(a+b\operatorname{csch}^{-1}(cx))+\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{x}-\frac{e\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{c}\right)}{\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcCsch[c*x]))/x) + e*x*(a + b*ArcCsch[c*x]) + (b*c*x*((d*Sqrt
[-1 - c^2*x^2])/x - (e*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/c))/Sqrt[-(c^2*x^
2)]
```

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.79.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12

method	result	size
parts	$a\left(ex-\frac{d}{x}\right)+bc\left(-\frac{\operatorname{arccsch}(cx)d}{xc}+\frac{\operatorname{arccsch}(cx)ex}{c}+\frac{\sqrt{c^{2}x^{2}+1}\left(d\,c^{2}\sqrt{c^{2}x^{2}+1}+e\,\operatorname{arcsinh}(cx)cx\right)}{c^{4}x^{2}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right)$	102
derivativedivides	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(c\,\operatorname{arccsch}(cx)ex-\frac{\operatorname{arccsch}(cx)dc}{x}+\frac{\sqrt{c^{2}x^{2}+1}\left(d\,c^{2}\sqrt{c^{2}x^{2}+1}+e\,\operatorname{arcsinh}(cx)cx\right)}{c^{2}x^{2}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right)}{c^{2}}\right)$	107
default	$c\left(\frac{a\left(cex-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(c\,\operatorname{arccsch}(cx)ex-\frac{\operatorname{arccsch}(cx)dc}{x}+\frac{\sqrt{c^{2}x^{2}+1}\left(d\,c^{2}\sqrt{c^{2}x^{2}+1}+e\,\operatorname{arcsinh}(cx)cx\right)}{c^{2}x^{2}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right)}{c^{2}}\right)$	107

input
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(e*x-d/x)+b*c*(-arccsch(c*x)*d/x/c+1/c*arccsch(c*x)*e*x+1/c^4*(c^2*x^2+1
)^(1/2)*(d*c^2*(c^2*x^2+1)^(1/2)+e*arcsinh(c*x)*c*x)/x^2/((c^2*x^2+1)/c^2/
x^2)^(1/2))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.44

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$
$$=\frac{bc^2dx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+bc^2dx+acex^2-bex\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-acd-(bcd-bce)x\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right.}{cx}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

output
```
(b*c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + b*c^2*d*x + a*c*e*x^2 - b*e*x*l
og(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - a*c*d - (b*c*d - b*c*e)*x*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c*d - b*c*e)*x*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*e*x^2 - b*c*d + (b*c*d -
b*c*e)*x)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)
```

3.79.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\,(d+ex^2)}{x^2}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x))/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)/x**2, x)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\begin{aligned}
&\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx \\
&= \left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)bd+aex \\
&\quad+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)be}{2\,c}-\frac{ad}{x}
\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d + a*e*x + 1/2*(2*c*x*arccsc
h(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*
b*e/c - a*d/x
```

3.79.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.80 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$

3.80.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{bc(2c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

output
```
-1/3*d*(a+b*arccsch(c*x))/x^3-e*(a+b*arccsch(c*x))/x-1/9*b*c*(2*c^2*d-9*e)
*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/9*b*c*d*(-c^2*x^2-1)^(1/2)/x^2/(-c^
2*x^2)^(1/2)
```

3.80.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \frac{-3a(d+3ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}x(d-2c^2dx^2+9ex^2)-3b(d+3ex^2)\text{csch}^{-1}(cx)}{9x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
(-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 9*e*x
^2) - 3*b*(d + 3*e*x^2)*ArcCsch[c*x])/(9*x^3)
```

3.80.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {6856, 27, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}dx$$

$$\Big\downarrow \text{ 6856}$$

$$-\frac{bcx\int -\frac{3ex^2+d}{3x^4\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx\int \frac{3ex^2+d}{x^4\sqrt{-c^2x^2-1}}dx}{3\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{3x^3}-\frac{1}{3}(2c^2d-9e)\int \frac{1}{x^2\sqrt{-c^2x^2-1}}dx\right)}{3\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}+\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{3x^3}-\frac{\sqrt{-c^2x^2-1}(2c^2d-9e)}{3x}\right)}{3\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
(b*c*x*((d*Sqrt[-1 - c^2*x^2])/(3*x^3) - ((2*c^2*d - 9*e)*Sqrt[-1 - c^2*x^
2])/(3*x)))/(3*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(3*x^3) - (e*(
a + b*ArcCsch[c*x]))/x
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.80.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

method	result	size
parts	$a\left(-\frac{e}{x}-\frac{d}{3x^3}\right)+b\,c^3\left(-\frac{\operatorname{arccsch}(cx)e}{c^3x}-\frac{\operatorname{arccsch}(cx)d}{3x^3c^3}-\frac{(c^2x^2+1)(2c^4d\,x^2-9e\,c^2x^2-c^2d)}{9c^6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x^4}\right)$	109
derivativedivides	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{cx}-\frac{\operatorname{arccsch}(cx)d}{3c\,x^3}-\frac{(c^2x^2+1)(2c^4d\,x^2-9e\,c^2x^2-c^2d)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	122
default	$c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{cx}-\frac{\operatorname{arccsch}(cx)d}{3c\,x^3}-\frac{(c^2x^2+1)(2c^4d\,x^2-9e\,c^2x^2-c^2d)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)}{c^2}\right)$	122

input
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-e/x-1/3*d/x^3)+b*c^3*(-1/c^3*arccsch(c*x)*e/x-1/3*arccsch(c*x)*d/x^3/c
^3-1/9/c^6*(c^2*x^2+1)*(2*c^4*d*x^2-9*c^2*e*x^2-c^2*d)/((c^2*x^2+1)/c^2/x^
2)^(1/2)/x^4)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$$
$$=-\frac{9\,aex^2+3\,ad+3\,(3\,bex^2+bd)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(bcdx-(2\,bc^3d-9\,bce)x^3)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{9\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 1)/(c*x)) - (b*c*d*x - (2*b*c^3*d - 9*b*c*e)*x^3)*sqrt((c^2*x
^2 + 1)/(c^2*x^2)))/x^3
```

3.80.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)}{x^4}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x))/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)/x**4, x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\begin{aligned}&\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx\\&=\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\text{arcsch}\,(cx)}{x}\right)be\\&\quad+\frac{1}{9}\,bd\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\text{arcsch}\,(cx)}{x^3}\right)-\frac{ae}{x}-\frac{ad}{3\,x^3}\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2
) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - a*e/
x - 1/3*a*d/x^3
```

3.80.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.81 $\int \frac{(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^6}\,dx$

3.81.1 Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{2bc^3(12c^2d-25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

output
```
-1/5*d*(a+b*arccsch(c*x))/x^5-1/3*e*(a+b*arccsch(c*x))/x^3+2/225*b*c^3*(12
*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/25*b*c*d*(-c^2*x^2-1)^(
1/2)/x^4/(-c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2
/(-c^2*x^2)^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{-15a(3d+5ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}x(25ex^2(1-2c^2x^2)+3d(3-4c^2x^2+8c^4x^4))-15b(3d+5ex^2)\operatorname{csch}^{-1}(c}{225x^5}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(25*e*x^2*(1 - 2*c^2*
x^2) + 3*d*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcCsch[c*x
])/(225*x^5)
```

3.81.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6856, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\,(a+b\text{csch}^{-1}(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{ 6856}$$

$$-\frac{bcx\int -\frac{5ex^2+3d}{15x^6\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d(a+b\text{csch}^{-1}(cx))}{5x^5}-\frac{e(a+b\text{csch}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx\int \frac{5ex^2+3d}{x^6\sqrt{-c^2x^2-1}}dx}{15\sqrt{-c^2x^2}}-\frac{d(a+b\text{csch}^{-1}(cx))}{5x^5}-\frac{e(a+b\text{csch}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{bcx\left(\frac{3d\sqrt{-c^2x^2-1}}{5x^5}-\frac{1}{5}(12c^2d-25e)\int \frac{1}{x^4\sqrt{-c^2x^2-1}}dx\right)}{15\sqrt{-c^2x^2}}-\frac{d(a+b\text{csch}^{-1}(cx))}{5x^5}-\frac{e(a+b\text{csch}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{bcx\left(\frac{3d\sqrt{-c^2x^2-1}}{5x^5}-\frac{1}{5}(12c^2d-25e)\left(\frac{\sqrt{-c^2x^2-1}}{3x^3}-\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{-c^2x^2-1}}dx\right)\right)}{15\sqrt{-c^2x^2}}-\frac{d(a+b\text{csch}^{-1}(cx))}{5x^5}-\frac{e(a+b\text{csch}^{-1}(cx))}{3x^3}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{d(a+b\operatorname{csch}^{-1}(cx))}{5x^5}-\frac{e(a+b\operatorname{csch}^{-1}(cx))}{3x^3}+\frac{bcx\left(\frac{3d\sqrt{-c^2x^2-1}}{5x^5}-\frac{1}{5}\left(\frac{\sqrt{-c^2x^2-1}}{3x^3}-\frac{2c^2\sqrt{-c^2x^2-1}}{3x}\right)(12c^2d-25e)\right)}{15\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
(b*c*x*((3*d*Sqrt[-1 - c^2*x^2])/(5*x^5) - ((12*c^2*d - 25*e)*(Sqrt[-1 - c
^2*x^2]/(3*x^3) - (2*c^2*Sqrt[-1 - c^2*x^2])/(3*x)))/5))/(15*Sqrt[-(c^2*x^
2)]) - (d*(a + b*ArcCsch[c*x]))/(5*x^5) - (e*(a + b*ArcCsch[c*x]))/(3*x^3)
```

3.81.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.81.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

method	result
parts	$a\left(-\frac{e}{3x^3}-\frac{d}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arccsch}(cx)e}{3c^5x^3}-\frac{\operatorname{arccsch}(cx)d}{5x^5c^5}+\frac{(c^2x^2+1)\left(24c^6d\,x^4-50c^4e\,x^4-12c^4d\,x^2+25e\,c^2x^2\right.}{225c^8\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x^6}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsch}(cx)d}{5c^3x^5}+\frac{(c^2x^2+1)\left(24c^6d\,x^4-50c^4e\,x^4-12c^4d\,x^2+25e\,c^2x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right.$
default	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsch}(cx)d}{5c^3x^5}+\frac{(c^2x^2+1)\left(24c^6d\,x^4-50c^4e\,x^4-12c^4d\,x^2+25e\,c^2x^2+9c^2d\right)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*e/x^3-1/5*d/x^5)+b*c^5*(-1/3/c^5*arccsch(c*x)*e/x^3-1/5*arccsch(c*
x)*d/x^5/c^5+1/225/c^8*(c^2*x^2+1)*(24*c^6*d*x^4-50*c^4*e*x^4-12*c^4*d*x^2
+25*c^2*e*x^2+9*c^2*d)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^6)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.80

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx =$$

$$-\frac{75\,aex^2+45\,ad+15\,(5\,bex^2+3\,bd)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(2\,(12\,bc^5d-25\,bc^3e)x^5+9\,bcdx-(12\,bc^3d-}{225\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*log((c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (2*(12*b*c^5*d - 25*b*c^3*e)*x^5 + 9*b*c*d
*x - (12*b*c^3*d - 25*b*c*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^5
```

3.81.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)}{x^6}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x))/x**6,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)/x**6, x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$$

$$= \frac{1}{75}\,bd\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\text{arcsch}\,(cx)}{x^5}\right)$$

$$+\frac{1}{9}\,be\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\text{arcsch}\,(cx)}{x^3}\right)-\frac{ae}{3\,x^3}-\frac{ad}{5\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/75*b*d*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2)
+ 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) + 1/9*b*e*((c^4*(
1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x
^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

3.81.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.82 $\int \frac{(d+ex^2)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^8}\,dx$

3.82.1 Optimal result

Integrand size = 19, antiderivative size = 205

$$\int \frac{(d+ex^2)\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{8bc^5(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{4bc^3(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675x^2\sqrt{-c^2x^2}} - \frac{d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5x^5}$$

output
```
-1/7*d*(a+b*arccsch(c*x))/x^7-1/5*e*(a+b*arccsch(c*x))/x^5-8/3675*b*c^5*(3
0*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/49*b*c*d*(-c^2*x^2-1)^
(1/2)/x^6/(-c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x
^4/(-c^2*x^2)^(1/2)+4/3675*b*c^3*(30*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^2/(-
c^2*x^2)^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.53

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$$
$$=\frac{-105a(5d+7ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}x(49ex^2(3-4c^2x^2+8c^4x^4)-15d(-5+6c^2x^2-8c^4x^4+16c^6x^6))-10}{3675x^7}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

output
```
(-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(49*e*x^2*(3 - 4*c^2
*x^2 + 8*c^4*x^4) - 15*d*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*
b*(5*d + 7*e*x^2)*ArcCsch[c*x])/(3675*x^7)
```

3.82.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6856, 27, 359, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$$

$$\Big\downarrow \ 6856$$

$$-\frac{bcx\int -\frac{7ex^2+5d}{35x^8\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}$$

$$\Big\downarrow \ 27$$

$$\frac{bcx\int \frac{7ex^2+5d}{x^8\sqrt{-c^2x^2-1}}dx}{35\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}$$

$$\Big\downarrow \ 359$$

$$\frac{bcx\left(\frac{5d\sqrt{-c^2x^2-1}}{7x^7}-\frac{1}{7}(30c^2d-49e)\int \frac{1}{x^6\sqrt{-c^2x^2-1}}dx\right)}{35\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}$$

$$\Big\downarrow \ 245$$

$$\frac{bcx\left(\frac{5d\sqrt{-c^2x^2-1}}{7x^7}-\frac{1}{7}(30c^2d-49e)\left(\frac{\sqrt{-c^2x^2-1}}{5x^5}-\frac{4}{5}c^2\int\frac{1}{x^4\sqrt{-c^2x^2-1}}dx\right)\right)}{35\sqrt{-c^2x^2}}-\frac{d(a+b\text{csch}^{-1}(cx))}{7x^7}-\frac{e(a+b\text{csch}^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{245}$$

$$\frac{bcx\left(\frac{5d\sqrt{-c^2x^2-1}}{7x^7}-\frac{1}{7}(30c^2d-49e)\left(\frac{\sqrt{-c^2x^2-1}}{5x^5}-\frac{4}{5}c^2\left(\frac{\sqrt{-c^2x^2-1}}{3x^3}-\frac{2}{3}c^2\int\frac{1}{x^2\sqrt{-c^2x^2-1}}dx\right)\right)\right)}{35\sqrt{-c^2x^2}}-\frac{d(a+b\text{csch}^{-1}(cx))}{7x^7}-\frac{e(a+b\text{csch}^{-1}(cx))}{5x^5}$$

$$\Big\downarrow \text{242}$$

$$-\frac{d(a+b\text{csch}^{-1}(cx))}{7x^7}-\frac{e(a+b\text{csch}^{-1}(cx))}{5x^5}+\frac{bcx\left(\frac{5d\sqrt{-c^2x^2-1}}{7x^7}-\frac{1}{7}\left(\frac{\sqrt{-c^2x^2-1}}{5x^5}-\frac{4}{5}c^2\left(\frac{\sqrt{-c^2x^2-1}}{3x^3}-\frac{2c^2\sqrt{-c^2x^2-1}}{3x}\right)\right)(30c^2d-49e)\right)}{35\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

output
```
(b*c*x*((5*d*Sqrt[-1 - c^2*x^2])/(7*x^7) - ((30*c^2*d - 49*e)*(Sqrt[-1 - c
^2*x^2]/(5*x^5) - (4*c^2*(Sqrt[-1 - c^2*x^2]/(3*x^3) - (2*c^2*Sqrt[-1 - c^
2*x^2])/(3*x)))/5))/7))/(35*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(
7*x^7) - (e*(a + b*ArcCsch[c*x]))/(5*x^5)
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.82.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.71

method	result
parts	$a\left(-\frac{e}{5x^5}-\frac{d}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arccsch}(cx)e}{5c^7x^5}-\frac{\operatorname{arccsch}(cx)d}{7x^7c^7}-\frac{(c^2x^2+1)\left(240c^8d\,x^6-392c^6e\,x^6-120c^6d\,x^4+196c\cdots\right.}{3675c^{10}\sqrt{\frac{c^2x^2+1}{c^2x^2}}\cdots}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsch}(cx)e}{5c^5x^5}-\frac{(c^2x^2+1)\left(240c^8d\,x^6-392c^6e\,x^6-120c^6d\,x^4+196c^4e\,x^4+90c^4\cdots\right.}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsch}(cx)e}{5c^5x^5}-\frac{(c^2x^2+1)\left(240c^8d\,x^6-392c^6e\,x^6-120c^6d\,x^4+196c^4e\,x^4+90c^4\cdots\right.}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

input
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/5*e/x^5-1/7*d/x^7)+b*c^7*(-1/5/c^7*arccsch(c*x)*e/x^5-1/7*arccsch(c*
x)*d/x^7/c^7-1/3675/c^10*(c^2*x^2+1)*(240*c^8*d*x^6-392*c^6*e*x^6-120*c^6*
d*x^4+196*c^4*e*x^4+90*c^4*d*x^2-147*c^2*e*x^2-75*c^2*d)/((c^2*x^2+1)/c^2/
x^2)^(1/2)/x^8)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.71

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{735\,aex^2+525\,ad+105\,(7\,bex^2+5\,bd)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(8\,(30\,bc^7d-49\,bc^5e)x^7-4\,(30\,bc^5d-49}{3675\,x^7}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*(30*b*c^7*d - 49*b*c^5*e)*x^7 - 4*(
30*b*c^5*d - 49*b*c^3*e)*x^5 - 75*b*c*d*x + 3*(30*b*c^3*d - 49*b*c*e)*x^3)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

3.82.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\,(d+ex^2)}{x^8}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x))/x**8,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)/x**8, x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx &= \frac{1}{245}\,bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\operatorname{arcsch}(cx)}{x^7}\right)\\ &\quad+\frac{1}{75}\,be\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsch}(cx)}{x^5}\right)\\ &\quad-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/245*b*d*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2)
 + 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*a
rccsch(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c
^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5
) - 1/5*a*e/x^5 - 1/7*a*d/x^7
```

3.82.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(ex^2+d)(b\operatorname{arcsch}(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^8, x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.83 $\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.83.1 Optimal result

Integrand size = 19, antiderivative size = 204

$$\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{b(4c^2d-3e)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}} + \frac{b(8c^2d-9e)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}} + \frac{b(4c^2d-9e)\,x(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}} - \frac{bex(-1-c^2x^2)^{7/2}}{56c^7\sqrt{-c^2x^2}} + \frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)$$

output
```
1/6*d*x^6*(a+b*arccsch(c*x))+1/8*e*x^8*(a+b*arccsch(c*x))+1/72*b*(8*c^2*d-
9*e)*x*(-c^2*x^2-1)^(3/2)/c^7/(-c^2*x^2)^(1/2)+1/120*b*(4*c^2*d-9*e)*x*(-c
^2*x^2-1)^(5/2)/c^7/(-c^2*x^2)^(1/2)-1/56*b*e*x*(-c^2*x^2-1)^(7/2)/c^7/(-c
^2*x^2)^(1/2)+1/24*b*(4*c^2*d-3*e)*x*(-c^2*x^2-1)^(1/2)/c^7/(-c^2*x^2)^(1/
2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{x\left(105ax^5(4d+3ex^2)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(-144e+8c^2(28d+9ex^2)-2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6)\right)}{c^7}+105bx^5(4d+3ex^2)\,\text{c}\right.}{2520}$$

input
```
Integrate[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-144*e + 8*c^2*(
28*d + 9*e*x^2) - 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6))
)/c^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsch[c*x]))/2520
```

3.83.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6856, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \ 6856$$
$$-\frac{bcx\int\frac{x^5(3ex^2+4d)}{24\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)$$
$$\Big\downarrow \ 27$$
$$-\frac{bcx\int\frac{x^5(3ex^2+4d)}{\sqrt{-c^2x^2-1}}dx}{24\sqrt{-c^2x^2}}+\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)$$
$$\Big\downarrow \ 354$$
$$-\frac{bcx\int\frac{x^4(3ex^2+4d)}{\sqrt{-c^2x^2-1}}dx^2}{48\sqrt{-c^2x^2}}+\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{86}$$

$$-\frac{bcx\int\left(-\frac{3e(-c^2x^2-1)^{5/2}}{c^6}+\frac{(4c^2d-9e)(-c^2x^2-1)^{3/2}}{c^6}+\frac{(8c^2d-9e)\sqrt{-c^2x^2-1}}{c^6}+\frac{4c^2d-3e}{c^6\sqrt{-c^2x^2-1}}\right)dx^2}{48\sqrt{-c^2x^2}}+$$
$$\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-$$
$$\frac{bcx\left(-\frac{2(-c^2x^2-1)^{5/2}(4c^2d-9e)}{5c^8}-\frac{2(-c^2x^2-1)^{3/2}(8c^2d-9e)}{3c^8}-\frac{2\sqrt{-c^2x^2-1}(4c^2d-3e)}{c^8}+\frac{6e(-c^2x^2-1)^{7/2}}{7c^8}\right)}{48\sqrt{-c^2x^2}}$$

input
```
Int[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
-1/48*(b*c*x*((-2*(4*c^2*d - 3*e)*Sqrt[-1 - c^2*x^2])/c^8 - (2*(8*c^2*d -
9*e)*(-1 - c^2*x^2)^(3/2))/(3*c^8) - (2*(4*c^2*d - 9*e)*(-1 - c^2*x^2)^(5/
2))/(5*c^8) + (6*e*(-1 - c^2*x^2)^(7/2))/(7*c^8)))/Sqrt[-(c^2*x^2)] + (d*x
^6*(a + b*ArcCsch[c*x]))/6 + (e*x^8*(a + b*ArcCsch[c*x]))/8
```

3.83.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.83.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.68

method	result
parts	$a\left(\frac{1}{8}e\,x^{8}+\frac{1}{6}d\,x^{6}\right)+\frac{b\left(\frac{c^{6}\operatorname{arccsch}(cx)e\,x^{8}}{8}+\frac{\operatorname{arccsch}(cx)d\,x^{6}c^{6}}{6}+\frac{\left(c^{2}x^{2}+1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}-54c^{4}e\,x^{4}-112c^{4}d\,x^{2}+72c\right.}{2520c^{3}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,x}\right.}{c^{6}}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arccsch}(cx)e\,c^{8}x^{8}}{8}+\frac{\left(c^{2}x^{2}+1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}-54c^{4}e\,x^{4}-112c^{4}d\,x^{2}+72e\,c^{2}x^{2}\right.}{2520\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,cx}\right.}{c^{2}}}{c^{6}}$
default	$\frac{\frac{a\left(\frac{1}{6}c^{8}d\,x^{6}+\frac{1}{8}e\,c^{8}x^{8}\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^{8}x^{6}}{6}+\frac{\operatorname{arccsch}(cx)e\,c^{8}x^{8}}{8}+\frac{\left(c^{2}x^{2}+1\right)\left(45c^{6}e\,x^{6}+84c^{6}d\,x^{4}-54c^{4}e\,x^{4}-112c^{4}d\,x^{2}+72e\,c^{2}x^{2}\right.}{2520\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,cx}\right.}{c^{2}}}{c^{6}}$

input
```
int(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e*x^8+1/6*d*x^6)+b/c^6*(1/8*c^6*arccsch(c*x)*e*x^8+1/6*arccsch(c*x)
*d*x^6*c^6+1/2520/c^3*(c^2*x^2+1)*(45*c^6*e*x^6+84*c^6*d*x^4-54*c^4*e*x^4-
112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d-144*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.81

$$\int x^5 (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx$$
$$= \frac{315\, ac^7ex^8 + 420\, ac^7dx^6 + 105\,(3\, bc^7ex^8 + 4\, bc^7dx^6)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + (45\, bc^6ex^7 + 6\,(14\, bc^6d - 9\, bc^4e}{2520\, c^7}$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d
*x^6)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (45*b*c^6*e*x^7
 + 6*(14*b*c^6*d - 9*b*c^4*e)*x^5 - 8*(14*b*c^4*d - 9*b*c^2*e)*x^3 + 16*(1
4*b*c^2*d - 9*b*e)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

3.83.6 Sympy [F]

$$\int x^5 (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx = \int x^5 (a+b\,\text{acsch}\,(cx))\,(d+ex^2)\, dx$$

input
```
integrate(x**5*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**5*(a + b*acsch(c*x))*(d + e*x**2), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

$$\int x^5 (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{1}{8}\, aex^8 + \frac{1}{6}\, adx^6$$
$$+ \frac{1}{90}\left(15\, x^6\, \text{arcsch}\,(cx) + \frac{3\, c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\, c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\, x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right) bd$$
$$+ \frac{1}{280}\left(35\, x^8\, \text{arcsch}\,(cx) + \frac{5\, c^6x^7\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21\, c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\, c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35\, x\sqrt{\frac{1}{c^2x^2}+1}}{c^7}\right.$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2
*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x
^2) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2)
+ 1)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2)
+ 1)^(3/2) - 35*x*sqrt(1/(c^2*x^2) + 1))/c^7)*b*e
```

3.83.8 Giac [F]

$$\int x^5\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (ex^2+d)(b\,\text{arcsch}\,(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int x^5\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^5\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.84 $\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.84.1 Optimal result

Integrand size = 19, antiderivative size = 159

$$\begin{aligned}\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx =& -\frac{b(3c^2d-2e)\,x\sqrt{-1-c^2x^2}}{12c^5\sqrt{-c^2x^2}} \\ &-\frac{b(3c^2d-4e)\,x(-1-c^2x^2)^{3/2}}{36c^5\sqrt{-c^2x^2}}+\frac{bex(-1-c^2x^2)^{5/2}}{30c^5\sqrt{-c^2x^2}} \\ &+\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)\end{aligned}$$

output
```
1/4*d*x^4*(a+b*arccsch(c*x))+1/6*e*x^6*(a+b*arccsch(c*x))-1/36*b*(3*c^2*d-
4*e)*x*(-c^2*x^2-1)^(3/2)/c^5/(-c^2*x^2)^(1/2)+1/30*b*e*x*(-c^2*x^2-1)^(5/
2)/c^5/(-c^2*x^2)^(1/2)-1/12*b*(3*c^2*d-2*e)*x*(-c^2*x^2-1)^(1/2)/c^5/(-c^
2*x^2)^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\begin{aligned}&\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\ &=\frac{1}{180}x\left(15ax^3(3d+2ex^2)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(16e-2c^2(15d+4ex^2)+3c^4(5dx^2+2ex^4)\right)}{c^5}\right. \\ &\left.+15bx^3(3d+2ex^2)\,\text{csch}^{-1}(cx)\right)\end{aligned}$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(16*e - 2*c^2*(15*
d + 4*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*
ArcCsch[c*x]))/180
```

3.84.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6856, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+e x^2\right) \left(a+b \text{csch}^{-1}(c x)\right) \, dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{b c x \int \frac{x^3 \left(2 e x^2+3 d\right)}{12 \sqrt{-c^2 x^2-1}}dx}{\sqrt{-c^2 x^2}}+\frac{1}{4} d x^4 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{6} e x^6 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{b c x \int \frac{x^3 \left(2 e x^2+3 d\right)}{\sqrt{-c^2 x^2-1}}dx}{12 \sqrt{-c^2 x^2}}+\frac{1}{4} d x^4 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{6} e x^6 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{354}$$

$$-\frac{b c x \int \frac{x^2 \left(2 e x^2+3 d\right)}{\sqrt{-c^2 x^2-1}}dx^2}{24 \sqrt{-c^2 x^2}}+\frac{1}{4} d x^4 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{6} e x^6 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{86}$$

$$-\frac{b c x \int \left(\frac{2 e \left(-c^2 x^2-1\right)^{3/2}}{c^4}+\frac{\left(4 e-3 c^2 d\right) \sqrt{-c^2 x^2-1}}{c^4}+\frac{2 e-3 c^2 d}{c^4 \sqrt{-c^2 x^2-1}}\right)dx^2}{24 \sqrt{-c^2 x^2}}+\frac{1}{4} d x^4 \left(a+b \text{csch}^{-1}(c x)\right)+\frac{1}{6} e x^6 \left(a+b \text{csch}^{-1}(c x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bcx\left(\frac{2\left(-c^2x^2-1\right)^{3/2}\left(3c^2d-4e\right)}{3c^6}+\frac{2\sqrt{-c^2x^2-1}\left(3c^2d-2e\right)}{c^6}-\frac{4e\left(-c^2x^2-1\right)^{5/2}}{5c^6}\right)}{24\sqrt{-c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
-1/24*(b*c*x*((2*(3*c^2*d - 2*e)*Sqrt[-1 - c^2*x^2])/c^6 + (2*(3*c^2*d - 4
*e)*(-1 - c^2*x^2)^(3/2))/(3*c^6) - (4*e*(-1 - c^2*x^2)^(5/2))/(5*c^6)))/S
qrt[-(c^2*x^2)] + (d*x^4*(a + b*ArcCsch[c*x]))/4 + (e*x^6*(a + b*ArcCsch[c
*x]))/6
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.84.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.76

method	result
parts	$a\left(\frac{1}{6}e\,x^{6}+\frac{1}{4}d\,x^{4}\right)+\frac{b\left(\frac{c^{4}\operatorname{arccsch}(cx)e\,x^{6}}{6}+\frac{\operatorname{arccsch}(cx)x^{4}c^{4}d}{4}+\frac{\left(c^{2}x^{2}+1\right)\left(6c^{4}e\,x^{4}+15c^{4}d\,x^{2}-8e\,c^{2}x^{2}-30c^{2}d+16e\right)}{180c^{3}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,x}\right)}{c^{4}}$
derivativedivides	$\frac{-\frac{a\left(\frac{c^{2}d\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^{6}d^{3}}{12e^{2}}+\frac{\operatorname{arccsch}(cx)c^{6}d\,x^{4}}{4}+\frac{e\operatorname{arccsch}(cx)c^{6}x^{6}}{6}+\frac{\sqrt{c^{2}x^{2}+1}\left(15c^{6}d^{3}a\right.}{}\right.}{}}{c^{4}}$
default	$\frac{-\frac{a\left(\frac{c^{2}d\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}{2}-\frac{\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{3}\right)}{2c^{2}e^{2}}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^{6}d^{3}}{12e^{2}}+\frac{\operatorname{arccsch}(cx)c^{6}d\,x^{4}}{4}+\frac{e\operatorname{arccsch}(cx)c^{6}x^{6}}{6}+\frac{\sqrt{c^{2}x^{2}+1}\left(15c^{6}d^{3}a\right.}{}\right.}{}}{c^{4}}$

input
```
int(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arccsch(c*x)*e*x^6+1/4*arccsch(c*x)
*x^4*c^4*d+1/180/c^3*(c^2*x^2+1)*(6*c^4*e*x^4+15*c^4*d*x^2-8*c^2*e*x^2-30*
c^2*d+16*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91

$$\int x^3\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{30\,ac^5ex^6+45\,ac^5dx^4+15\left(2\,bc^5ex^6+3\,bc^5dx^4\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(6\,bc^4ex^5+\left(15\,bc^4d-8\,bc^2e\right)x^3-\right.}{180\,c^5}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4
)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e*x^5 + (1
5*b*c^4*d - 8*b*c^2*e)*x^3 - 2*(15*b*c^2*d - 8*b*e)*x)*sqrt((c^2*x^2 + 1)/
(c^2*x^2)))/c^5
```

3.84.6 Sympy [F]

$$\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^3(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\begin{aligned}&\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx\\&=\frac{1}{6}\,aex^6+\frac{1}{4}\,adx^4+\frac{1}{12}\left(3\,x^4\,\text{arcsch}\,(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd\\&\quad+\frac{1}{90}\left(15\,x^6\,\text{arcsch}\,(cx)+\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)be\end{aligned}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^
2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsch
(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^
(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e
```

3.84.8 Giac [F]

$$\int x^3 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b \operatorname{arcsch}(cx)+a)x^3 \, dx$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int x^3 \left(e\,x^2+d\right) \left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int(x^3*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.85 $\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.85.1 Optimal result

Integrand size = 17, antiderivative size = 146

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex(-1-c^2x^2)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcd^2x\arctan\left(\sqrt{-1-c^2x^2}\right)}{4e\sqrt{-c^2x^2}}$$

output
```
1/4*(e*x^2+d)^2*(a+b*arccsch(c*x))/e-1/12*b*e*x*(-c^2*x^2-1)^(3/2)/c^3/(-c
^2*x^2)^(1/2)-1/4*b*c*d^2*x*arctan((-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+
1/4*b*(2*c^2*d-e)*x*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)
```

3.85.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.53

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{x\left(3ac^3x(2d+ex^2)+b\sqrt{1+\frac{1}{c^2x^2}}(-2e+c^2(6d+ex^2))+3bc^3x(2d+ex^2)\,\text{csch}^{-1}(cx)\right)}{12c^3}$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-2*e + c^2*(6*d + e
*x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsch[c*x]))/(12*c^3)
```

3.85.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {6854, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{ 6854}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcx\int \frac{\left(ex^2+d\right)^2}{x\sqrt{-c^2x^2-1}}dx}{4e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcx\int \frac{\left(ex^2+d\right)^2}{x^2\sqrt{-c^2x^2-1}}dx^2}{8e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcx\int \left(\frac{d^2}{x^2\sqrt{-c^2x^2-1}}-\frac{e^2\sqrt{-c^2x^2-1}}{c^2}-\frac{e\left(e-2c^2d\right)}{c^2\sqrt{-c^2x^2-1}}\right)dx^2}{8e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcx\left(2d^2\arctan\left(\sqrt{-c^2x^2-1}\right)-\frac{2e\sqrt{-c^2x^2-1}\left(2c^2d-e\right)}{c^4}+\frac{2e^2\left(-c^2x^2-1\right)^{3/2}}{3c^4}\right)}{8e\sqrt{-c^2x^2}}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/(4*e) - (b*c*x*((-2*(2*c^2*d - e)*e*S
qrt[-1 - c^2*x^2])/c^4 + (2*e^2*(-1 - c^2*x^2)^(3/2))/(3*c^4) + 2*d^2*ArcT
an[Sqrt[-1 - c^2*x^2]]))/(8*e*Sqrt[-(c^2*x^2)])
```

3.85.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.85.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.24

method	result
parts	$\frac{a\left(e\,x^{2}+d\right)^{2}}{4e}+\frac{b\left(\frac{c^{2}e\ \operatorname{arccsch}(cx)x^{4}}{4}+\frac{\operatorname{arccsch}(cx)x^{2}c^{2}d}{2}+\frac{c^{2}\operatorname{arccsch}(cx)d^{2}}{4e}-\frac{\sqrt{c^{2}x^{2}+1}\left(3c^{4}d^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)-e^{2}c^{2}x^{2}\sqrt{\ldots}\right.}{12c^{3}e\sqrt{\frac{c^{2}x^{2}\ldots}{c^{2}x\ldots}}}\ldots\right)}{c^{2}}$ …
derivativedivides	$\frac{\frac{a\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}{4c^{2}e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^{4}d^{2}}{4e}+\frac{\operatorname{arccsch}(cx)c^{4}d\,x^{2}}{2}+\frac{e\ \operatorname{arccsch}(cx)c^{4}x^{4}}{4}+\frac{\sqrt{c^{2}x^{2}+1}\left(-3c^{4}d^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)+6c^{2}de\sqrt{\ldots}\right.}{12e\sqrt{\frac{c^{2}x^{2}\ldots}{c^{2}x^{2}\ldots}}}\ldots\right)}{c^{2}}}{c^{2}}$ …
default	$\frac{\frac{a\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}{4c^{2}e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^{4}d^{2}}{4e}+\frac{\operatorname{arccsch}(cx)c^{4}d\,x^{2}}{2}+\frac{e\ \operatorname{arccsch}(cx)c^{4}x^{4}}{4}+\frac{\sqrt{c^{2}x^{2}+1}\left(-3c^{4}d^{2}\operatorname{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)+6c^{2}de\sqrt{\ldots}\right.}{12e\sqrt{\frac{c^{2}x^{2}\ldots}{c^{2}x^{2}\ldots}}}\ldots\right)}{c^{2}}}{c^{2}}$ …

input
```
int(x*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+b/c^2*(1/4*c^2*e*arccsch(c*x)*x^4+1/2*arccsch(c*x)*x^2
*c^2*d+1/4*c^2/e*arccsch(c*x)*d^2-1/12/c^3/e*(c^2*x^2+1)^(1/2)*(3*c^4*d^2*
arctanh(1/(c^2*x^2+1)^(1/2))-e^2*c^2*x^2*(c^2*x^2+1)^(1/2)-6*c^2*d*e*(c^2*
x^2+1)^(1/2)+2*e^2*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.84

$$\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{3\,ac^3ex^4+6\,ac^3dx^2+3\,(bc^3ex^4+2\,bc^3dx^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(bc^2ex^3+2\,(3\,bc^2d-be)x)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log(
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*e*x^3 + 2*(3*b*c^2
*d - b*e)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

3.85.6 Sympy [F]

$$\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral(x*(a + b*acsch(c*x))*(d + e*x**2), x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.65

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd$$
$$+\frac{1}{12}\left(3x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1
)/c)*b*d + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e
```

3.85.8 Giac [F]

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int\left(ex^2+d\right)\left(b\operatorname{arcsch}(cx)+a\right)x\,dx$$

input
```
integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x\left(e\,x^2+d\right)\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.86 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

3.86.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx =& \frac{be\sqrt{1+\frac{1}{c^2x^2}}x}{2c}+\frac{1}{2}bd\text{csch}^{-1}(cx)^2 \\ &+\frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) \\ &-bd\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\ &+bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right)-d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ &-\frac{1}{2}bd\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\end{aligned}$$

output
```
1/2*b*d*arccsch(c*x)^2+1/2*e*x^2*(a+b*arccsch(c*x))-b*d*arccsch(c*x)*ln(1-
(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*d*arccsch(c*x)*ln(1/x)-d*(a+b*arccsch(c*x
))*ln(1/x)-1/2*b*d*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1+1
/c^2/x^2)^(1/2)/c
```

3.86.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\frac{1}{2}aex^2+\frac{bex\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bex^2\text{csch}^{-1}(cx)+\frac{1}{2}bd\text{csch}^{-1}(cx)^2-bd\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+ad\log(x)-\frac{1}{2}bd\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
(a*e*x^2)/2 + (b*e*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*e*x^2*ArcCs
ch[c*x])/2 + (b*d*ArcCsch[c*x]^2)/2 - b*d*ArcCsch[c*x]*Log[1 - E^(2*ArcCsc
h[c*x])] + a*d*Log[x] - (b*d*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

3.86.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6858, 6237, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

$$\Big\downarrow \ 6858$$

$$-\int \left(\frac{d}{x^2}+e\right)x^3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \ 6237$$

$$\frac{b\int -\frac{ex^2-2d\log\left(\frac{1}{x}\right)}{2\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-d\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int\frac{ex^2-2d\log\left(\frac{1}{x}\right)}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int\left(\frac{ex^2}{\sqrt{1+\frac{1}{c^2x^2}}}-\frac{2d\log\left(\frac{1}{x}\right)}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-d\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-d\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}ex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)-\frac{b\left(cd\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)-cd\text{arcsinh}\left(\frac{1}{cx}\right)^2+2cd\text{arcsinh}\left(\frac{1}{cx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)-2cd\log\left(\frac{1}{x}\right)\text{arcsinh}\left(\frac{1}{cx}\right)\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
(e*x^2*(a + b*ArcSinh[1/(c*x)]))/2 - d*(a + b*ArcSinh[1/(c*x)])*Log[x^(-1)
] - (b*(-(e*Sqrt[1 + 1/(c^2*x^2)]*x) - c*d*ArcSinh[1/(c*x)]^2 + 2*c*d*ArcS
inh[1/(c*x)]*Log[1 - E^(2*ArcSinh[1/(c*x)])] - 2*c*d*ArcSinh[1/(c*x)]*Log[
x^(-1)] + c*d*PolyLog[2, E^(2*ArcSinh[1/(c*x)])]))/(2*c)
```

3.86.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6237
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1
+ c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d]
 && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.86.4 Maple [F]

$$\int \frac{(e\,x^2+d)\,(a+b\,\operatorname{arccsch}\,(cx))}{x}dx$$

input
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

output
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

3.86.5 Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\operatorname{csch}^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\,\operatorname{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))/x, x)
```

3.86.6 Sympy [F]

$$\int \frac{(d+ex^2)\,(a+b\operatorname{csch}^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\operatorname{acsch}\,(cx))\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x))/x,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)/x, x)
```

3.86.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

output
```
2*b*c^2*d*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x) - 1/2*b*e*x^2*log(c) - 1/2*b*e*x^2*log(x) + 1/2*a
*e*x^2 - b*d*log(c)*log(x) - 1/2*b*d*log(x)^2 - 1/4*(2*log(c^2*x^2 + 1)*lo
g(x) + dilog(-c^2*x^2))*b*d + a*d*log(x) + 1/2*(b*e*x^2 + 2*b*d*log(x))*lo
g(sqrt(c^2*x^2 + 1) + 1) + 1/4*b*e*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1)
)/c^2 + 1/4*b*e*log(c^2*x^2 + 1)/c^2
```

3.86.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.87 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$

3.87.1 Optimal result

Integrand size = 19, antiderivative size = 128

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2d\text{csch}^{-1}(cx)+\frac{1}{2}be\text{csch}^{-1}(cx)^2-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}-be\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+be\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right)-e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right)-\frac{1}{2}be\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)$$

output
```
-1/4*b*c^2*d*arccsch(c*x)+1/2*b*e*arccsch(c*x)^2-1/2*d*(a+b*arccsch(c*x))/
x^2-b*e*arccsch(c*x)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*e*arccsch(c*x)*
ln(1/x)-e*(a+b*arccsch(c*x))*ln(1/x)-1/2*b*e*polylog(2,(1/c/x+(1+1/c^2/x^2
)^(1/2))^2)+1/4*b*c*d*(1+1/c^2/x^2)^(1/2)/x
```

3.87.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = -\frac{ad}{2x^2}+\frac{bcd\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{4x}-\frac{bd\text{csch}^{-1}(cx)}{2x^2} +\frac{1}{2}be\text{csch}^{-1}(cx)^2-\frac{1}{4}bc^2d\,\text{arcsinh}\left(\frac{1}{cx}\right) -be\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) +ae\log(x)-\frac{1}{2}be\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 + (b*c*d*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*d*ArcCsc
h[c*x])/(2*x^2) + (b*e*ArcCsch[c*x]^2)/2 - (b*c^2*d*ArcSinh[1/(c*x)])/4 -
b*e*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + a*e*Log[x] - (b*e*PolyLog[2
, E^(2*ArcCsch[c*x])])/2
```

3.87.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {6858, 6237, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \left(\frac{d}{x^2}+e\right)x\left(a+b\,\text{arcsinh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{6237}$$

$$\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{2\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-\frac{d\left(a+b\,\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\,\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{b\int \frac{\frac{d}{x^2}+2e\log\left(\frac{1}{x}\right)}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 7293$$

$$\frac{b\int \left(\frac{d}{\sqrt{1+\frac{1}{c^2x^2}}x^2}+\frac{2e\log\left(\frac{1}{x}\right)}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \; 2009$$

$$-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+$$
$$\frac{b\left(-\frac{1}{2}c^3d\text{arcsinh}\left(\frac{1}{cx}\right)-ce\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)+ce\text{arcsinh}\left(\frac{1}{cx}\right)^2-2ce\text{arcsinh}\left(\frac{1}{cx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)+\cdots\right.}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcSinh[1/(c*x)]))/x^2 - e*(a + b*ArcSinh[1/(c*x)])*Log[x^(
-1)] + (b*((c^2*d*Sqrt[1 + 1/(c^2*x^2)])/(2*x) - (c^3*d*ArcSinh[1/(c*x)])/
2 + c*e*ArcSinh[1/(c*x)]^2 - 2*c*e*ArcSinh[1/(c*x)]*Log[1 - E^(2*ArcSinh[1
/(c*x)])] + 2*c*e*ArcSinh[1/(c*x)]*Log[x^(-1)] - c*e*PolyLog[2, E^(2*ArcSi
nh[1/(c*x)])]))/(2*c)
```

3.87.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6237
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1
 + c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d]
 && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.87.4 Maple [F]

$$\int \frac{(e\,x^2+d)\,(a+b\ \mathrm{arccsch}\,(cx))}{x^3}dx$$

input
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

output
```
int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

3.87.5 Fricas [F]

$$\int \frac{(d+ex^2)\left(a+b\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(ex^2+d)(b\,\mathrm{arcsch}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))/x^3, x)
```

3.87.6 Sympy [F]

$$\int \frac{(d+ex^2)\left(a+b\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{(a+b\,\mathrm{acsch}\,(cx))\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*acsch(c*x))/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)/x**3, x)
```

3.87.7 Maxima [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log
(x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x)
)*log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) +
 2*integrate(log(x)/(c^2*x^3 + x), x))*b*e + 1/8*b*d*((2*c^4*x*sqrt(1/(c^2
*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2)
+ 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2
) + a*e*log(x) - 1/2*a*d/x^2
```

3.87.8 Giac [F]

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(ex^2+d)(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(e\,x^2+d)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.88 $\int x^2(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 260

$$\begin{aligned}\int x^2(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx &= \frac{b(280c^4d^2-252c^2de+75e^2)\,x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}} \\ &+ \frac{b(84c^2d-25e)\,ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}} \\ &+ \frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}} + \frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right) \\ &+ \frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right) \\ &+ \frac{b(280c^4d^2-252c^2de+75e^2)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{1680c^6\sqrt{-c^2x^2}}\end{aligned}$$

output
```
1/3*d^2*x^3*(a+b*arccsch(c*x))+2/5*d*e*x^5*(a+b*arccsch(c*x))+1/7*e^2*x^7*
(a+b*arccsch(c*x))+1/1680*b*(280*c^4*d^2-252*c^2*d*e+75*e^2)*x*arctan(c*x/
(-c^2*x^2-1)^(1/2))/c^6/(-c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2-252*c^2*d*e
+75*e^2)*x^2*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)+1/840*b*(84*c^2*d-25*
e)*e*x^4*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(-c^2*x^2-
1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.70

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{c^2x^2\left(16ac^5x(35d^2+42dex^2+15e^2x^4)+b\sqrt{1+\frac{1}{c^2x^2}}(75e^2-2c^2e(126d+25ex^2)+8c^4(35d^2+21dex^2+\cdots$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 + 1/(c^
2*x^2)]*(75*e^2 - 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2
+ 5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c
*x] + b*(-280*c^4*d^2 + 252*c^2*d*e - 75*e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2
)])*x])/(1680*c^7)
```

3.88.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.85, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6856, 27, 1590, 27, 363, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{6856}$$
$$-\frac{bcx\int \frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{105\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$
$$\Big\downarrow \text{27}$$
$$-\frac{bcx\int \frac{x^2\left(15e^2x^4+42dex^2+35d^2\right)}{\sqrt{-c^2x^2-1}}dx}{105\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$\Big\downarrow$ 1590

$$-\frac{bcx\left(-\frac{\int -\frac{3x^2\left(70c^2d^2+\left(84c^2d-25e\right)ex^2\right)}{\sqrt{-c^2x^2-1}}dx}{6c^2}-\frac{5e^2x^5\sqrt{-c^2x^2-1}}{2c^2}\right)}{105\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)$$

$\Big\downarrow$ 27

$$-\frac{bcx\left(\frac{\int \frac{x^2\left(70c^2d^2+\left(84c^2d-25e\right)ex^2\right)}{\sqrt{-c^2x^2-1}}dx}{2c^2}-\frac{5e^2x^5\sqrt{-c^2x^2-1}}{2c^2}\right)}{105\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)$$

$\Big\downarrow$ 363

$$-\frac{bcx\left(\frac{\frac{\left(280c^4d^2-252c^2de+75e^2\right)\int \frac{x^2}{\sqrt{-c^2x^2-1}}dx}{4c^2}-\frac{ex^3\sqrt{-c^2x^2-1}\left(84c^2d-25e\right)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{-c^2x^2-1}}{2c^2}\right)}{105\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)$$

$\Big\downarrow$ 262

$$-\frac{bcx\left(\frac{\frac{\left(280c^4d^2-252c^2de+75e^2\right)\left(-\frac{\int \frac{1}{\sqrt{-c^2x^2-1}}dx}{2c^2}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)}{4c^2}-\frac{ex^3\sqrt{-c^2x^2-1}\left(84c^2d-25e\right)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{-c^2x^2-1}}{2c^2}\right)}{105\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)$$

$\Big\downarrow$ 224

$$-\frac{bcx\left(\frac{\frac{\left(280c^4d^2-252c^2de+75e^2\right)\left(-\frac{\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}}{2c^2}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)}{4c^2}-\frac{ex^3\sqrt{-c^2x^2-1}\left(84c^2d-25e\right)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{-c^2x^2-1}}{2c^2}\right)}{105\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)$$

3.88. $\int x^2(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\big\downarrow 216$$

$$\frac{1}{3}d^2x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{bcx\left(\frac{\frac{\left(-\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{2c^3}-\frac{x\sqrt{-c^2x^2-1}}{2c^2}\right)\left(280c^4d^2-252c^2de+75e^2\right)}{4c^2}-\frac{ex^3\sqrt{-c^2x^2-1}\left(84c^2d-25e\right)}{4c^2}}{2c^2}-\frac{5e^2x^5\sqrt{-c^2x^2-1}}{2c^2}\right)}{105\sqrt{-c^2x^2}}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcCsch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsch[c*x]))/5 + (e
^2*x^7*(a + b*ArcCsch[c*x]))/7 - (b*c*x*((-5*e^2*x^5*Sqrt[-1 - c^2*x^2])/(
2*c^2) + (-1/4*((84*c^2*d - 25*e)*e*x^3*Sqrt[-1 - c^2*x^2])/c^2 + ((280*c^
4*d^2 - 252*c^2*d*e + 75*e^2)*(-1/2*(x*Sqrt[-1 - c^2*x^2])/c^2 - ArcTan[(c
*x)/Sqrt[-1 - c^2*x^2]]/(2*c^3)))/(4*c^2))/(2*c^2)))/(105*Sqrt[-(c^2*x^2)]
)
```

3.88.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.88.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.04

method	result
parts	$a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}x^3d^2\right)+\frac{b\left(\frac{c^3\operatorname{arccsch}(cx)e^2x^7}{7}+\frac{2c^3\operatorname{arccsch}(cx)de\,x^5}{5}+\frac{\operatorname{arccsch}(cx)d^2x^3c^3}{3}-\frac{\sqrt{c^2x^2+1}\left(-40e\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d^2c^7x^3}{3}+\frac{2\operatorname{arccsch}(cx)dc^7e\,x^5}{5}+\frac{\operatorname{arccsch}(cx)e^2c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\left(280d^2c^5x\sqrt{c^2\right.}{}\right.}{}}{}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d^2c^7x^3}{3}+\frac{2\operatorname{arccsch}(cx)dc^7e\,x^5}{5}+\frac{\operatorname{arccsch}(cx)e^2c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\left(280d^2c^5x\sqrt{c^2\right.}{}\right.}{}}{}$

input
```
int(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*x^3*d^2)+b/c^3*(1/7*c^3*arccsch(c*x)*e^2*x^
7+2/5*c^3*arccsch(c*x)*d*e*x^5+1/3*arccsch(c*x)*d^2*x^3*c^3-1/1680/c^5*(c^
2*x^2+1)^(1/2)*(-40*e^2*c^5*x^5*(c^2*x^2+1)^(1/2)-168*d*c^5*e*x^3*(c^2*x^2
+1)^(1/2)-280*d^2*c^5*x*(c^2*x^2+1)^(1/2)+280*d^2*c^4*arcsinh(c*x)+50*e^2*
c^3*x^3*(c^2*x^2+1)^(1/2)+252*d*c^3*e*x*(c^2*x^2+1)^(1/2)-252*d*c^2*e*arcs
inh(c*x)-75*e^2*c*x*(c^2*x^2+1)^(1/2)+75*e^2*arcsinh(c*x))/((c^2*x^2+1)/c^
2/x^2)^(1/2)/x)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.49

$$\int x^2 (d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right) dx$$

$$= \frac{240\, ac^7e^2x^7 + 672\, ac^7dex^5 + 560\, ac^7d^2x^3 + 16\left(35\, bc^7d^2 + 42\, bc^7de + 15\, bc^7e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right)}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 + 16*(35
*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) - c*x + 1) + (280*b*c^4*d^2 - 252*b*c^2*d*e + 75*b*e^2)*log(c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*
c^7*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 16*(15*b*c^7*e
^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e
 - 15*b*c^7*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (40*
b*c^6*e^2*x^6 + 2*(84*b*c^6*d*e - 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 - 252
*b*c^4*d*e + 75*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

3.88.6 Sympy [F]

$$\int x^2 (d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right) dx = \int x^2 (a+b\operatorname{acsch}(cx)) \left(d+ex^2\right)^2 dx$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.52

$$\int x^2(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{1}{7}ae^2x^7+\frac{2}{5}adex^5+\frac{1}{3}ad^2x^3$$
$$+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2$$
$$+\frac{1}{40}\left(16\,x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde$$
$$+\frac{1}{672}\left(96\,x^7\operatorname{arcsch}(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3\,c^6\left(\frac{1}{c^2x^2}+1\right)^2+3\,c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right.$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsch(c*x) +
 (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*
x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(
16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) +
 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqr
t(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d
*e + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 40*(1/(
c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1)^3 -
 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(sqrt(
1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^
2
```

3.88.8 Giac [F]

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^2(b\operatorname{arcsch}(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^2, x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^2\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.89 $\int (d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right) dx$

3.89.1 Optimal result

Integrand size = 18, antiderivative size = 197

$$\begin{aligned}\int (d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right) dx &= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}} \\ &+ \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}} + d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right) \\ &+ \frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right) \\ &- \frac{b(120c^4d^2-40c^2de+9e^2)\,x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{120c^4\sqrt{-c^2x^2}}\end{aligned}$$

output

```
d^2*x*(a+b*arccsch(c*x))+2/3*d*e*x^3*(a+b*arccsch(c*x))+1/5*e^2*x^5*(a+b*a
rccsch(c*x))-1/120*b*(120*c^4*d^2-40*c^2*d*e+9*e^2)*x*arctan(c*x/(-c^2*x^2
-1)^(1/2))/c^4/(-c^2*x^2)^(1/2)+1/120*b*(40*c^2*d-9*e)*e*x^2*(-c^2*x^2-1)^
(1/2)/c^3/(-c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(-c^2*x^2-1)^(1/2)/c/(-c^2*x^2)^
(1/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= \frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)+be\sqrt{1+\frac{1}{c^2x^2}}x(-9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2}{120c^5}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/(c^2*x^
2)]*x*(-9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*
e^2*x^4)*ArcCsch[c*x] + b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt
[1 + 1/(c^2*x^2)])*x])/(120*c^5)
```

3.89.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6846, 27, 1473, 25, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6846}$$

$$-\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{15\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\int \frac{3e^2x^4+10dex^2+15d^2}{\sqrt{-c^2x^2-1}}dx}{15\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 1473}$$

$$-\frac{bcx\left(-\frac{\int -\frac{60c^2d^2+(40c^2d-9e)ex^2}{\sqrt{-c^2x^2-1}}dx}{4c^2}-\frac{3e^2x^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}+d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$\downarrow$ 25

$$-\frac{bcx\left(\frac{\int \frac{60c^2d^2+(40c^2d-9e)ex^2}{\sqrt{-c^2x^2-1}}dx}{4c^2}-\frac{3e^2x^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}+d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$\downarrow$ 299

$$-\frac{bcx\left(\frac{\frac{(120c^4d^2-40c^2de+9e^2)\int \frac{1}{\sqrt{-c^2x^2-1}}dx}{2c^2}-\frac{ex\sqrt{-c^2x^2-1}(40c^2d-9e)}{2c^2}}{4c^2}-\frac{3e^2x^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}+d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$\downarrow$ 224

$$-\frac{bcx\left(\frac{\frac{(120c^4d^2-40c^2de+9e^2)\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}}{2c^2}-\frac{ex\sqrt{-c^2x^2-1}(40c^2d-9e)}{2c^2}}{4c^2}-\frac{3e^2x^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}+d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$\downarrow$ 216

$$d^2x\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{bcx\left(\frac{\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(120c^4d^2-40c^2de+9e^2)}{2c^3}-\frac{ex\sqrt{-c^2x^2-1}(40c^2d-9e)}{2c^2}}{4c^2}-\frac{3e^2x^3\sqrt{-c^2x^2-1}}{4c^2}\right)}{15\sqrt{-c^2x^2}}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

3.89. $\int (d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

output
```
d^2*x*(a + b*ArcCsch[c*x]) + (2*d*e*x^3*(a + b*ArcCsch[c*x]))/3 + (e^2*x^5
*(a + b*ArcCsch[c*x]))/5 - (b*c*x*((-3*e^2*x^3*Sqrt[-1 - c^2*x^2])/(4*c^2)
 + (-1/2*((40*c^2*d - 9*e)*e*x*Sqrt[-1 - c^2*x^2])/c^2 + ((120*c^4*d^2 - 4
0*c^2*d*e + 9*e^2)*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(2*c^3))/(4*c^2)))/(1
5*Sqrt[-(c^2*x^2)])
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 6846
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsch[c*x])   u
, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[-1 -
 c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ
[p + 1/2, 0])
```

3.89.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.99

method	result
parts	$a\left(\frac{1}{5}e^{2}x^{5}+\frac{2}{3}de\,x^{3}+x\,d^{2}\right)+\frac{b\left(\frac{c\,\operatorname{arccsch}(cx)e^{2}x^{5}}{5}+\frac{2c\,\operatorname{arccsch}(cx)de\,x^{3}}{3}+\operatorname{arccsch}(cx)xc\,d^{2}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(120d^{2}c^{4}\operatorname{ar}\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a\left(d^{2}c^{5}x+\frac{2}{3}d\,c^{5}e\,x^{3}+\frac{1}{5}e^{2}c^{5}x^{5}\right)}{c^{4}}+\frac{b\left(\operatorname{arccsch}(cx)d^{2}c^{5}x+\frac{2\operatorname{arccsch}(cx)d\,c^{5}e\,x^{3}}{3}+\frac{\operatorname{arccsch}(cx)e^{2}c^{5}x^{5}}{5}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(120d^{2}c^{4}\operatorname{arcsinh}(cx\right.}{}\right.}{}}{c}$
default	$\frac{\frac{a\left(d^{2}c^{5}x+\frac{2}{3}d\,c^{5}e\,x^{3}+\frac{1}{5}e^{2}c^{5}x^{5}\right)}{c^{4}}+\frac{b\left(\operatorname{arccsch}(cx)d^{2}c^{5}x+\frac{2\operatorname{arccsch}(cx)d\,c^{5}e\,x^{3}}{3}+\frac{\operatorname{arccsch}(cx)e^{2}c^{5}x^{5}}{5}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(120d^{2}c^{4}\operatorname{arcsinh}(cx\right.}{}\right.}{}}{c}$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+x*d^2)+b/c*(1/5*c*arccsch(c*x)*e^2*x^5+2/3*c*ar
ccsch(c*x)*d*e*x^3+arccsch(c*x)*x*c*d^2+1/120/c^5*(c^2*x^2+1)^(1/2)*(120*d
^2*c^4*arcsinh(c*x)+40*d*c^3*e*x*(c^2*x^2+1)^(1/2)+6*e^2*c^3*x^3*(c^2*x^2+
1)^(1/2)-40*d*c^2*e*arcsinh(c*x)-9*e^2*c*x*(c^2*x^2+1)^(1/2)+9*e^2*arcsinh
(c*x))/x/((c^2*x^2+1)/c^2/x^2)^(1/2))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(175) = 350.

Time = 0.33 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.79

$$\int \left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{24\,ac^5e^2x^5+80\,ac^5dex^3+120\,ac^5d^2x+8\left(15\,bc^5d^2+10\,bc^5de+3\,bc^5e^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-(1}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x + 8*(15*b*c^5
*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) -
 c*x + 1) - (120*b*c^4*d^2 - 40*b*c^2*d*e + 9*b*e^2)*log(c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) - c*x) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(3*b*c^5*e^2*x^5 + 10*b
*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)
*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*e^2*x^4 + (
40*b*c^4*d*e - 9*b*c^2*e^2)*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

3.89.6 Sympy [F]

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \left(a+b\,\text{acsch}\,(cx)\right)\left(d+ex^2\right)^2\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.46

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx &= \frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3\\&+\frac{1}{6}\left(4\,x^3\,\text{arcsch}\,(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde\\&+\frac{1}{80}\left(16\,x^5\,\text{arcsch}\,(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\,\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be^2\\&+ad^2x+\frac{\left(2\,cx\,\text{arcsch}\,(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^2}{2\,c}\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x
^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^
2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/80*(16*x^5*arccsch(c*
x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2
*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1
) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2 + a*d^2*x + 1/
2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x
^2) + 1) - 1))*b*d^2/c
```

3.89.8 Giac [F]

$$\int (d+ex^2)^2 (a+b\text{csch}^{-1}(cx))\ dx = \int (ex^2+d)^2 (b\,\text{arcsch}\,(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)^2 (a+b\text{csch}^{-1}(cx))\ dx = \int (e\,x^2+d)^2 \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\ dx$$

input
```
int((d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

output
```
int((d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.90 $\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 2dex\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{3}e^2x^3\left(a+b\text{csch}^{-1}(cx)\right) - \frac{b(12c^2d-e)\,ex\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6c^2\sqrt{-c^2x^2}}$$

output
```
-d^2*(a+b*arccsch(c*x))/x+2*d*e*x*(a+b*arccsch(c*x))+1/3*e^2*x^3*(a+b*arcc
sch(c*x))-1/6*b*(12*c^2*d-e)*e*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^2/(-c^2*
x^2)^(1/2)+b*c*d^2*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(-c^2
*x^2-1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2\,(a+b\text{csch}^{-1}(cx))}{x^2}\,dx$$
$$=\frac{c^2\left(b\sqrt{1+\frac{1}{c^2x^2}}x(6c^2d^2+e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\,\text{csch}^{-1}(cx)+}{6c^3x}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
(c^2*(b*Sqrt[1 + 1/(c^2*x^2)]*x*(6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*
d*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsch[c*x]
+ b*(12*c^2*d - e)*e*x*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3*x)
```

3.90.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.89, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6856, 27, 1588, 25, 27, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\text{csch}^{-1}(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{6856}$$
$$-\frac{bcx\int -\frac{-e^2x^4-6dex^2+3d^2}{3x^2\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$
$$\Big\downarrow \text{27}$$
$$\frac{bcx\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\sqrt{-c^2x^2-1}}dx}{3\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$
$$\Big\downarrow \text{1588}$$

$$\frac{bcx\left(\int -\frac{e(ex^2+6d)}{\sqrt{-c^2x^2-1}}dx+\frac{3d^2\sqrt{-c^2x^2-1}}{x}\right)}{3\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$

$$\Big\downarrow 25$$

$$\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{x}-\int \frac{e(ex^2+6d)}{\sqrt{-c^2x^2-1}}dx\right)}{3\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$

$$\Big\downarrow 27$$

$$\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{x}-e\int \frac{ex^2+6d}{\sqrt{-c^2x^2-1}}dx\right)}{3\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$

$$\Big\downarrow 299$$

$$\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{x}-e\left(\frac{1}{2}\left(12d-\frac{e}{c^2}\right)\int \frac{1}{\sqrt{-c^2x^2-1}}dx-\frac{ex\sqrt{-c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$

$$\Big\downarrow 224$$

$$\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{x}-e\left(\frac{1}{2}\left(12d-\frac{e}{c^2}\right)\int \frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}-\frac{ex\sqrt{-c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{-c^2x^2}}-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))$$

$$\Big\downarrow 216$$

$$-\frac{d^2(a+b\text{csch}^{-1}(cx))}{x}+2dex(a+b\text{csch}^{-1}(cx))+\frac{1}{3}e^2x^3(a+b\text{csch}^{-1}(cx))+\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{x}-e\left(\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)\left(12d-\frac{e}{c^2}\right)}{2c}-\frac{ex\sqrt{-c^2x^2-1}}{2c^2}\right)\right)}{3\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

3.90. $\int \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{x^2}\,dx$

output
```
-((d^2*(a + b*ArcCsch[c*x]))/x) + 2*d*e*x*(a + b*ArcCsch[c*x]) + (e^2*x^3*
(a + b*ArcCsch[c*x]))/3 + (b*c*x*((3*d^2*Sqrt[-1 - c^2*x^2])/x - e*(-1/2*(
e*x*Sqrt[-1 - c^2*x^2])/c^2 + ((12*d - e/c^2)*ArcTan[(c*x)/Sqrt[-1 - c^2*x
^2]])/(2*c))))/(3*Sqrt[-(c^2*x^2)])
```

3.90.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.90.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02

method	result
parts	$a\left(\frac{e^2x^3}{3}+2dex-\frac{d^2}{x}\right)+bc\left(\frac{\operatorname{arccsch}(cx)x^3e^2}{3c}+\frac{2\operatorname{arccsch}(cx)dex}{c}-\frac{\operatorname{arccsch}(cx)d^2}{xc}+\frac{\sqrt{c^2x^2+1}\left(6c^4d^2\sqrt{\cdots}\right.}{\cdots}\right.$
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arccsch}(cx)c^3dex+\frac{\operatorname{arccsch}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arccsch}(cx)c^3d^2}{x}+\frac{\sqrt{c^2x^2+1}\left(6c^4d^2\sqrt{c^2x^2\cdots}\right.}{\cdots}\right.}{c^4}\right.$
default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\operatorname{arccsch}(cx)c^3dex+\frac{\operatorname{arccsch}(cx)e^2c^3x^3}{3}-\frac{\operatorname{arccsch}(cx)c^3d^2}{x}+\frac{\sqrt{c^2x^2+1}\left(6c^4d^2\sqrt{c^2x^2\cdots}\right.}{\cdots}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3/c*arccsch(c*x)*x^3*e^2+2/c*arccsch(
c*x)*d*e*x-arccsch(c*x)*d^2/x/c+1/6/c^6*(c^2*x^2+1)^(1/2)*(6*c^4*d^2*(c^2*
x^2+1)^(1/2)+12*c^3*d*e*arcsinh(c*x)*x+e^2*c^2*x^2*(c^2*x^2+1)^(1/2)-arcsi
nh(c*x)*e^2*c*x)/x^2/((c^2*x^2+1)/c^2/x^2)^(1/2))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(152) = 304$.

Time = 0.32 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \frac{2\,ac^3e^2x^4 + 6\,bc^4d^2x + 12\,ac^3dex^2 - 6\,ac^3d^2 - 2\,(3\,bc^3d^2 - 6\,bc^3de - bc^3e^2)x\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - }{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*
(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) - c*x + 1) - (12*b*c^2*d*e - b*e^2)*x*log(c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) - c*x) + 2*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 -
 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log((c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^4*d^2*x + b*c^2*e^2*x^3)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)))/(c^3*x)
```

3.90.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^2}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**2, x)
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^2} dx$$
$$= \frac{1}{3}ae^2x^3 + \left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\operatorname{arcsch}(cx)}{x}\right)bd^2$$
$$+ \frac{1}{12}\left(4x^3\operatorname{arcsch}(cx) + \frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2} - \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2} + \frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2$$
$$+ 2\,adex + \frac{\left(2\,cx\operatorname{arcsch}(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bde}{c} - \frac{ad^2}{x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d^2 + 1/12*(4
*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2)
 - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2
)/c)*b*e^2 + 2*a*d*e*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) +
 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d*e/c - a*d^2/x
```

3.90.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^2} dx = \int \frac{(ex^2+d)^2(b\operatorname{arcsch}(cx)+a)}{x^2} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^2, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^2 \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.91 $\int \frac{(d+ex^2)^2(a+b\text{csch}^{-1}(cx))}{x^4} dx$

3.91.1 Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\text{csch}^{-1}(cx)\right)-\frac{be^2x\arctan\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

output
```
-1/3*d^2*(a+b*arccsch(c*x))/x^3-2*d*e*(a+b*arccsch(c*x))/x+e^2*x*(a+b*arcc
sch(c*x))-b*e^2*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/(-c^2*x^2)^(1/2)-2/9*b*c*
d*(c^2*d-9*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/9*b*c*d^2*(-c^2*x^2-1)
^(1/2)/x^2/(-c^2*x^2)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}x(d-2c^2dx^2+18ex^2)-3a(d^2+6dex^2-3e^2x^4)}{9x^3} - \frac{b(d^2+6dex^2-3e^2x^4)\text{csch}^{-1}(cx)}{3x^3}+\frac{be^2\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{c}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
(b*c*d*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2 + 6
*d*e*x^2 - 3*e^2*x^4))/(9*x^3) - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsch[
c*x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/c
```

3.91.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6856, 27, 1588, 25, 358, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 6856}$$

$$-\frac{bcx \int -\frac{-3e^2x^4+6dex^2+d^2}{3x^4\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx \int \frac{-3e^2x^4+6dex^2+d^2}{x^4\sqrt{-c^2x^2-1}}dx}{3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 1588}$$

$$\frac{bcx\left(\frac{1}{3}\int -\frac{9e^2x^2+2d\left(c^2d-9e\right)}{x^2\sqrt{-c^2x^2-1}}dx+\frac{d^2\sqrt{-c^2x^2-1}}{3x^3}\right)}{3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{bcx\left(\frac{d^2\sqrt{-c^2x^2-1}}{3x^3}-\frac{1}{3}\int \frac{9e^2x^2+2d\left(c^2d-9e\right)}{x^2\sqrt{-c^2x^2-1}}dx\right)}{3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{ 358}$$

$$\frac{bcx\left(\frac{1}{3}\left(-9e^2\int\frac{1}{\sqrt{-c^2x^2-1}}dx-\frac{2d\sqrt{-c^2x^2-1}(c^2d-9e)}{x}\right)+\frac{d^2\sqrt{-c^2x^2-1}}{3x^3}\right)}{3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{bcx\left(\frac{1}{3}\left(-9e^2\int\frac{1}{\frac{c^2x^2}{-c^2x^2-1}+1}d\frac{x}{\sqrt{-c^2x^2-1}}-\frac{2d\sqrt{-c^2x^2-1}(c^2d-9e)}{x}\right)+\frac{d^2\sqrt{-c^2x^2-1}}{3x^3}\right)}{3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcx\left(\frac{1}{3}\left(-\frac{9e^2\arctan\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{c}-\frac{2d\sqrt{-c^2x^2-1}(c^2d-9e)}{x}\right)+\frac{d^2\sqrt{-c^2x^2-1}}{3x^3}\right)}{3\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcCsch[c*x]))/x^3 - (2*d*e*(a + b*ArcCsch[c*x]))/x + e^2
*x*(a + b*ArcCsch[c*x]) + (b*c*x*((d^2*Sqrt[-1 - c^2*x^2])/(3*x^3) + ((-2*
d*(c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/x - (9*e^2*ArcTan[(c*x)/Sqrt[-1 - c^2*
x^2]])/c)/3))/(3*Sqrt[-(c^2*x^2)])
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.91. $\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.91.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.14

method	result
parts	$a\left(e^2x-\frac{2de}{x}-\frac{d^2}{3x^3}\right)+b\,c^3\left(\frac{\operatorname{arccsch}(cx)e^2x}{c^3}-\frac{2\operatorname{arccsch}(cx)de}{c^3x}-\frac{\operatorname{arccsch}(cx)d^2}{3x^3c^3}-\frac{\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2+1}\right.}{}\right.$
derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arccsch}(cx)e^2cx-\frac{\operatorname{arccsch}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arccsch}(cx)cde}{x}-\frac{\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2+1}\,c^6d^2x^2-c^4d^2\right.}{}\right.}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arccsch}(cx)e^2cx-\frac{\operatorname{arccsch}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arccsch}(cx)cde}{x}-\frac{\sqrt{c^2x^2+1}\left(2\sqrt{c^2x^2+1}\,c^6d^2x^2-c^4d^2\right.}{}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(e^2*x-2*d*e/x-1/3*d^2/x^3)+b*c^3*(1/c^3*arccsch(c*x)*e^2*x-2/c^3*arccsc
h(c*x)*d*e/x-1/3*arccsch(c*x)*d^2/x^3/c^3-1/9/c^8*(c^2*x^2+1)^(1/2)*(2*(c^
2*x^2+1)^(1/2)*c^6*d^2*x^2-c^4*d^2*(c^2*x^2+1)^(1/2)-18*c^4*d*e*(c^2*x^2+1
)^(1/2)*x^2-9*e^2*arcsinh(c*x)*c^3*x^3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^4)
```

3.91.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(146) = 292$.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

$$=\frac{9\,ace^2x^4-9\,be^2x^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-18\,acdex^2-3\,(bcd^2+6\,bcde-3\,bce^2)x^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-c\right.}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/9*(9*a*c*e^2*x^4 - 9*b*e^2*x^3*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x) - 18*a*c*d*e*x^2 - 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*log(c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 3*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2
)*x^3*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*a*c*d^2 - 2*(b*
c^4*d^2 - 9*b*c^2*d*e)*x^3 + 3*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2 +
(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 1)/(c*x)) + (b*c^2*d^2*x - 2*(b*c^4*d^2 - 9*b*c^2*d*e)*x^3)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/(c*x^3)
```

3.91.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^4} dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^2}{x^4} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**4, x)
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^4} dx \\
&= 2\left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\text{arcsch}\,(cx)}{x}\right)bde + ae^2x \\
&\quad + \frac{1}{9}bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\text{arcsch}\,(cx)}{x^3}\right) \\
&\quad + \frac{\left(2\,cx\,\text{arcsch}\,(cx) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)be^2}{2\,c} - \frac{2\,ade}{x} - \frac{ad^2}{3\,x^3}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

output
```
2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d*e + a*e^2*x + 1/9*b*d^2*(
(c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(
c*x)/x^3) + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log
(sqrt(1/(c^2*x^2) + 1) - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

3.91.8 Giac [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^4, x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(e\,x^2+d)^2\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.92 $\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$

3.92.1 Optimal result

Integrand size = 21, antiderivative size = 189

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{bc(24c^4d^2-100c^2de+225e^2)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd(6c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

output
```
-1/5*d^2*(a+b*arccsch(c*x))/x^5-2/3*d*e*(a+b*arccsch(c*x))/x^3-e^2*(a+b*ar
ccsch(c*x))/x+1/225*b*c*(24*c^4*d^2-100*c^2*d*e+225*e^2)*(-c^2*x^2-1)^(1/2
)/(-c^2*x^2)^(1/2)+1/25*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)-2/
225*b*c*d*(6*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*x^2)^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx$$
$$=\frac{-15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}x(225e^2x^4-50dex^2(-1+2c^2x^2)+3d^2(3-4c^2x^2+8c^4x^4)}{225x^5}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
(-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(22
5*e^2*x^4 - 50*d*e*x^2*(-1 + 2*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4
)) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x])/(225*x^5)
```

3.92.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6856, 27, 1588, 25, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx$$
$$\Big\downarrow \text{6856}$$
$$-\frac{bcx\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^6\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$
$$\Big\downarrow \text{27}$$
$$\frac{bcx\int \frac{15e^2x^4+10dex^2+3d^2}{x^6\sqrt{-c^2x^2-1}}dx}{15\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}$$
$$\Big\downarrow \text{1588}$$

$$\frac{bcx\left(\frac{1}{5}\int -\frac{2d(6c^2d-25e)-75e^2x^2}{x^4\sqrt{-c^2x^2-1}}dx+\frac{3d^2\sqrt{-c^2x^2-1}}{5x^5}\right)}{15\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \ 25$$

$$\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{5x^5}-\frac{1}{5}\int \frac{2d(6c^2d-25e)-75e^2x^2}{x^4\sqrt{-c^2x^2-1}}dx\right)}{15\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \ 359$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(24c^4d^2-100c^2de+225e^2\right)\int \frac{1}{x^2\sqrt{-c^2x^2-1}}dx-\frac{2d\sqrt{-c^2x^2-1}(6c^2d-25e)}{3x^3}\right)+\frac{3d^2\sqrt{-c^2x^2-1}}{5x^5}\right)}{15\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

$$\Big\downarrow \ 242$$

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}+\frac{bcx\left(\frac{3d^2\sqrt{-c^2x^2-1}}{5x^5}+\frac{1}{5}\left(\frac{\sqrt{-c^2x^2-1}\left(24c^4d^2-100c^2de+225e^2\right)}{3x}-\frac{2d\sqrt{-c^2x^2-1}(6c^2d-25e)}{3x^3}\right)\right)}{15\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
(b*c*x*((3*d^2*Sqrt[-1 - c^2*x^2])/(5*x^5) + ((-2*d*(6*c^2*d - 25*e)*Sqrt[
-1 - c^2*x^2])/(3*x^3) + ((24*c^4*d^2 - 100*c^2*d*e + 225*e^2)*Sqrt[-1 - c
^2*x^2])/(3*x))/5))/(15*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/(5*
x^5) - (2*d*e*(a + b*ArcCsch[c*x]))/(3*x^3) - (e^2*(a + b*ArcCsch[c*x]))/x
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.92.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

method	result
parts	$a\left(-\frac{e^2}{x}-\frac{2de}{3x^3}-\frac{d^2}{5x^5}\right)+b\,c^5\left(-\frac{\operatorname{arccsch}(cx)e^2}{c^5x}-\frac{2\operatorname{arccsch}(cx)de}{3c^5x^3}-\frac{\operatorname{arccsch}(cx)d^2}{5x^5c^5}+\frac{(c^2x^2+1)(24c^8d^2x^4}{}\right.$
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{cx}-\frac{\operatorname{arccsch}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arccsch}(cx)de}{3c\,x^3}+\frac{(c^2x^2+1)(24c^8d^2x^4-100c^6de\,x^4-12c}{225\sqrt{\frac{c^2}{c}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}-\frac{2de}{3c\,x^3}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{cx}-\frac{\operatorname{arccsch}(cx)d^2}{5c\,x^5}-\frac{2\operatorname{arccsch}(cx)de}{3c\,x^3}+\frac{(c^2x^2+1)(24c^8d^2x^4-100c^6de\,x^4-12c}{225\sqrt{\frac{c^2x}{c^2}}}\right.}{c^4}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/x-2/3*d*e/x^3-1/5*d^2/x^5)+b*c^5*(-1/c^5*arccsch(c*x)*e^2/x-2/3/c^
5*arccsch(c*x)*d*e/x^3-1/5*arccsch(c*x)*d^2/x^5/c^5+1/225/c^10*(c^2*x^2+1)
*(24*c^8*d^2*x^4-100*c^6*d*e*x^4-12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e
*x^2+9*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^6)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx =$$

$$-\frac{225\,ae^2x^4+150\,adex^2+45\,ad^2+15\,(15\,be^2x^4+10\,bdex^2+3\,bd^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-((24\,bc^5d^2-10}{225\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b
*d*e*x^2 + 3*b*d^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (
(24*b*c^5*d^2 - 100*b*c^3*d*e + 225*b*c*e^2)*x^5 + 9*b*c*d^2*x - 2*(6*b*c^
3*d^2 - 25*b*c*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^5
```

3.92.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^6} dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^2}{x^6} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**6,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**6, x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^6} dx$$
$$= \left(c\sqrt{\frac{1}{c^2x^2}+1} - \frac{\text{arcsch}\,(cx)}{x}\right)be^2 + \frac{1}{75}bd^2\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{15\,\text{arcsch}\,(cx)}{x^5}\right) + \frac{2}{9}bde\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c} - \frac{3\,\text{arcsch}\,(cx)}{x^3}\right) - \frac{ae^2}{x} - \frac{2\,ade}{3\,x^3} - \frac{ad^2}{5\,x^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

output
```
(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(
c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*
x^2) + 1))/c - 15*arccsch(c*x)/x^5) + 2/9*b*d*e*((c^4*(1/(c^2*x^2) + 1)^(3
/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3) - a*e^2/x - 2/3
*a*d*e/x^3 - 1/5*a*d^2/x^5
```

3.92.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^6} dx = \int \frac{(ex^2+d)^2(b \operatorname{arcsch}(cx)+a)}{x^6} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^6, x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^6} dx = \int \frac{(e x^2+d)^2 \left(a+b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{x^6} dx$$

input
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.93 $\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^8} dx$

3.93.1 Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^8} dx = -\frac{2bc^3(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{bc(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}$$

output
```
-1/7*d^2*(a+b*arccsch(c*x))/x^7-2/5*d*e*(a+b*arccsch(c*x))/x^5-1/3*e^2*(a+
b*arccsch(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^
2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/49*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^6/(-c^2*
x^2)^(1/2)-2/1225*b*c*d*(15*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^
(1/2)+1/11025*b*c*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*x^2-1)^(1/2)/x
^2/(-c^2*x^2)^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.61

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx$$
$$= \frac{-105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}x(1225e^2x^4(1-2c^2x^2)+294dex^2(3-4c^2x^2+8c^4x^4)-4}{11025x^7}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

output
```
(-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(
1225*e^2*x^4*(1 - 2*c^2*x^2) + 294*d*e*x^2*(3 - 4*c^2*x^2 + 8*c^4*x^4) - 4
5*d^2*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*
x^2 + 35*e^2*x^4)*ArcCsch[c*x])/(11025*x^7)
```

3.93.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6856, 27, 1588, 25, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx$$
$$\Big\downarrow \text{6856}$$
$$-\frac{bcx\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^8\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$
$$\Big\downarrow \text{27}$$
$$\frac{bcx\int \frac{35e^2x^4+42dex^2+15d^2}{x^8\sqrt{-c^2x^2-1}}dx}{105\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$
$$\Big\downarrow \text{1588}$$

$$\frac{bcx\left(\frac{1}{7}\int -\frac{6d(15c^2d-49e)-245e^2x^2}{x^6\sqrt{-c^2x^2-1}}dx+\frac{15d^2\sqrt{-c^2x^2-1}}{7x^7}\right)}{105\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \; 25$$

$$\frac{bcx\left(\frac{15d^2\sqrt{-c^2x^2-1}}{7x^7}-\frac{1}{7}\int \frac{6d(15c^2d-49e)-245e^2x^2}{x^6\sqrt{-c^2x^2-1}}dx\right)}{105\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \; 359$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2-1176c^2de+1225e^2\right)\int \frac{1}{x^4\sqrt{-c^2x^2-1}}dx-\frac{6d\sqrt{-c^2x^2-1}(15c^2d-49e)}{5x^5}\right)+\frac{15d^2\sqrt{-c^2x^2-1}}{7x^7}\right)}{105\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \; 245$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(360c^4d^2-1176c^2de+1225e^2\right)\left(\frac{\sqrt{-c^2x^2-1}}{3x^3}-\frac{2}{3}c^2\int \frac{1}{x^2\sqrt{-c^2x^2-1}}dx\right)-\frac{6d\sqrt{-c^2x^2-1}(15c^2d-49e)}{5x^5}\right)+\frac{15d^2\sqrt{-c^2x^2-1}}{7x^7}\right)}{105\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

$$\Big\downarrow \; 242$$

$$-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}+\frac{bcx\left(\frac{15d^2\sqrt{-c^2x^2-1}}{7x^7}+\frac{1}{7}\left(\frac{1}{5}\left(\frac{\sqrt{-c^2x^2-1}}{3x^3}-\frac{2c^2\sqrt{-c^2x^2-1}}{3x}\right)\left(360c^4d^2-1176c^2de+1225e^2\right)-\frac{6d\sqrt{-c^2x^2-1}(15c^2d-49e)}{5x^5}\right)\right)}{105\sqrt{-c^2x^2}}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

output
```
(b*c*x*((15*d^2*Sqrt[-1 - c^2*x^2])/(7*x^7) + ((-6*d*(15*c^2*d - 49*e)*Sqr
t[-1 - c^2*x^2])/(5*x^5) + ((360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*(Sqrt[
-1 - c^2*x^2]/(3*x^3) - (2*c^2*Sqrt[-1 - c^2*x^2])/(3*x)))/5)/7))/(105*Sqr
t[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/(7*x^7) - (2*d*e*(a + b*ArcCsc
h[c*x]))/(5*x^5) - (e^2*(a + b*ArcCsch[c*x]))/(3*x^3)
```

3.93. $\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx$

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.93. $\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$

3.93.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.83

method	result
parts	$a\left(-\frac{e^2}{3x^3}-\frac{2de}{5x^5}-\frac{d^2}{7x^7}\right)+b\,c^7\left(-\frac{\operatorname{arccsch}(cx)e^2}{3c^7x^3}-\frac{2\operatorname{arccsch}(cx)de}{5c^7x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7x^7c^7}-\frac{(c^2x^2+1)(720c^{10}\cdots}{\cdots}\right.$
derivativedivides	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arccsch}(cx)de}{5c^3x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arccsch}(cx)e^2}{3c^3x^3}-\frac{(c^2x^2+1)(720c^{10}d^2x^6-2352c^8\cdots}{\cdots}\right.}{\cdots}\right.$
default	$c^7\left(\frac{a\left(-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}-\frac{e^2}{3c^3x^3}\right)}{c^4}+\frac{b\left(-\frac{2\operatorname{arccsch}(cx)de}{5c^3x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7c^3x^7}-\frac{\operatorname{arccsch}(cx)e^2}{3c^3x^3}-\frac{(c^2x^2+1)(720c^{10}d^2x^6-2352c^8\cdots}{\cdots}\right.}{\cdots}\right.$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3/x^3*e^2-2/5*d*e/x^5-1/7*d^2/x^7)+b*c^7*(-1/3/c^7*arccsch(c*x)/x^3*
e^2-2/5/c^7*arccsch(c*x)*d*e/x^5-1/7*arccsch(c*x)*d^2/x^7/c^7-1/11025/c^12
*(c^2*x^2+1)*(720*c^10*d^2*x^6-2352*c^8*d*e*x^6-360*c^8*d^2*x^4+2450*c^6*e
^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2-1225*c^4*e^2*x^4-882*c^4*d*e*x^2-2
25*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x^8)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx =$$

$$-\frac{3675\,ae^2x^4+4410\,adex^2+1575\,ad^2+105\left(35\,be^2x^4+42\,bdex^2+15\,bd^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(2\left(360\,\cdots\right.\right.}{\cdots}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) + (2*(360*b*c^7*d^2 - 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 - (360*b*c
^5*d^2 - 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 - 225*b*c*d^2*x + 18*(15*b*c^3
*d^2 - 49*b*c*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

3.93.6 Sympy [F]

$$\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\,(d+ex^2)^2}{x^8}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**8,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**8, x)
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.93

$$\begin{aligned}
&\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx \\
&= \frac{1}{245}bd^2\left(\frac{5c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\operatorname{arcsch}(cx)}{x^7}\right) \\
&\quad+\frac{2}{75}bde\left(\frac{3c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsch}(cx)}{x^5}\right) \\
&\quad+\frac{1}{9}be^2\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arcsch}(cx)}{x^3}\right)-\frac{ae^2}{3x^3}-\frac{2ade}{5x^5}-\frac{ad^2}{7x^7}
\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/245*b*d^2*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/
2) + 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35
*arccsch(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(
1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)
/x^5) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) +
 1))/c - 3*arccsch(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x
^7
```

3.93.8 Giac [F]

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx=\int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x^8}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^8, x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx=\int \frac{(e\,x^2+d)^2\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.94 $\int x^3(d+ex^2)^2\,(a+b\operatorname{csch}^{-1}(cx))\,dx$

3.94.1 Optimal result

Integrand size = 21, antiderivative size = 250

$$\begin{aligned}\int x^3(d+ex^2)^2\,(a+b\operatorname{csch}^{-1}(cx))\,dx &= -\frac{b(6c^4d^2-8c^2de+3e^2)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}} - \frac{b(6c^4d^2-16c^2de+9e^2)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}} \\ &\quad + \frac{b(8c^2d-9e)\,ex(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}} - \frac{be^2x(-1-c^2x^2)^{7/2}}{56c^7\sqrt{-c^2x^2}} \\ &\quad + \frac{1}{4}d^2x^4(a+b\operatorname{csch}^{-1}(cx)) + \frac{1}{3}dex^6(a+b\operatorname{csch}^{-1}(cx)) + \frac{1}{8}e^2x^8(a+b\operatorname{csch}^{-1}(cx))\end{aligned}$$

output
```
1/4*d^2*x^4*(a+b*arccsch(c*x))+1/3*d*e*x^6*(a+b*arccsch(c*x))+1/8*e^2*x^8*
(a+b*arccsch(c*x))-1/72*b*(6*c^4*d^2-16*c^2*d*e+9*e^2)*x*(-c^2*x^2-1)^(3/2
)/c^7/(-c^2*x^2)^(1/2)+1/120*b*(8*c^2*d-9*e)*e*x*(-c^2*x^2-1)^(5/2)/c^7/(-
c^2*x^2)^(1/2)-1/56*b*e^2*x*(-c^2*x^2-1)^(7/2)/c^7/(-c^2*x^2)^(1/2)-1/24*b
*(6*c^4*d^2-8*c^2*d*e+3*e^2)*x*(-c^2*x^2-1)^(1/2)/c^7/(-c^2*x^2)^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.64

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{x\left(105ax^3(6d^2+8dex^2+3e^2x^4)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(-144e^2+8c^2e(56d+9ex^2)-2c^4(210d^2+112dex^2+27e^2x^4)+3c^6(70d^2x^2+56dex^4+\cdots\right.}{c^7}\right.}{2520}$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(
-144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) - 2*c^4*(210*d^2 + 112*d*e*x^2 + 27*e^
2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6
*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCsch[c*x]))/2520
```

3.94.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6856, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int\frac{x^3(3e^2x^4+8dex^2+6d^2)}{24\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int\frac{x^3(3e^2x^4+8dex^2+6d^2)}{\sqrt{-c^2x^2-1}}dx}{24\sqrt{-c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

$$\Big\downarrow \text{1578}$$

$$-\frac{bcx\int \frac{x^2(3e^2x^4+8dex^2+6d^2)}{\sqrt{-c^2x^2-1}}dx^2}{48\sqrt{-c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)$$

$\big\downarrow$ 1195

$$-\frac{bcx\int \left(-\frac{3e^2(-c^2x^2-1)^{5/2}}{c^6}+\frac{(8c^2d-9e)e(-c^2x^2-1)^{3/2}}{c^6}+\frac{(-6d^2c^4+16dec^2-9e^2)\sqrt{-c^2x^2-1}}{c^6}+\frac{-6d^2c^4+8dec^2-3e^2}{c^6\sqrt{-c^2x^2-1}}\right)dx^2}{48\sqrt{-c^2x^2}}+\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)$$

$\big\downarrow$ 2009

$$\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bcx\left(-\frac{2e(-c^2x^2-1)^{5/2}(8c^2d-9e)}{5c^8}+\frac{6e^2(-c^2x^2-1)^{7/2}}{7c^8}+\frac{2(-c^2x^2-1)^{3/2}(6c^4d^2-16c^2de+9e^2)}{3c^8}+\frac{2\sqrt{-c^2x^2-1}(6c^4d^2-8c^2de+3e^2)}{c^8}\right)}{48\sqrt{-c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
-1/48*(b*c*x*((2*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*Sqrt[-1 - c^2*x^2])/c^8 +
 (2*(6*c^4*d^2 - 16*c^2*d*e + 9*e^2)*(-1 - c^2*x^2)^(3/2))/(3*c^8) - (2*(8
*c^2*d - 9*e)*e*(-1 - c^2*x^2)^(5/2))/(5*c^8) + (6*e^2*(-1 - c^2*x^2)^(7/2
))/(7*c^8)))/Sqrt[-(c^2*x^2)] + (d^2*x^4*(a + b*ArcCsch[c*x]))/4 + (d*e*x^
6*(a + b*ArcCsch[c*x]))/3 + (e^2*x^8*(a + b*ArcCsch[c*x]))/8
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.94.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.79

method	result
parts	$a\left(\frac{1}{8}e^{2}x^{8}+\frac{1}{3}de\,x^{6}+\frac{1}{4}x^{4}d^{2}\right)+\frac{b\left(\frac{c^{4}\operatorname{arccsch}(cx)e^{2}x^{8}}{8}+\frac{c^{4}\operatorname{arccsch}(cx)de\,x^{6}}{3}+\frac{\operatorname{arccsch}(cx)d^{2}x^{4}c^{4}}{4}+\frac{\left(c^{2}x^{2}+1\right)\left(45c^{6}e\cdots\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$-\frac{a\left(\frac{c^{2}d\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{3}-\frac{\left(e\,c^{2}x^{2}+c^{2}d\right)^{4}}{4}\right)}{2c^{4}e^{2}}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^{8}d^{4}}{24e^{2}}+\frac{\operatorname{arccsch}(cx)c^{8}d^{2}x^{4}}{4}+\frac{e\operatorname{arccsch}(cx)c^{8}d\,x^{6}}{3}+\frac{e^{2}\operatorname{arccsch}(cx)c^{8}x\cdots}{8}\right.}{\cdots}$
default	$-\frac{a\left(\frac{c^{2}d\left(e\,c^{2}x^{2}+c^{2}d\right)^{3}}{3}-\frac{\left(e\,c^{2}x^{2}+c^{2}d\right)^{4}}{4}\right)}{2c^{4}e^{2}}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^{8}d^{4}}{24e^{2}}+\frac{\operatorname{arccsch}(cx)c^{8}d^{2}x^{4}}{4}+\frac{e\operatorname{arccsch}(cx)c^{8}d\,x^{6}}{3}+\frac{e^{2}\operatorname{arccsch}(cx)c^{8}x\cdots}{8}\right.}{\cdots}$

input
```
int(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*x^4*d^2)+b/c^4*(1/8*c^4*arccsch(c*x)*e^2*x^
8+1/3*c^4*arccsch(c*x)*d*e*x^6+1/4*arccsch(c*x)*d^2*x^4*c^4+1/2520/c^5*(c^
2*x^2+1)*(45*c^6*e^2*x^6+168*c^6*d*e*x^4+210*c^6*d^2*x^2-54*c^4*e^2*x^4-22
4*c^4*d*e*x^2-420*c^4*d^2+72*c^2*e^2*x^2+448*c^2*d*e-144*e^2)/((c^2*x^2+1)
/c^2/x^2)^(1/2)/x)
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.90

$$\int x^3(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{315\,ac^7e^2x^8+840\,ac^7dex^6+630\,ac^7d^2x^4+105\,(3\,bc^7e^2x^8+8\,bc^7dex^6+6\,bc^7d^2x^4)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(\cdots}{\cdots}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3
*b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) + (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e - 9*b*c
^4*e^2)*x^5 + 2*(105*b*c^6*d^2 - 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 - 4*(10
5*b*c^4*d^2 - 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/
c^7
```

3.94.6 Sympy [F]

$$\int x^3(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^3(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{8}ae^2x^8 + \frac{1}{3}adex^6 + \frac{1}{4}ad^2x^4 + \frac{1}{12}\left(3\,x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2 + \frac{1}{45}\left(15\,x^6 \operatorname{arcsch}(cx) + \frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)bde + \frac{1}{280}\left(35\,x^8 \operatorname{arcsch}(cx) + \frac{5\,c^6x^7\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}} - 21\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} + 35\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 35\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^7}\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsch(c*x) +
 (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d^2
+ 1/45*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*
x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/2
80*(35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1)^(7/2) - 21*c^4*x^5*
(1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 35*x*sqrt(1
/(c^2*x^2) + 1))/c^7)*b*e^2
```

3.94.8 Giac [F]

$$\int x^3 \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 \left(b \operatorname{arcsch}(cx)+a\right)x^3\,dx$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^3, x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^3\left(e\,x^2+d\right)^2\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.95 $\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.95.1 Optimal result

Integrand size = 19, antiderivative size = 203

$$\begin{aligned}\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx = {} & \frac{b(3c^4d^2-3c^2de+e^2)\,x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} \\ & - \frac{b(3c^2d-2e)\,ex(-1-c^2x^2)^{3/2}}{18c^5\sqrt{-c^2x^2}} \\ & + \frac{be^2x(-1-c^2x^2)^{5/2}}{30c^5\sqrt{-c^2x^2}} \\ & + \frac{(d+ex^2)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e} \\ & - \frac{bcd^3x\arctan\left(\sqrt{-1-c^2x^2}\right)}{6e\sqrt{-c^2x^2}}\end{aligned}$$

output
```
1/6*(e*x^2+d)^3*(a+b*arccsch(c*x))/e-1/18*b*(3*c^2*d-2*e)*e*x*(-c^2*x^2-1)
^(3/2)/c^5/(-c^2*x^2)^(1/2)+1/30*b*e^2*x*(-c^2*x^2-1)^(5/2)/c^5/(-c^2*x^2)
^(1/2)-1/6*b*c*d^3*x*arctan((-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/6*b*(
3*c^4*d^2-3*c^2*d*e+e^2)*x*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.61

$$\int x\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}(8e^2-2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4))}{c^5}+15bx\left(3d^2+3dex^2+e^2x^4\right)\text{csch}^{-1}(cx)\right)$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(8*e^2
 - 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 +
 15*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsch[c*x]))/90
```

3.95.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6854, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6854}$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}-\frac{bcx\int\frac{\left(ex^2+d\right)^3}{x\sqrt{-c^2x^2-1}}dx}{6e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}-\frac{bcx\int\frac{\left(ex^2+d\right)^3}{x^2\sqrt{-c^2x^2-1}}dx^2}{12e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{99}$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}-\frac{bcx\int\left(\frac{d^3}{x^2\sqrt{-c^2x^2-1}}+\frac{e^3\left(-c^2x^2-1\right)^{3/2}}{c^4}-\frac{\left(3c^2d-2e\right)e^2\sqrt{-c^2x^2-1}}{c^4}+\frac{e\left(3d^2c^4-3dec^2+e^2\right)}{c^4\sqrt{-c^2x^2-1}}\right)dx^2}{12e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)}{6e}-\frac{bcx\left(2d^3\arctan\left(\sqrt{-c^2x^2-1}\right)+\frac{2e^2\left(-c^2x^2-1\right)^{3/2}\left(3c^2d-2e\right)}{3c^6}-\frac{2e^3\left(-c^2x^2-1\right)^{5/2}}{5c^6}-\frac{2e\sqrt{-c^2x^2-1}\left(3c^4d^2-3c^2de+e^2\right)}{c^6}\right)}{12e\sqrt{-c^2x^2}}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcCsch[c*x]))/(6*e) - (b*c*x*((-2*e*(3*c^4*d^2 - 3*
c^2*d*e + e^2)*Sqrt[-1 - c^2*x^2])/c^6 + (2*(3*c^2*d - 2*e)*e^2*(-1 - c^2*
x^2)^(3/2))/(3*c^6) - (2*e^3*(-1 - c^2*x^2)^(5/2))/(5*c^6) + 2*d^3*ArcTan[
Sqrt[-1 - c^2*x^2]]))/(12*e*Sqrt[-(c^2*x^2)])
```

3.95.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.95.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31

method	result
parts	$\frac{a(e x^2+d)^3}{6e} + \frac{b\left(\frac{c^2e^2 \operatorname{arccsch}(cx)x^6}{6} + \frac{c^2 e \operatorname{arccsch}(cx)x^4 d}{2} + \frac{\operatorname{arccsch}(cx)x^2c^2d^2}{2} + \frac{c^2 \operatorname{arccsch}(cx)d^3}{6e} - \frac{\sqrt{c^2x^2+1}\left(15c^6d^3 \operatorname{arctanh} \ldots\right.}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{\frac{a\left(e c^2x^2+c^2d\right)^3}{6c^4e} + b\left(\frac{\operatorname{arccsch}(cx)c^6d^3}{6e} + \frac{\operatorname{arccsch}(cx)c^6d^2x^2}{2} + \frac{e \operatorname{arccsch}(cx)c^6 d x^4}{2} + \frac{e^2 \operatorname{arccsch}(cx)c^6x^6}{6} - \frac{\sqrt{c^2x^2+1}\left(15c^6d^3 \operatorname{arctanh} \ldots\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{a\left(e c^2x^2+c^2d\right)^3}{6c^4e} + b\left(\frac{\operatorname{arccsch}(cx)c^6d^3}{6e} + \frac{\operatorname{arccsch}(cx)c^6d^2x^2}{2} + \frac{e \operatorname{arccsch}(cx)c^6 d x^4}{2} + \frac{e^2 \operatorname{arccsch}(cx)c^6x^6}{6} - \frac{\sqrt{c^2x^2+1}\left(15c^6d^3 \operatorname{arctanh} \ldots\right.}{\ldots}\right.}{\ldots}$

input
```
int(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*(e*x^2+d)^3/e+b/c^2*(1/6*c^2*e^2*arccsch(c*x)*x^6+1/2*c^2*e*arccsch(
c*x)*x^4*d+1/2*arccsch(c*x)*x^2*c^2*d^2+1/6*c^2/e*arccsch(c*x)*d^3-1/90/c^
5/e*(c^2*x^2+1)^(1/2)*(15*c^6*d^3*arctanh(1/(c^2*x^2+1)^(1/2))-45*c^4*d^2*
e*(c^2*x^2+1)^(1/2)-15*c^4*d*e^2*x^2*(c^2*x^2+1)^(1/2)-3*e^3*c^4*x^4*(c^2*
x^2+1)^(1/2)+30*c^2*d*e^2*(c^2*x^2+1)^(1/2)+4*e^3*c^2*x^2*(c^2*x^2+1)^(1/2
)-8*e^3*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/x)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

$$\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{15\,ac^5e^2x^6+45\,ac^5dex^4+45\,ac^5d^2x^2+15\,(bc^5e^2x^6+3\,bc^5dex^4+3\,bc^5d^2x^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(3\,bc^4e^2}{90\,c^5}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^5*e^2*x^6 + 45*a*c^5*d*e*x^4 + 45*a*c^5*d^2*x^2 + 15*(b*c^5*e
^2*x^6 + 3*b*c^5*d*e*x^4 + 3*b*c^5*d^2*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) + 1)/(c*x)) + (3*b*c^4*e^2*x^5 + (15*b*c^4*d*e - 4*b*c^2*e^2)*x^3
 + (45*b*c^4*d^2 - 30*b*c^2*d*e + 8*b*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
))/c^5
```

3.95.6 Sympy [F]

$$\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

output
```
Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90

$$\int x(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx$$
$$= \frac{1}{6} ae^2x^6 + \frac{1}{2} adex^4 + \frac{1}{2} ad^2x^2 + \frac{1}{2} \left(x^2 \operatorname{arcsch}(cx) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right) bd^2$$
$$+ \frac{1}{6} \left(3x^4 \operatorname{arcsch}(cx) + \frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} - 3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right) bde$$
$$+ \frac{1}{90} \left(15x^6 \operatorname{arcsch}(cx) + \frac{3c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}} - 10c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}} + 15x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right) be^2$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsch(c*x) + x*
sqrt(1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^
2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*a
rccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2)
 + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e^2
```

3.95.8 Giac [F]

$$\int x(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = \int (ex^2+d)^2(b \operatorname{arcsch}(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x, x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x\,\left(e\,x^2+d\right)^2\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.96 $\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x} \, dx$

3.96.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x} \, dx = {} & \frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c} \\ & +\frac{1}{2}bd^2\text{csch}^{-1}(cx)^2+dex^2\left(a+b\text{csch}^{-1}(cx)\right) \\ & +\frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right) \\ & -bd^2\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\ & +bd^2\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\ & -d^2\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ & -\frac{1}{2}bd^2\,\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\end{aligned}$$

output
```
1/2*b*d^2*arccsch(c*x)^2+d*e*x^2*(a+b*arccsch(c*x))+1/4*e^2*x^4*(a+b*arccs
ch(c*x))-b*d^2*arccsch(c*x)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*d^2*arcc
sch(c*x)*ln(1/x)-d^2*(a+b*arccsch(c*x))*ln(1/x)-1/2*b*d^2*polylog(2,(1/c/x
+(1+1/c^2/x^2)^(1/2))^2)+1/6*b*(6*c^2*d-e)*e*x*(1+1/c^2/x^2)^(1/2)/c^3+1/1
2*b*e^2*x^3*(1+1/c^2/x^2)^(1/2)/c
```

3.96.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = adex^2+\frac{1}{4}ae^2x^4+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x(-2+c^2x^2)}{12c^3} + \frac{1}{4}be^2x^4\operatorname{csch}^{-1}(cx) + \frac{bdex\left(\sqrt{1+\frac{1}{c^2x^2}}+cx\operatorname{csch}^{-1}(cx)\right)}{c} + ad^2\log(x)+\frac{1}{2}bd^2\left(\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx) - 2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)\right)-\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
a*d*e*x^2 + (a*e^2*x^4)/4 + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + c^2*x^2))
/(12*c^3) + (b*e^2*x^4*ArcCsch[c*x])/4 + (b*d*e*x*(Sqrt[1 + 1/(c^2*x^2)] +
 c*x*ArcCsch[c*x]))/c + a*d^2*Log[x] + (b*d^2*(ArcCsch[c*x]*(ArcCsch[c*x]
- 2*Log[1 - E^(2*ArcCsch[c*x])]) - PolyLog[2, E^(2*ArcCsch[c*x])]))/2
```

3.96.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6858, 6237, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \left(\frac{d}{x^2}+e\right)^2 x^5 \left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{6237}$$

$$\frac{b\int -\frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{4\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 27$$

$$-\frac{b\int \frac{e\left(\frac{4d}{x^2}+e\right)x^4-4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 7293$$

$$-\frac{b\int \left(\frac{e\left(\frac{4d}{x^2}+e\right)x^4}{\sqrt{1+\frac{1}{c^2x^2}}}-\frac{4d^2\log\left(\frac{1}{x}\right)}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{4c}-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \ 2009$$

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+dex^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{4}e^2x^4\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)-\frac{b\left(2cd^2\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)-2cd^2\text{arcsinh}\left(\frac{1}{cx}\right)^2+4cd^2\text{arcsinh}\left(\frac{1}{cx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)-4cd^2\log\left(\frac{1}{x}\right)\text{arcs}\right.}{4c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
d*e*x^2*(a + b*ArcSinh[1/(c*x)]) + (e^2*x^4*(a + b*ArcSinh[1/(c*x)]))/4 -
d^2*(a + b*ArcSinh[1/(c*x)])*Log[x^(-1)] - (b*((-2*e*(6*d - e/c^2)*Sqrt[1
+ 1/(c^2*x^2)]*x)/3 - (e^2*Sqrt[1 + 1/(c^2*x^2)]*x^3)/3 - 2*c*d^2*ArcSinh[
1/(c*x)]^2 + 4*c*d^2*ArcSinh[1/(c*x)]*Log[1 - E^(2*ArcSinh[1/(c*x)])] - 4*
c*d^2*ArcSinh[1/(c*x)]*Log[x^(-1)] + 2*c*d^2*PolyLog[2, E^(2*ArcSinh[1/(c*
x)])]))/(4*c)
```

3.96. $\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6237
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1
+ c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d]
 && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.96.4 Maple [F]

$$\int \frac{\left(e\,x^{2}+d\right)^{2}\left(a+b\,\mathrm{arccsch}\left(c x\right)\right)}{x}dx$$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

output
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

3.96.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccsch(c*x))/x, x)
```

3.96.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x, x)
```

3.96.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + 4*b*c^2*d^2*integrate(1/4*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + a*d*e*x^2 - b*d^2*log(c)*log(
x) - 1/4*(2*log(c^2*x^2 + 1)*log(x) + dilog(-c^2*x^2))*b*d^2 + a*d^2*log(x
) + 1/2*b*d*e*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2 - 1/24*(3*c^2*x
^2 - 2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)
*b*e^2/c^4 - 1/8*(2*b*c^2*e^2*x^4*log(c) + 4*b*c^2*d^2*log(x)^2 + (8*c^2*d
*e*log(c) - e^2)*b*x^2 + 2*(b*c^2*e^2*x^4 + 4*b*c^2*d*e*x^2)*log(x) - 2*(b
*c^2*e^2*x^4 + 4*b*c^2*d*e*x^2 + 4*b*c^2*d^2*log(x))*log(sqrt(c^2*x^2 + 1)
 + 1))/c^2 + 1/8*(4*c^2*d*e - e^2)*b*log(c^2*x^2 + 1)/c^4
```

3.96.8 Giac [F]

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{(ex^2+d)^2(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x, x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{(e\,x^2+d)^2\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x, x)
```

3.97 $\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^3} dx$

3.97.1 Optimal result

Integrand size = 21, antiderivative size = 178

$$\begin{aligned}\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^3} dx &= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}x}{2c} \\ &\quad - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\text{csch}^{-1}(cx)^2 \\ &\quad - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) \\ &\quad - 2bde\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) \\ &\quad + 2bde\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) \\ &\quad - 2de\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) \\ &\quad - bde\,\text{PolyLog}\left(2, e^{2\text{csch}^{-1}(cx)}\right)\end{aligned}$$

output
```
-1/4*b*c^2*d^2*arccsch(c*x)+b*d*e*arccsch(c*x)^2-1/2*d^2*(a+b*arccsch(c*x)
)/x^2+1/2*e^2*x^2*(a+b*arccsch(c*x))-2*b*d*e*arccsch(c*x)*ln(1-(1/c/x+(1+1
/c^2/x^2)^(1/2))^2)+2*b*d*e*arccsch(c*x)*ln(1/x)-2*d*e*(a+b*arccsch(c*x))*
ln(1/x)-b*d*e*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/4*b*c*d^2*(1+1/c^
2/x^2)^(1/2)/x+1/2*b*e^2*x*(1+1/c^2/x^2)^(1/2)/c
```

3.97.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2\,(a+b\text{csch}^{-1}(cx))}{x^3}\,dx$$
$$= \frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2bd^2\text{csch}^{-1}(cx)}{x^2}+\frac{2be^2x\left(\sqrt{1+\frac{1}{c^2x^2}}+cx\text{csch}^{-1}(cx)\right)}{c}\right.$$
$$-\frac{bd^2\left(-1-c^2x^2+c^2x^2\sqrt{1+c^2x^2}\text{arctanh}\left(\sqrt{1+c^2x^2}\right)\right)}{c\sqrt{1+\frac{1}{c^2x^2}}x^3}+8ade\log(x)$$
$$\left.+4bde\left(\text{csch}^{-1}(cx)\left(\text{csch}^{-1}(cx)-2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsch[c*x])/x^2 + (2*b*e^2*x*(S
qrt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c*x]))/c - (b*d^2*(-1 - c^2*x^2 + c^2*x
^2*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]]))/(c*Sqrt[1 + 1/(c^2*x^2)]
*x^3) + 8*a*d*e*Log[x] + 4*b*d*e*(ArcCsch[c*x]*(ArcCsch[c*x] - 2*Log[1 - E
^(2*ArcCsch[c*x])]) - PolyLog[2, E^(2*ArcCsch[c*x])]))/4
```

3.97.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6858, 6237, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2\,(a+b\text{csch}^{-1}(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{ 6858}$$
$$-\int \left(\frac{d}{x^2}+e\right)^2 x^3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{6237}$$

$$\frac{b\int -\frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{2\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{c}-\frac{d^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{-\frac{d^2}{x^2}-4e\log\left(\frac{1}{x}\right)d+e^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}}d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{7293}$$

$$-\frac{b\int \left(-\frac{d^2}{\sqrt{1+\frac{1}{c^2x^2}}x^2}-\frac{4e\log\left(\frac{1}{x}\right)d}{\sqrt{1+\frac{1}{c^2x^2}}}+\frac{e^2x^2}{\sqrt{1+\frac{1}{c^2x^2}}}\right)d\frac{1}{x}}{2c}-\frac{d^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)+\frac{1}{2}e^2x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)-\frac{b\left(\frac{1}{2}c^3d^2\text{arcsinh}\left(\frac{1}{cx}\right)+2cde\,\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)-2cde\text{arcsinh}\left(\frac{1}{cx}\right)^2+4cde\text{arcsinh}\left(\frac{1}{cx}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{1}{cx}\right)}\right.\right.}{2c}$$

input

```
Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

output

```
-1/2*(d^2*(a + b*ArcSinh[1/(c*x)]))/x^2 + (e^2*x^2*(a + b*ArcSinh[1/(c*x)]
))/2 - 2*d*e*(a + b*ArcSinh[1/(c*x)])*Log[x^(-1)] - (b*(-1/2*(c^2*d^2*Sqrt
[1 + 1/(c^2*x^2)])/x - e^2*Sqrt[1 + 1/(c^2*x^2)]*x + (c^3*d^2*ArcSinh[1/(c
*x)])/2 - 2*c*d*e*ArcSinh[1/(c*x)]^2 + 4*c*d*e*ArcSinh[1/(c*x)]*Log[1 - E^
(2*ArcSinh[1/(c*x)])] - 4*c*d*e*ArcSinh[1/(c*x)]*Log[x^(-1)] + 2*c*d*e*Pol
yLog[2, E^(2*ArcSinh[1/(c*x)])]))/(2*c)
```

3.97. $\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6237
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Sim
p[(a + b*ArcSinh[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/Sqrt[1
+ c^2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d]
 && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.97.4 Maple [F]

$$\int \frac{(e\,x^2+d)^2\,(a+b\,\operatorname{arccsch}(cx))}{x^3}dx$$

input
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

3.97.5 Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccsch(c*x))/x^3, x)
```

3.97.6 Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^3} dx = \int \frac{(a+b\,\text{acsch}(cx))(d+ex^2)^2}{x^3} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**3, x)
```

3.97.7 Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2(b\,\text{arcsch}(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

output
```
4*b*c^2*d*e*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 +
sqrt(c^2*x^2 + 1) + 1), x) - 1/2*b*e^2*x^2*log(c) - 1/2*b*e^2*x^2*log(x) +
 1/2*a*e^2*x^2 - 2*b*d*e*log(c)*log(x) - b*d*e*log(x)^2 - 1/2*(2*log(c^2*x
^2 + 1)*log(x) + dilog(-c^2*x^2))*b*d*e + 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*
x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) +
 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2)
 + 2*a*d*e*log(x) + 1/4*b*e^2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))/c^2
 + 1/4*b*e^2*log(c^2*x^2 + 1)/c^2 + 1/2*(b*e^2*x^2 + 4*b*d*e*log(x))*log(s
qrt(c^2*x^2 + 1) + 1) - 1/2*a*d^2/x^2
```

3.97.8 Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^3} dx = \int \frac{(ex^2+d)^2 (b \operatorname{arcsch}(cx)+a)}{x^3} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^3, x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^3} dx = \int \frac{(e\,x^2+d)^2 \left(a+b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3} dx$$

input
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.98 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.98.1 Optimal result

Integrand size = 21, antiderivative size = 512

$$
\begin{aligned}
\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx &= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{b\,\text{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} \\
&+\frac{\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&-\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}} \\
&+\frac{b\sqrt{-d}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^{3/2}}
\end{aligned}
$$

output
```
x*(a+b*arccsch(c*x))/e+b*arctanh((1+1/c^2/x^2)^(1/2))/c/e+1/2*(a+b*arccsch
(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(
1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a
+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-
c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3
/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c
^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*pol
ylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)
))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

3.98.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 1239, normalized size of antiderivative = 2.42

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

output
```
(4*a*c*Sqrt[e]*x + 4*b*c*Sqrt[e]*x*ArcCsch[c*x] - 4*a*c*Sqrt[d]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]
/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/
Sqrt[-(c^2*d) + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d]
)]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4]
)/Sqrt[-(c^2*d) + e]] + b*c*Sqrt[d]*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[
1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*
c*Sqrt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - b*c*Sqrt[d]*Pi*Log
[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*
I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - b*c*Sqrt[d]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*(Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d]*Ar
cSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c
^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[1 + (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt...
```

3.98.3 Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^{3/2}}+\frac{x\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{e}-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\operatorname{arctanh}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{ce}$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

output
```
(x*(a + b*ArcSinh[1/(c*x)]))/e + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e)
+ (Sqrt[-d]*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)
])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSinh
[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d
) + e])])/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt
[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (S
qrt[-d]*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(
Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*
Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*e^(3/2))
 + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcS
inh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*
PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])]
)/(2*e^(3/2))
```

3.98. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.98.4 Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arccsch}\,(cx))}{e\,x^2+d}dx$$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

output
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

3.98.5 Fricas [F]

$$\int \frac{x^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)/(e*x^2 + d), x)
```

3.98.6 Sympy [F]

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{d + ex^2} dx = \int \frac{x^2(a + b\,\text{acsch}\,(cx))}{d + ex^2} dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2), x)
```

3.98.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{d + ex^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.98.8 Giac [F]

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{d + ex^2} dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^2}{ex^2 + d} dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.99 $\int \frac{x\left(a+b\,\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.99.1 Optimal result

Integrand size = 19, antiderivative size = 467

$$\begin{aligned}\int \frac{x(a+b\,\text{csch}^{-1}(cx))}{d+ex^2}\,dx = & -\frac{(a+b\,\text{csch}^{-1}(cx))^2}{be}-\frac{(a+b\,\text{csch}^{-1}(cx))\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)}{e}\\
&+\frac{(a+b\,\text{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{(a+b\,\text{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{(a+b\,\text{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{(a+b\,\text{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)}{2e}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\\
&+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e}\end{aligned}$$

output
```
-(a+b*arccsch(c*x))^2/b/e-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)/e+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*(a+b*arcc
sch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e
)^(1/2)))/e+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2
)^(1/2))^2)/e+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
```

3.99.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 1103, normalized size of antiderivative = 2.36

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

output
```
(b*Pi^2 - (4*I)*b*Pi*ArcCsch[c*x] - 8*b*ArcCsch[c*x]^2 + 16*b*ArcSin[Sqrt[
1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi +
(2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCs
ch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[
c*x])] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x]
)/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-
(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 -
 Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[...
```

3.99.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \ 6858$$

$$-\int \frac{x\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \ 6238$$

$$-\int \left(\frac{x\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{be}-\frac{\log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{1}{cx}\right)}\right)\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{e}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}\left(\frac{1}{cx}\right)}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

output
```
-((a + b*ArcSinh[1/(c*x)])^2/(b*e)) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - E^
(-2*ArcSinh[1/(c*x)])])/e + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*
E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcSi
nh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2
*d) + e])])/(2*e) + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSin
h[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcSinh[1/(c*
x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(2*e) + (b*PolyLog[2, E^(-2*ArcSinh[1/(c*x)])])/(2*e) + (b*PolyLog[2,
-((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*e)
+ (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) +
 e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] +
Sqrt[-(c^2*d) + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)
])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e)
```

3.99. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.99.4 Maple [F]

$$\int \frac{x(a+b\ \mathrm{arccsch}\left(cx\right))}{e\,x^{2}+d}dx$$

input
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

output
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

3.99.5 Fricas [F]

$$\int \frac{x\left(a+b\mathrm{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\mathrm{arcsch}\left(cx\right)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arccsch(c*x) + a*x)/(e*x^2 + d), x)
```

3.99.6 Sympy [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{d + ex^2} dx = \int \frac{x(a + b\,\text{acsch}\,(cx))}{d + ex^2} dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*acsch(c*x))/(d + e*x**2), x)
```

3.99.7 Maxima [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{d + ex^2} dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{ex^2 + d} dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d), x) + 1/2*a
*log(e*x^2 + d)/e
```

3.99.8 Giac [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{d + ex^2} dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{ex^2 + d} dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.100 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx$

3.100.1 Optimal result

Integrand size = 18, antiderivative size = 477

$$\begin{aligned}\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx &= \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2
)/e^(1/2)+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccsch(c*x))
*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))
/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/
x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 1055, normalized size of antiderivative = 2.21

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx$$

$$= \frac{4a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+8ib\arcsin\left(\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right)\arctan\left(\frac{\left(c\sqrt{d}-\sqrt{e}\right)\cot\left(\frac{1}{4}\left(\pi+2i\operatorname{csch}^{-1}(cx)\right)\right)}{\sqrt{-c^2d+e}}\right)+8ib\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right.}{}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

output
```
(4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt
[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])
/4])/Sqrt[-(c^2*d) + e]] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sq
rt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-
(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] - b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + Sqrt[e]/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs...
```

3.100.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6848, 6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{6848}$$

$$-\int \frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{\frac{d}{x^2}+e}d\frac{1}{x}$$

$$\Big\downarrow \text{6208}$$

$$-\int \left(\frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}+\frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{2\sqrt{e}\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}\right)d\frac{1}{x}$$

$$\downarrow 2009$$

$$\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

output
```
((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e]
 - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSinh[1/(c*x)])*
Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(
2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcS
inh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a
+ b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + S
qrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^
ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[e]) +
 (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)
])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2,
(c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-
d]*Sqrt[e])
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

3.100. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{d+ex^2}\,dx$

rule 6848
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

3.100.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{e\,x^{2}+d}dx$$

input
```
int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

output
```
int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

3.100.5 Fricas [F]

$$\int \frac{a+b\mathrm{csch}^{-1}(cx)}{d+ex^2}\,dx=\int \frac{b\,\mathrm{arcsch}\left(cx\right)+a}{ex^{2}+d}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

3.100.6 Sympy [F]

$$\int \frac{a+b\mathrm{csch}^{-1}(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\mathrm{acsch}\left(cx\right)}{d+ex^{2}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x**2), x)
```

3.100.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{d + ex^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.100.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{d + ex^2} \, dx = \int \frac{b \,\text{arcsch}\,(cx) + a}{ex^2 + d} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{d + ex^2} \, dx = \int \frac{a + b \,\text{asinh}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2), x)
```

3.101 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)}\,dx$

3.101.1 Optimal result

Integrand size = 21, antiderivative size = 425

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)}\,dx &= \frac{(a+b\text{csch}^{-1}(cx))^2}{2bd} - \frac{(a+b\text{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\ &- \frac{(a+b\text{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\ &- \frac{(a+b\text{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\ &- \frac{(a+b\text{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} \\ &- \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d} \\ &- \frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d} - \frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d}\end{aligned}$$

output
```
1/2*(a+b*arccsch(c*x))^2/b/d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x
))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)
))/d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*
(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*b
*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1
/2)))/d-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
+(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d
```

3.101.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 1141, normalized size of antiderivative = 2.68

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\,(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

output
```
-1/8*(b*Pi^2 - (4*I)*b*Pi*ArcCsch[c*x] - 12*b*ArcCsch[c*x]^2 + 16*b*ArcSin
[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[
(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - S
qrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)
*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*Ar
cCsch[c*x])] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c
^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 -
 Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sq
rt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*
(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*A...
```

3.101.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\,(d+ex^2)}\,dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x}\,d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{2d\left(\frac{\sqrt{-d}}{x}+\sqrt{e}\right)}-\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{2d\left(\sqrt{e}-\frac{\sqrt{-d}}{x}\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{2bd}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a + b*ArcSinh[1/(c*x)])^2/(2*b*d) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*
Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a
+ b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - S
qrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]
*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcS
inh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^
2*d) + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt
[e] - Sqrt[-(c^2*d) + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1
/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[
-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*d) - (b*PolyL
og[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*
d)
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

3.101. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex^2)}\,dx$

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.101.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x\left(e\,x^{2}+d\right)}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
```

3.101.5 Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{b\,\mathrm{arcsch}\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e*x^3 + d*x), x)
```

3.101.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx=\int \frac{a+b\,\mathrm{acsch}\left(cx\right)}{x\left(d+ex^2\right)}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*(d + e*x**2)), x)
```

3.101.7 Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c^2*x^2)
+ 1) + 1/(c*x))/(e*x^3 + d*x), x)
```

3.101.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x), x)
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\,(d + ex^2)}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2 + d\right)}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.102 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.102.1 Optimal result

Integrand size = 21, antiderivative size = 518

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx = {} & \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\text{csch}^{-1}(cx)}{dx} \\ & +\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & -\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}} \\ & +\frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}}\end{aligned}$$

output
```
-a/d/x-b*arccsch(c*x)/d/x+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a
+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-
c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3
/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(1
+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/
2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*poly
log(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))
*e^(1/2)/(-d)^(3/2)+b*c*(1+1/c^2/x^2)^(1/2)/d
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.52 (sec) , antiderivative size = 1211, normalized size of antiderivative = 2.34

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)} \, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
-(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[
1 + 1/(c^2*x^2)] - ArcCsch[c*x]/x)/d - ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*Ar
cCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sq
rt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*
Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1
- (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*
Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
+ 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1
 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*
Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*Po
lyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])]))/d^(3/2) + ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsc
h[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqr
t[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*
ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[...
```

3.102.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)}dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{d}-\frac{e\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2(-d)^{3/2}}-\frac{a}{dx}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{dx}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{d}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(b*c*Sqrt[1 + 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcSinh[1/(c*x)])/(d*x) + (Sq
rt[e]*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sq
rt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSinh[1/(
c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) +
e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-
d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (
Sqrt[e]*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(
Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((
c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*(-d)^(
3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - S
qrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*
E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*(-d)^(3/2)) + (b*
Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d
) + e])])/(2*(-d)^(3/2))
```

3.102. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.102.4 Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2\left(e\,x^2 + d\right)}dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

3.102.5 Fricas [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\left(d + ex^2\right)}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e*x^4 + d*x^2), x)
```

3.102.6 Sympy [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx=\int \frac{a+b\operatorname{acsch}(cx)}{x^2(d+ex^2)}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)), x)
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.102.8 Giac [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx=\int \frac{b\operatorname{arcsch}(cx)+a}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x^2), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{asinh}\left(\frac{1}{c x}\right)}{x^2 \left(e x^2 + d\right)} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.103 $\int \frac{x^5\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.103.1 Optimal result

Integrand size = 21, antiderivative size = 591

$$\begin{aligned}\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx =& \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2e^2}\\
&+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3}-\frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{5/2}}\\
&+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}\\
&-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{bd\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}\\
&-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}\\
&-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{e^3}\end{aligned}$$

output
```
1/2*d*(a+b*arccsch(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsch(c*x))/e^2+2*d*
(a+b*arccsch(c*x))^2/b/e^3+2*d*(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x
^2)^(1/2))^2)/e^3-d*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsch(c*x))*ln(1+c*(1/
c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-d*(a+b
*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^
2*d+e)^(1/2)))/e^3-d*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))
*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,1/(1/c/x+(1+1/c^
2/x^2)^(1/2))^2)/e^3-b*d*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,-c*(1/c/x+(1
+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(
2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3
-1/2*b*d*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))/e^(5/2)/(
c^2*d-e)^(1/2)+1/2*b*x*(1+1/c^2/x^2)^(1/2)/c/e^2
```

3.103.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 1447, normalized size of antiderivative = 2.45

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(d*Pi^
2 - (2*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsch[c*x] - 2*e*x^2*Ar
cCsch[c*x] + (d^(3/2)*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*Arc
Csch[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 8*d*ArcCsch[c*x]^2 - 2*d*ArcSinh[1/(c
*x)] + 16*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[
d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*d
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e
])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*d*ArcCsch[c*x
]*Log[1 - E^(-2*ArcCsch[c*x])] + (2*I)*d*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSi
n[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(-Sqrt[e] +
 Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1
 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)
*d*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 - (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x
]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
 (8*I)*d*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt...
```

3.103.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^2} \, dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{x^3 \left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2} d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right) x^3}{e^2}-\frac{2d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right) x}{e^3}+\frac{2d^2\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right) x}+\frac{d^2\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2 x}\right) d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{e^3}-\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{e^3}+\frac{2d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{be^3}+\frac{2d\log\left(1-e^{-2\operatorname{arcsinh}\left(\frac{1}{cx}\right)}\right)\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{2e^2}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\operatorname{PolyLog}\left(2,e^{-2\operatorname{arcsinh}\left(\frac{1}{cx}\right)}\right)}{e^3}-\frac{bd\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{5/2}\sqrt{c^2d-e}}+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2ce^2}$$

input
```
Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
(b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcSinh[1/(c*x)]))/(2*e^
2*(e + d/x^2)) + (x^2*(a + b*ArcSinh[1/(c*x)]))/(2*e^2) + (2*d*(a + b*ArcS
inh[1/(c*x)])^2)/(b*e^3) - (b*d*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 +
 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d - e]*e^(5/2)) + (2*d*(a + b*ArcSinh[1/(c*
x)])*Log[1 - E^(-2*ArcSinh[1/(c*x)])])/e^3 - (d*(a + b*ArcSinh[1/(c*x)])*L
og[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^
3 - (d*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(S
qrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a + b*ArcSinh[1/(c*x)])*Log[1 - (
c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3 - (d*(
a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, E^(-2*ArcSinh[1/(c*x)])])/e^3
 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2
*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e
] - Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/
(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d
]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3
```

3.103. $\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.103.4 Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^2}dx$$

input
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

output
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.103.5 Fricas [F]

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx=\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^5}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^5*arccsch(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.103.6 Sympy [F]

$$\int \frac{x^5 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{x^5 (a + b\,\text{acsch}\,(cx))}{(d + ex^2)^2} \, dx$$

input
```
integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**5*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

3.103.7 Maxima [F]

$$\int \frac{x^5 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^5}{(ex^2 + d)^2} \, dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*inte
grate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2)
, x)
```

3.103.8 Giac [F]

$$\int \frac{x^5 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^5}{(ex^2 + d)^2} \, dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a + b \operatorname{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^2} \, dx$$

input
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.104.1 Optimal result

Integrand size $=21$, antiderivative size $=553$

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^2}+\frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2\sqrt{c^2d-e}e^{3/2}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^2}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^2}$$

output
```
1/2*(-a-b*arccsch(c*x))/e/(e+d/x^2)-(a+b*arccsch(c*x))^2/b/e^2-(a+b*arccsc
h(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^2+1/2*(a+b*arccsch(c*x))*l
n(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e
^2+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)-(-c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsch(
c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1
/2)))/e^2+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^2+1/2*b*polyl
og(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))
/e^2+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c
^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d-e)^
(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))/e^(3/2)/(c^2*d-e)^(1/2)
```

3.104.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 1410, normalized size of antiderivative = 2.55

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
(b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsch[c*x] + (2*b*Sqrt[d]*Arc
Csch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] +
 I*Sqrt[e]*x) - 8*b*ArcCsch[c*x]^2 - 4*b*ArcSinh[1/(c*x)] + 16*b*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
 + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*Ar
cCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCs
ch[c*x])] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c
*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sq
rt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e]
 + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[
1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[...
```

3.104.3 Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{x\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{x\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^2}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^2}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^2}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{be^2}-\frac{\log\left(1-e^{-2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)}{2e^2}+\frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{3/2}\sqrt{c^2d-e}}$$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcSinh[1/(c*x)])/(e*(e + d/x^2)) - (a + b*ArcSinh[1/(c*x)])^2
/(b*e^2) + (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])
/(2*Sqrt[c^2*d - e]*e^(3/2)) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - E^(-2*Arc
Sinh[1/(c*x)])])/e^2 + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^Arc
Sinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcSinh[
1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d)
 + e])])/(2*e^2) + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh
[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcSinh[1/(c
*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])])/(2*e^2) + (b*PolyLog[2, E^(-2*ArcSinh[1/(c*x)])])/(2*e^2) + (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(
2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c
^2*d) + e])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(S
qrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcS
inh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2)
```

3.104. $\int \frac{x^3\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.104.4 Maple [F]

$$\int \frac{x^3(a + b\,\mathrm{arccsch}\,(cx))}{(e\,x^2 + d)^2}dx$$

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

output
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.104.5 Fricas [F]

$$\int \frac{x^3(a + b\mathrm{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx) + a)x^3}{(ex^2 + d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arccsch(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.104.6 Sympy [F]

$$\int \frac{x^3 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{x^3 (a + b \operatorname{acsch}(cx))}{\left(d + ex^2\right)^2} \, dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

3.104.7 Maxima [F]

$$\int \frac{x^3 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\left(ex^2 + d\right)^2} \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqr
t(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.104.8 Giac [F]

$$\int \frac{x^3 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^2} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{\left(ex^2 + d\right)^2} \, dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.105 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 139

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bcx\arctan\left(\sqrt{-1-c^2x^2}\right)}{2de\sqrt{-c^2x^2}}+\frac{bcx\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\sqrt{e}\sqrt{-c^2x^2}}$$

output
```
1/2*(-a-b*arccsch(c*x))/e/(e*x^2+d)+1/2*b*c*x*arctan((-c^2*x^2-1)^(1/2))/d
/e/(-c^2*x^2)^(1/2)+1/2*b*c*x*arctanh(e^(1/2)*(-c^2*x^2-1)^(1/2)/(c^2*d-e)
^(1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)/(-c^2*x^2)^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=$$
$$-\frac{\frac{2a}{d+ex^2}+\frac{2b\text{csch}^{-1}(cx)}{d+ex^2}-\frac{2b\,\text{arcsinh}\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}\log\left(-\frac{4\left(ide+cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b\sqrt{-c^2d+e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d+e}}+\frac{b\sqrt{e}\log\left(\frac{4i\left(de+cd\sqrt{e}\left(ic\sqrt{d}+\cdots\right.\right.}{b\sqrt{-c^2d\cdots}}\right.}{d\sqrt{-c^2\cdots}}\cdots}{4e}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsch[c*x])/(d + e*x^2) - (2*b*ArcSinh[1/
(c*x)])/d + (b*Sqrt[e]*Log[(-4*(I*d*e + c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(
c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^2*d) + e]*(Sqrt[d] - I*
Sqrt[e]*x))]])/(d*Sqrt[-(c^2*d) + e]) + (b*Sqrt[e]*Log[((4*I)*(d*e + c*d*Sq
rt[e]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt
[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]])/(d*Sqrt[-(c^2*d) + e]))/e
```

3.105.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {6854, 354, 97, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 6854}$$

$$\frac{bcx\int \frac{1}{x\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx}{2e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx^2}{4e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 97}$$

$$\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{-c^2x^2-1}}dx^2}{d}-\frac{e\int \frac{1}{\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}\right)}{4e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{bcx\left(\frac{2e\int \frac{1}{-\frac{ex^4}{c^2}+d-\frac{e}{c^2}}d\sqrt{-c^2x^2-1}}{c^2d}-\frac{2\int \frac{1}{-\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{-c^2x^2-1}}{c^2d}\right)}{4e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{bcx\left(\frac{2e\int \frac{1}{-\frac{ex^4}{c^2}+d-\frac{e}{c^2}}d\sqrt{-c^2x^2-1}}{c^2d}+\frac{2\arctan\left(\sqrt{-c^2x^2-1}\right)}{d}\right)}{4e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{bcx\left(\frac{2\arctan\left(\sqrt{-c^2x^2-1}\right)}{d}+\frac{2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}\right)}{4e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcCsch[c*x])/(e*(d + e*x^2)) + (b*c*x*((2*ArcTan[Sqrt[-1 - c^
2*x^2]])/d + (2*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sqrt[c^2*d -
e]])/(d*Sqrt[c^2*d - e])))/(4*e*Sqrt[-(c^2*x^2)])
```

3.105.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(120) = 240$.

Time = 6.22 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.95

method	result
parts	$-\frac{a}{2e(e x^2+d)}+\frac{b\left(-\frac{c^4 \operatorname{arccsch}(cx)}{2e\left(e c^2x^2+c^2d\right)}+\frac{c\sqrt{c^2x^2+1}\left(2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2+1}\sqrt{-\frac{c^2d-e}{e}}\,e+\sqrt{-c}\cdots\right.}{-cex+\sqrt{-c^2de}}\cdots\right.\right.}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd\sqrt{-\frac{c^2}{\cdots}}}\cdots\right)}{c^2}$
derivativedivides	$\frac{-\frac{a c^4}{2e\left(e c^2x^2+c^2d\right)}+b c^4\left(-\frac{\operatorname{arccsch}(cx)}{2e\left(e c^2x^2+c^2d\right)}+\frac{\sqrt{c^2x^2+1}\left(2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2+1}\sqrt{-\frac{c^2d-e}{e}}\,e+\sqrt{\cdots}\right.}{-cex+\sqrt{-c^2de}}\cdots\right.\right.}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3xd\sqrt{-\cdots}}\cdots\right)}{c^2}$
default	$\frac{-\frac{a c^4}{2e\left(e c^2x^2+c^2d\right)}+b c^4\left(-\frac{\operatorname{arccsch}(cx)}{2e\left(e c^2x^2+c^2d\right)}+\frac{\sqrt{c^2x^2+1}\left(2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}-\ln\left(-\frac{2\left(\sqrt{c^2x^2+1}\sqrt{-\frac{c^2d-e}{e}}\,e+\sqrt{\cdots}\right.}{-cex+\sqrt{-c^2de}}\cdots\right.\right.}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3xd\sqrt{-\cdots}}\cdots\right)}{c^2}$

input
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)+b/c^2*(-1/2*c^4/e/(c^2*e*x^2+c^2*d)*arccsch(c*x)+1/4*c/
e*(c^2*x^2+1)^(1/2)*(2*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)-l
n(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c
*e*x+(-c^2*d*e)^(1/2)))-ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(
-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2))))/((c^2*x^2+1)/c^2/x^2)^(1
/2)/x/d/(-(c^2*d-e)/e)^(1/2))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(117) = 234$.

Time = 0.29 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.42

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$$

$$= \left[-\frac{2\,ac^2d^2 - 2\,ade + \sqrt{-c^2de+e^2}(bex^2+bd)\log\left(\frac{c^2ex^2-c^2d-2\sqrt{-c^2de+e^2}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,e}{ex^2+d}\right) - 2\,(bc^2d^2 - bde +}{}\right.$$

$$-\frac{ac^2d^2 - ade + \sqrt{c^2de-e^2}(bex^2+bd)\arctan\left(-\frac{\sqrt{c^2de-e^2}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{c^2d-e}\right) - (bc^2d^2 - bde + (bc^2de - be^2)x^2)}{2\,(c^2d^3}$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 - 2*a*d*e + sqrt(-c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c
^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)
) + 2*e)/(e*x^2 + d)) - 2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*lo
g(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^2*d^2 - b*d*e + (b
*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) +
2*(b*c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/
(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*e
 + sqrt(c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d - e)) - (b*c^2*d^2 - b*d*e + (b*c^2*d*e
- b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (b*c^2*d^
2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 - c*x - 1) + (b*c^2*d^2 - b*d*e)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)))/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

3.105.6 Sympy [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{x(a + b\,\text{acsch}\,(cx))}{(d + ex^2)^2} \, dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

3.105.7 Maxima [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{(ex^2 + d)^2} \, dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*c^2*integrate(1/2*x/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e + (c^
2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(c^2*x^2 + 1)), x) - (2*c^2*d^2
*log(c) - 2*(c^2*d*e - e^2)*x^2*log(x) - 2*d*e*log(c) + (c^2*d*e*x^2 + c^2
*d^2)*log(c^2*x^2 + 1) - 2*(c^2*d^2 - d*e)*log(sqrt(c^2*x^2 + 1) + 1))/(c^
2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2) + log(e*x^2 + d)/(c^2*d^2 -
 d*e))*b - 1/2*a/(e^2*x^2 + d*e)
```

3.105.8 Giac [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{(ex^2 + d)^2} \, dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^2, x)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{x\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^2}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.106 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

3.106.1 Optimal result

Integrand size = 21, antiderivative size = 515

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^2}\,dx = -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{2d^2\sqrt{c^2d-e}}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^2}$$

output
```
-1/2*e*(a+b*arccsch(c*x))/d^2/(e+d/x^2)+1/2*(a+b*arccsch(c*x))^2/b/d^2-1/2
*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
-(-c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsch(c*x))*
ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/
d^2-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(
e^(1/2)+(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(1
+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylo
g(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/
d^2-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^
2*d+e)^(1/2)))/d^2+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^
(1/2))*e^(1/2)/d^2/(c^2*d-e)^(1/2)
```

3.106.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 1428, normalized size of antiderivative = 2.77

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
a/(2*d^2 + 2*d*e*x^2) + (a*Log[x])/d^2 - (a*Log[d + e*x^2])/(2*d^2) - (b*(
Pi^2 - (4*I)*Pi*ArcCsch[c*x] - (2*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[
e]*x) - (2*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 12*ArcCsch[c*x]
^2 + 4*ArcSinh[1/(c*x)] + 16*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^
2*d) + e]] - 16*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*S
qrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] -
8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (2*I)*Pi*Log[1 - (I*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*Log[
1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I
)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*Pi*Log[1 + (I*(-Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*Lo
g[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8
*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*Pi*Log[1 - (I*(Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*ArcCsch[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8
*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*Pi*Log[1 + (I*(S...
```

3.106.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 6858}$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6238}$$

$$-\int \left(\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)x}-\frac{e\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2x}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d^2}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^2}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d^2}-\frac{e\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{2bd^2}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^2}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2d^2\sqrt{c^2d-e}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(e*(a + b*ArcSinh[1/(c*x)]))/(d^2*(e + d/x^2)) + (a + b*ArcSinh[1/(c*
x)])^2/(2*b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1
/(c^2*x^2)]*x)])/(2*d^2*Sqrt[c^2*d - e]) - ((a + b*ArcSinh[1/(c*x)])*Log[1
 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^2
) - ((a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqr
t[e] - Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - (
c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^2) -
((a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1
/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt
[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^2) - (b*Pol
yLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))]
)/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[
-(c^2*d) + e])])/(2*d^2)
```

3.106. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.106.4 Maple [F]

$$\int \frac{a+b \operatorname{arccsch}(cx)}{x\,(e\,x^2+d)^2}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

3.106.5 Fricas [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\,(d+ex^2)^2}\,dx = \int \frac{b\,\operatorname{arcsch}(cx)+a}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.106.6 Sympy [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\operatorname{acsch}\left(cx\right)}{x\left(d+ex^2\right)^2}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acsch(c*x))/(x*(d + e*x**2)**2), x)
```

3.106.7 Maxima [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\operatorname{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrat
e(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

3.106.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\operatorname{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\left(d + ex^2\right)^2}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\left(e\,x^2 + d\right)^2}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.107 $\int \frac{x^4\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.107.1 Optimal result

Integrand size = 21, antiderivative size = 756

$$\int \frac{x^4(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^2}\,dx = -\frac{d(a+b\operatorname{csch}^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{d(a+b\operatorname{csch}^{-1}(cx))}{4e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{x(a+b\operatorname{csch}^{-1}(cx))}{e^2} + \frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2} + \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2} + \frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2} + \frac{3\sqrt{-d}(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d}(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}} + \frac{3\sqrt{-d}(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}} - \frac{3\sqrt{-d}(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}} - \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4e^{5/2}} - \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}} + \frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4e^{5/2}}$$

output
```
x*(a+b*arccsch(c*x))/e^2+b*arctanh((1+1/c^2/x^2)^(1/2))/c/e^2+3/4*(a+b*arc
csch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+
e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/
4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/
e^(5/2)-3/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)
-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b
*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(
1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+1/4*b*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))*d^
(1/2)/e^2/(c^2*d-e)^(1/2)+1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(
1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d-e)^(1/2)-1/4*
d*(a+b*arccsch(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsch(c*x)
)/e^2/(d/x+(-d)^(1/2)*e^(1/2))
```

3.107.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.07 (sec) , antiderivative size = 1593, normalized size of antiderivative = 2.11

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/
Sqrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*
x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c
*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*
d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/e^2 - (d*
(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[
e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e
]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]
/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(4*e^2) - (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I
)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/
4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4
*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]
]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])...
```

3.107.3 Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{x^2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)x^2}{e^2}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}+\frac{3\sqrt{-d}\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{3\sqrt{-d}\log\left(\frac{\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e^{5/2}}-\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{b\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}+\frac{b\sqrt{d}\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{c^2d-e}e^2}-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4e^{5/2}}-\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4e^{5/2}}+\frac{3b\sqrt{-d}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

3.107. $\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

output
```
-1/4*(d*(a + b*ArcSinh[1/(c*x)]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a +
 b*ArcSinh[1/(c*x)]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSinh
[1/(c*x)]))/e^2 + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*
ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 +
 1/(c^2*x^2)])])/(4*Sqrt[c^2*d - e]*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sq
rt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*
Sqrt[c^2*d - e]*e^2) + (3*Sqrt[-d]*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqr
t[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (
3*Sqrt[-d]*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)]
)/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSin
h[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*
d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*S
qrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) -
 (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqr
t[-(c^2*d) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^A
rcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*b*Sqrt[
-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d)
+ e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c
*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2))
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.107.4 Maple [F]

$$\int \frac{x^4(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^2}dx$$

input
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

output
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.107.5 Fricas [F]

$$\int \frac{x^4(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^4}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsch(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.107.6 Sympy [F]

$$\int \frac{x^4(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^4(a+b\,\mathrm{acsch}\,(cx))}{(d+ex^2)^2}\,dx$$

input
```
integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**4*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

3.107.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.107.8 Giac [F]

$$\int \frac{x^4(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^4}{(ex^2 + d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{x^4\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^2}\,dx$$

input
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.108 $\int \frac{x^2\left(a+b\,\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.108.1 Optimal result

Integrand size = 21, antiderivative size = 719

$$\int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^2}\,dx = \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\sqrt{c^2d-e}e} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4\sqrt{-d}e^{3/2}}$$

output
```
1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1+c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(
-d)^(1/2)+1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccsch(c*x))
*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))
/e^(3/2)/(-d)^(1/2)-1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/
x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)
^(1/2)-1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+
(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^
2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*
arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^
2)^(1/2))/e/d^(1/2)/(c^2*d-e)^(1/2)-1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2
)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d-e)^(1
/2)+1/4*(a+b*arccsch(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsch(c
*x))/e/(d/x+(-d)^(1/2)*e^(1/2))
```

3.108.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 1442, normalized size of antiderivative = 2.01

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d]
+ b*((2*ArcCsch[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsch[c*x])/(I*Sqrt[d
] + Sqrt[e]*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcT
an[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d)
+ e]])/Sqrt[d] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcT
an[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d)
+ e]])/Sqrt[d] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])])/Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] - (4*ArcSin[Sqrt[
1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] + (Pi*Log[1 + (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] - ((2*I)*ArcCsch[c*x
]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])
/Sqrt[d] + (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[d] + (Pi*L
og[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqr
t[d] - ((2*I)*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])])/Sqrt[d] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]
/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])])/Sqrt[d] - (Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs...
```

3.108.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6208}$$

$$-\int \left(-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{2e\left(-\frac{d^2}{x^2}-ed\right)}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{4e\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4\sqrt{-d}e^{3/2}}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4\sqrt{-d}e^{3/2}}+\frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{4e\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{4e\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4\sqrt{d}e\sqrt{c^2d-e}}-\frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4\sqrt{d}e\sqrt{c^2d-e}}$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a + b*ArcSinh[1/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSinh[1
/(c*x)])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sq
rt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*S
qrt[c^2*d - e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d - e]*e) + ((
a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] -
 Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSinh[1/(c*x)])*Lo
g[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*
Sqrt[-d]*e^(3/2)) + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSin
h[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a +
b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqr
t[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^Ar
cSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*Sqrt[-d]*e^(3/2)) + (
b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]
)])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])
/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c
*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]
*e^(3/2))
```

3.108. $\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.108.4 Maple [F]

$$\int \frac{x^{2} \left(a +b \,\mathrm{arccsch}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{2}}d x$$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

output
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.108.5 Fricas [F]

$$\int \frac{x^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx = \int \frac{{\left(b \operatorname{arcsch}\left(c x\right) + a\right)} x^{2}}{{\left(e x^{2} + d\right)}^{2}} \,d x$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.108.6 Sympy [F]

$$\int \frac{x^2(a + b\operatorname{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{x^2(a + b\operatorname{acsch}(cx))}{(d + ex^2)^2}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

3.108.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\operatorname{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.108.8 Giac [F]

$$\int \frac{x^2(a + b\operatorname{csch}^{-1}(cx))}{(d + ex^2)^2}\,dx = \int \frac{(b\operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.109 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx$

3.109.1 Optimal result

Integrand size = 18, antiderivative size = 713

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx = & -\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}\\
&+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}\\
&-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\\
&+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}}\end{aligned}$$

output
```
1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/
c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(1/2)+1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^
(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^
(1/2)-1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccsch(c*x))*ln(
1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d
)^(3/2)/e^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*
(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccsch
(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(
1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c
*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(
3/2)/e^(1/2)+1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+
1/4*(-a-b*arccsch(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsch(c*x))
/d/(d/x+(-d)^(1/2)*e^(1/2))
```

3.109.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 4.78 (sec) , antiderivative size = 1437, normalized size of antiderivative = 2.02

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{(d + ex^2)^2}\, dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

output
```
((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
 + (b*((2*Sqrt[d]*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*
ArcCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch
[c*x])/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch
[c*x])/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*ArcCsch[c*x]
*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/
Sqrt[e] - (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + (Pi*Lo
g[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqr
t[e] - ((2*I)*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])])/Sqrt[e] + (Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])])/Sqrt[e] - ((2*I)*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - (4*ArcSin[Sqrt
[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - (Pi*Log[1 + (I*(Sqrt[e] + Sq...
```

3.109.3 Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6848, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^2}dx$$

$$\Big\downarrow \ 6848$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^2}d\frac{1}{x}$$

$$\Big\downarrow \ 6238$$

$$-\int \left(\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)}-\frac{e\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^2}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{4d\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{4d\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d-e}}\right)}{4d^{3/2}\sqrt{c^2d-e}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

output
```
-1/4*(a + b*ArcSinh[1/(c*x)])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSin
h[1/(c*x)])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]
*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2
)*Sqrt[c^2*d - e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*
Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d - e]) - ((a
 + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] -
Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSinh[1/(c*x)])*L
og[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4
*(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^Arc
Sinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (
(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e]
+ Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-
d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqr
t[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^
2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSin
h[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*
PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])]
)/(4*(-d)^(3/2)*Sqrt[e])
```

3.109. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6848
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

3.109.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{\left(e\,x^{2}+d\right)^{2}}dx$$

input
```
int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

output
```
int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.109.5 Fricas [F]

$$\int \frac{a+b\mathrm{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\,\mathrm{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.109.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^2}\, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{(d + ex^2)^2}\, dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^2}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.109.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^2}\, dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)^2}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^2, x)
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

input

```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^2,x)
```

output

```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^2, x)
```

3.110 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

3.110.1 Optimal result

Integrand size = 21, antiderivative size = 758

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\operatorname{csch}^{-1}(cx)}{d^2x}+\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{be\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}}-\frac{be\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}}-\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{4(-d)^{5/2}}$$

output
```
-a/d^2/x-b*arccsch(c*x)/d^2/x-1/4*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)
/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)-1/
4*b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/
c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)-3/4*(a+b*arccsch(c*x))*ln(1-c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d
)^(5/2)+3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/
2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arccsch(c*x))*l
n(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e
^(1/2)/(-d)^(5/2)+3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(
2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^
(1/2)/(-d)^(5/2)-3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5
/2)-3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^
2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsch(c*x))/d^2/(-d/x+(-d)^
(1/2)*e^(1/2))-1/4*e*(a+b*arccsch(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+b*c*(
1+1/c^2/x^2)^(1/2)/d^2
```

3.110.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 2.83 (sec) , antiderivative size = 1487, normalized size of antiderivative = 1.96

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(S
qrt[e]*x)/Sqrt[d]] + b*(8*c*Sqrt[d]*Sqrt[1 + 1/(c^2*x^2)] - (8*Sqrt[d]*Arc
Csch[c*x])/x - (2*Sqrt[d]*e*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (
2*Sqrt[d]*e*ArcCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (24*I)*Sqrt[e]*ArcSi
n[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot
[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - (24*I)*Sqrt[e]*ArcSin
[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[
(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] + 3*Sqrt[e]*Pi*Log[1 - (
I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqr
t[e]*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
 3*Sqrt[e]*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt[e
]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] - 3*Sqrt[e]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 - (I*
(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 12*Sqrt[e]*A
rcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt...
```

3.110.3 Rubi [A] (verified)

Time = 2.56 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 6858}$$

$$-\int \frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^2x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6238}$$

$$-\int \left(\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^2}-\frac{2\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)}+\frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d^2}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$-\frac{a}{d^2x}-\frac{b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d^2x}+\frac{e\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b e \operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}}-\frac{b e \operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{5/2}\sqrt{c^2d-e}}-\frac{3\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}+\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}-\frac{3b\sqrt{e}\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{5/2}}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

3.110. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

output
```
(b*c*Sqrt[1 + 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcSinh[1/(c*x)])/(d^2*x)
 + (e*(a + b*ArcSinh[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a +
 b*ArcSinh[1/(c*x)]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2
*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)
])])/(4*d^(5/2)*Sqrt[c^2*d - e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c
^2*d - e]) - (3*Sqrt[e]*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^Arc
Sinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e
]*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e
] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSinh[1/(c*
x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-
d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (
3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^Ar
cSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*b*Sqr
t[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d)
 + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1
/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2))
```

3.110.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.110.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x^{2}\left(e\,x^{2}+d\right)^{2}}dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

3.110.5 Fricas [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\int \frac{b\,\mathrm{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^2x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.110.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^2 x^2} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{a + b \operatorname{asinh}\left(\frac{1}{cx}\right)}{x^2 (e x^2 + d)^2} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.111 $\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$

3.111.1 Optimal result

Integrand size = 21, antiderivative size = 694

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}$$
$$- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be^3} + \frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{8\left(c^2d-e\right)^{3/2}e^{5/2}}$$
$$+ \frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}e^{5/2}}$$
$$- \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e^3} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$
$$+ \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2e^3}$$

output
```
1/4*(-a-b*arccsch(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsch(c*x))/e^2/(e+d/x^2
)-(a+b*arccsch(c*x))^2/b/e^3+1/8*b*(c^2*d-2*e)*arctan((c^2*d-e)^(1/2)/c/x/
e^(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e^(5/2)-(a+b*arccsch(c*x))*ln
(1-1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*(a+
b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c
^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1
+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+
1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*b
*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1
/2)))/e^3+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/
e^(1/2)/(1+1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d-e)^(1/2)+1/8*b*c*d*(1+1/c^2/x^
2)^(1/2)/(c^2*d-e)/e^2/(e+d/x^2)/x
```

3.111.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.89 (sec) , antiderivative size = 2023, normalized size of antiderivative = 2.91

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\text{Result too large to show}$$

input
```
Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*
x^2])/(2*e^3) + b*(-1/16*(d*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d
]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*Sq
rt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Lo
g[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]
*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(
c^2*d - e)^(3/2))))/e^(5/2) - (d*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)
/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*
Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*
Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*
d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))]
)/(d*(c^2*d - e)^(3/2))))/(16*e^(5/2)) - (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*
x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt
[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c
^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d)
+ e]))/Sqrt[d]))/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/((-I)*Sqrt[
d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e
]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x)
)/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[
d]))/e^(5/2) + (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*Ar...
```

3.111.3 Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{x \left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{x\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^3}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^3\left(\frac{d}{x^2}+e\right)x}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^2\left(\frac{d}{x^2}+e\right)^2x}-\frac{d\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e\left(\frac{d}{x^2}+e\right)^3x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^3}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^3}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^3}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{2e^2\left(\frac{d}{x^2}+e\right)}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{4e\left(\frac{d}{x^2}+e\right)^2}-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{be^3}-\frac{\log\left(1-e^{-2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{e^3}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^3}+\frac{b\,\text{PolyLog}\left(2,e^{-2\text{arcsinh}\left(\frac{1}{cx}\right)}\right)}{2e^3}+\frac{b\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{5/2}\sqrt{c^2d-e}}+\frac{b(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{8e^{5/2}\left(c^2d-e\right)^{3/2}}+\frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{8e^2x\left(c^2d-e\right)\left(\frac{d}{x^2}+e\right)}$$

input
```
Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

3.111. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

output
```
(b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(8*(c^2*d - e)*e^2*(e + d/x^2)*x) - (a + b*A
rcSinh[1/(c*x)])/(4*e*(e + d/x^2)^2) - (a + b*ArcSinh[1/(c*x)])/(2*e^2*(e
+ d/x^2)) - (a + b*ArcSinh[1/(c*x)])^2/(b*e^3) + (b*(c^2*d - 2*e)*ArcTan[S
qrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*(c^2*d - e)^(3/2)*
e^(5/2)) + (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])
/(2*Sqrt[c^2*d - e]*e^(5/2)) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - E^(-2*Arc
Sinh[1/(c*x)])])/e^3 + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^Arc
Sinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcSinh[
1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d)
 + e])])/(2*e^3) + ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh
[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcSinh[1/(c
*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])])/(2*e^3) + (b*PolyLog[2, E^(-2*ArcSinh[1/(c*x)])])/(2*e^3) + (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(
2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c
^2*d) + e])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(S
qrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcS
inh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3)
```

3.111.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.111. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.111.4 Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^3}dx$$

input
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

output
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

3.111.5 Fricas [F]

$$\int \frac{x^5(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arccsch(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.111.7 Maxima [F]

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^3} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{\left(ex^2 + d\right)^3} \, dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + b*integrate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e^3*x^6
 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

3.111.8 Giac [F]

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^3} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{\left(ex^2 + d\right)^3} \, dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^3} \, dx = \int \frac{x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{\left(e \, x^2 + d\right)^3} \, dx$$

input
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.112 $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)}+\frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bc(c^2d-2e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d\left(c^2d-e\right)^{3/2}e^{3/2}\sqrt{-c^2x^2}}$$

output
```
1/4*x^4*(a+b*arccsch(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d-2*e)*x*arctanh(e^(
1/2)*(-c^2*x^2-1)^(1/2)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(3/2)/e^(3/2)/(-c^2*x
^2)^(1/2)-1/8*b*c*x*(-c^2*x^2-1)^(1/2)/(c^2*d-e)/e/(e*x^2+d)/(-c^2*x^2)^(1
/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.25

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx =$$

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2bce\sqrt{1+\frac{1}{c^2x^2}}\,x}{(-c^2d+e)(d+ex^2)}+\frac{4b(d+2ex^2)\operatorname{csch}^{-1}(cx)}{(d+ex^2)^2}-\frac{4b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}(-c^2d+2e)\log\left(\frac{16de^{3/2}\sqrt{-c^2d+e}\cdots}{b\cdots}\right)}{d(-c^2\cdots}}{16e^2}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 + 1/(c
^2*x^2)]*x)/((-(c^2*d) + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcCsch[c*x]
)/(d + e*x^2)^2 - (4*b*ArcSinh[1/(c*x)])/d + (b*Sqrt[e]*(-(c^2*d) + 2*e)*L
og[(16*d*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(-(c^2*d) + 2*e)*(I*Sqrt[d] + Sq
rt[e]*x))])/(d*(-(c^2*d) + e)^(3/2)) + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[((-
16*I)*d*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(c^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x
))])/(d*(-(c^2*d) + e)^(3/2)))/e^2
```

3.112.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6856, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3} \, dx$$

$$\Big\downarrow \text{6856}$$

$$\frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{4d \left(d+ex^2\right)^2}-\frac{bcx \int \frac{x^3}{4d\sqrt{-c^2x^2-1}\left(ex^2+d\right)^2}dx}{\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{4d \left(d+ex^2\right)^2}-\frac{bcx \int \frac{x^3}{\sqrt{-c^2x^2-1}\left(ex^2+d\right)^2}dx}{4d\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{4d \left(d+ex^2\right)^2}-\frac{bcx \int \frac{x^2}{\sqrt{-c^2x^2-1}\left(ex^2+d\right)^2}dx^2}{8d\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{87}$$

$$\frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{(c^2d-2e)\int\frac{1}{\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx^2}{2e(c^2d-e)}+\frac{d\sqrt{-c^2x^2-1}}{e(c^2d-e)(d+ex^2)}\right)}{8d\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{e(c^2d-e)(d+ex^2)}-\frac{(c^2d-2e)\int\frac{1}{-\frac{ex^4}{c^2}+d-\frac{e}{c^2}}d\sqrt{-c^2x^2-1}}{c^2e(c^2d-e)}\right)}{8d\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}}{e(c^2d-e)(d+ex^2)}-\frac{(c^2d-2e)\text{arctanh}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{e^{3/2}(c^2d-e)^{3/2}}\right)}{8d\sqrt{-c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcCsch[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*x*((d*Sqrt[-1 - c^2*
x^2])/((c^2*d - e)*e*(d + e*x^2)) - ((c^2*d - 2*e)*ArcTanh[(Sqrt[e]*Sqrt[-
1 - c^2*x^2])/Sqrt[c^2*d - e]])/((c^2*d - e)^(3/2)*e^(3/2))))/(8*d*Sqrt[-(
c^2*x^2)])
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(145) = 290.

Time = 7.63 (sec) , antiderivative size = 967, normalized size of antiderivative = 5.79

method	result
parts	$a\left(\frac{d}{4e^{2}(e\,x^{2}+d)^{2}}-\frac{1}{2e^{2}(e\,x^{2}+d)}\right)+\frac{b\left(\frac{c^{8}\,\mathrm{arccsch}(cx)\,d}{4e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}-\frac{c^{6}\,\mathrm{arccsch}(cx)}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}-\frac{c^{3}\sqrt{c^{2}x^{2}+1}\left(4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)\sqrt{-\right.}{}}{}\right.}{}$
derivativedivides	$\frac{a\,c^{6}\left(\frac{d\,c^{2}}{4e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}-\frac{1}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}\right)+b\,c^{6}\left(\frac{\mathrm{arccsch}(cx)\,d\,c^{2}}{4e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}-\frac{\mathrm{arccsch}(cx)}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}+\frac{\sqrt{c^{2}x^{2}+1}\left(-4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}-}}\right.\right.}{}\right.}{}$
default	$\frac{a\,c^{6}\left(\frac{d\,c^{2}}{4e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}-\frac{1}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}\right)+b\,c^{6}\left(\frac{\mathrm{arccsch}(cx)\,d\,c^{2}}{4e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)^{2}}-\frac{\mathrm{arccsch}(cx)}{2e^{2}\left(e\,c^{2}x^{2}+c^{2}d\right)}+\frac{\sqrt{c^{2}x^{2}+1}\left(-4\,\mathrm{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}-}}\right.\right.}{}\right.}{}$

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

3.112. $\int \frac{x^{3}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex^{2})^{3}}\,dx$

output
```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(1/4*c^8*arccsch(c*x)*d/
e^2/(c^2*e*x^2+c^2*d)^2-1/2*c^6*arccsch(c*x)/e^2/(c^2*e*x^2+c^2*d)-1/16*c^
3*(c^2*x^2+1)^(1/2)/e*(4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)
*c^4*d*e*x^2+4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*c^4*d^2-l
n(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c
*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e
)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-ln(-2
*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x
+(-c^2*d*e)^(1/2)))*x^2*c^4*d*e-ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(
1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-4*arctanh
(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*e^2*c^2*x^2-4*arctanh(1/(c^2*x^
2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*c^2*d*e-2*(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e
)^(1/2)*c^2*d*e+2*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*
e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2+2*ln(-2*((c^2*x^2+1
)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^
(1/2)))*c^2*d*e+2*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d
*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^2*e^2+2*ln(-2*(-(c^2*x^2+
1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^
(1/2)))*c^2*d*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)/(c^2
*d-e)/(c*e*x+(-c^2*d*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(145) = 290$.

Time = 0.43 (sec) , antiderivative size = 1381, normalized size of antiderivative = 8.27

$$\int \frac{x^3(a + b\operatorname{csch}^{-1}(cx))}{(d + ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 8*(a*c^4*d^3*e - 2*a*c
^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b
*d^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e + e^2)*log((c^2*e*
x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2
*e)/(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e
^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e
^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 4*(b*c^4*d^4 -
 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 +
 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b
*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)) + 2*((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e
- b*c*d^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^5*e^2 - 2*c^2*d^4*
e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3
 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*
d^2*e^2 + 4*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 + (b*c^2*d^3 + (
b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(b*c^2*d^2*e - 2*b*d*e^2)*x^2)*
sqrt(c^2*d*e - e^2)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2))/(c^2*d - e)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d
^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 ...
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.112.7 Maxima [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/8*b*((2*c^4*d^4*log(c) - 2*(c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4*log(x)
+ 2*d^2*e^2*log(c) + d^2*e^2 - (4*d^3*e*log(c) + d^3*e)*c^2 + (4*c^4*d^3*e
*log(c) + 4*d*e^3*log(c) + d*e^3 - (8*d^2*e^2*log(c) + d^2*e^2)*c^2)*x^2 +
 (c^4*d^4 - 2*c^2*d^3*e + (c^4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + 2*(c^4*d^3*e -
 2*c^2*d^2*e^2)*x^2)*log(c^2*x^2 + 1) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2
 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*x^2)*log(sqrt(c^2*x^2 + 1) + 1))/
(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*
e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2) + log(e*x^2 + d)
/(c^4*d^3 - 2*c^2*d^2*e + d*e^2) - 8*integrate(1/4*(2*c^2*e*x^3 + c^2*d*x)
/(c^2*e^4*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 + 2*d*e^3
)*x^2 + (c^2*e^4*x^6 + (2*c^2*d*e^3 + e^4)*x^4 + d^2*e^2 + (c^2*d^2*e^2 +
2*d*e^3)*x^2)*sqrt(c^2*x^2 + 1)), x)) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d
*e^3*x^2 + d^2*e^2)
```

3.112.8 Giac [F]

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.113 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.113.1 Optimal result

Integrand size = 19, antiderivative size = 205

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{bcx\arctan\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}} + \frac{bc(3c^2d-2e)\,x\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d^2\left(c^2d-e\right)^{3/2}\sqrt{e}\sqrt{-c^2x^2}}$$

output
```
1/4*(-a-b*arccsch(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*arctan((-c^2*x^2-1)^(1/2))
/d^2/e/(-c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d-2*e)*x*arctanh(e^(1/2)*(-c^2*x^2-
1)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(3/2)/e^(1/2)/(-c^2*x^2)^(1/2)+1/8
*b*c*x*(-c^2*x^2-1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)/(-c^2*x^2)^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.80

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$$
$$= \frac{1}{16}\left(-\frac{4a}{e\,(d+ex^2)^2}+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}x}{d\,(c^2d-e)\,(d+ex^2)}-\frac{4b\operatorname{csch}^{-1}(cx)}{e\,(d+ex^2)^2}+\frac{4b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d^2e}\right.$$
$$+\frac{b(3c^2d-2e)\log\left(\frac{16d^2\sqrt{e}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b(-3c^2d+2e)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}\,(-c^2d+e)^{3/2}}$$
$$\left.+\frac{b(3c^2d-2e)\log\left(-\frac{16id^2\sqrt{e}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b(3c^2d-2e)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2\sqrt{e}\,(-c^2d+e)^{3/2}}\right)$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output
```
((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d - e)
*(d + e*x^2)) - (4*b*ArcCsch[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSinh[1/(c*x
)])/(d^2*e) + (b*(3*c^2*d - 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(S
qrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))
/(b*(-3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))]])/(d^2*Sqrt[e]*(-(c^2*d) + e
)^(3/2)) + (b*(3*c^2*d - 2*e)*Log[((-16*I)*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*
(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/
(b*(3*c^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))]])/(d^2*Sqrt[e]*(-(c^2*d) + e)^
(3/2)))/16
```

3.113.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {6854, 354, 114, 27, 174, 73, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \; 6854$$

$$\frac{bcx\int \frac{1}{x\sqrt{-c^2x^2-1}(ex^2+d)^2}dx}{4e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 354$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{-c^2x^2-1}(ex^2+d)^2}dx^2}{8e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 114$$

$$\frac{bcx\left(\frac{\int \frac{-ex^2c^2+2dc^2-2e}{2x^2\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx^2}{d(c^2d-e)}+\frac{e\sqrt{-c^2x^2-1}}{d(c^2d-e)(d+ex^2)}\right)}{8e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\left(\frac{\int \frac{2\left(c^2d-e\right)-c^2ex^2}{x^2\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx^2}{2d(c^2d-e)}+\frac{e\sqrt{-c^2x^2-1}}{d(c^2d-e)(d+ex^2)}\right)}{8e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 174$$

$$\frac{bcx\left(\frac{\frac{2\left(c^2d-e\right)\int \frac{1}{x^2\sqrt{-c^2x^2-1}}dx^2}{d}-\frac{e\left(3c^2d-2e\right)\int \frac{1}{\sqrt{-c^2x^2-1}\left(ex^2+d\right)}dx^2}{d}}{2d(c^2d-e)}+\frac{e\sqrt{-c^2x^2-1}}{d(c^2d-e)(d+ex^2)}\right)}{8e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{bcx\left(\frac{\frac{2e(3c^2d-2e)\int \frac{1}{-\frac{ex^4}{c^2}+d-\frac{e}{c^2}}d\sqrt{-c^2x^2-1}}{c^2d}-\frac{4(c^2d-e)\int \frac{1}{-\frac{x^4}{c^2}-\frac{1}{c^2}}d\sqrt{-c^2x^2-1}}{c^2d}}{2d(c^2d-e)}+\frac{e\sqrt{-c^2x^2-1}}{d(c^2d-e)(d+ex^2)}\right)}{8e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{bcx\left(\frac{\frac{2e(3c^2d-2e)\int \frac{1}{-\frac{ex^4}{c^2}+d-\frac{e}{c^2}}d\sqrt{-c^2x^2-1}}{c^2d}+\frac{4\arctan\left(\sqrt{-c^2x^2-1}\right)(c^2d-e)}{d}}{2d(c^2d-e)}+\frac{e\sqrt{-c^2x^2-1}}{d(c^2d-e)(d+ex^2)}\right)}{8e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$\frac{bcx\left(\frac{\frac{4\arctan\left(\sqrt{-c^2x^2-1}\right)(c^2d-e)}{d}+\frac{2\sqrt{e}(3c^2d-2e)\operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}}{2d(c^2d-e)}+\frac{e\sqrt{-c^2x^2-1}}{d(c^2d-e)(d+ex^2)}\right)}{8e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}$$

input

```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output

```
-1/4*(a + b*ArcCsch[c*x])/(e*(d + e*x^2)^2) + (b*c*x*((e*Sqrt[-1 - c^2*x^2
])/(d*(c^2*d - e)*(d + e*x^2)) + ((4*(c^2*d - e)*ArcTan[Sqrt[-1 - c^2*x^2]
])/d + (2*(3*c^2*d - 2*e)*Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sqr
t[c^2*d - e]])/(d*Sqrt[c^2*d - e]))/(2*d*(c^2*d - e))))/(8*e*Sqrt[-(c^2*x^
2)])
```

3.113. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

3.113. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.113.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(180) = 360$.

Time = 6.26 (sec) , antiderivative size = 916, normalized size of antiderivative = 4.47

method	result
parts	$-\frac{a}{4e(e x^2+d)^2}+\frac{b\left(-\frac{c^6 \operatorname{arccsch}(cx)}{4e\left(e c^2x^2+c^2d\right)^2}-\frac{c\sqrt{c^2x^2+1}\left(4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}\, c^4 d e x^2+4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{-\frac{a c^6}{4e\left(e c^2x^2+c^2d\right)^2}+b c^6\left(-\frac{\operatorname{arccsch}(cx)}{4e\left(e c^2x^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2+1}\left(4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}\, c^4 d^2+4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{a c^6}{4e\left(e c^2x^2+c^2d\right)^2}+b c^6\left(-\frac{\operatorname{arccsch}(cx)}{4e\left(e c^2x^2+c^2d\right)^2}-\frac{\sqrt{c^2x^2+1}\left(4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\frac{c^2d-e}{e}}\, c^4 d^2+4 \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$

input
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*c^6/e/(c^2*e*x^2+c^2*d)^2*arccsch(c*x)-1/
16*c*(c^2*x^2+1)^(1/2)*(4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2
)*c^4*d*e*x^2+4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c^2*d-e)/e)^(1/2)*c^4*d^2-
3*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/
(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d*e*x^2-3*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d
-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2-
3*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)
/(c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^4*d*e-3*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*
d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*c^4*d^2+
2*(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*c^2*d*e-4*arctanh(1/(c^2*x^2+1)^(
1/2))*(-(c^2*d-e)/e)^(1/2)*e^2*c^2*x^2-4*arctanh(1/(c^2*x^2+1)^(1/2))*(-(c
^2*d-e)/e)^(1/2)*c^2*d*e+2*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e
+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-c^2*d*e)^(1/2)))*e^2*c^2*x^2+2*ln(-2*((
c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-c*e*x+(-
c^2*d*e)^(1/2)))*c^2*d*e+2*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*
e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-c^2*d*e)^(1/2)))*x^2*c^2*e^2+2*ln(-2*(-
(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*e*x+(-
c^2*d*e)^(1/2)))*c^2*d*e)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d^2/(-(c^2*d-e)/e)
^(1/2)/(c^2*d-e)/(c*e*x+(-c^2*d*e)^(1/2))/(-c*e*x+(-c^2*d*e)^(1/2)))
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(177) = 354$.

Time = 0.41 (sec) , antiderivative size = 1256, normalized size of antiderivative = 6.13

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + (3*b*c^2*d^3 + (3*b*c^
2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sq
rt(-c^2*d*e + e^2)*log((c^2*e*x^2 - c^2*d - 2*sqrt(-c^2*d*e + e^2)*c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + 2*e)/(e*x^2 + d)) - 4*(b*c^4*d^4 - 2*b*c^2*d^
3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d
^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2
)) - c*x + 1) + 4*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2
- 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d^2*e^2 + b*d*e^3)
*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4 - 2*
b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) - 2*((b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (b*c^3*d^3*e - b*c*d^2*e^2)*x)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d
^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d
^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 + (3*b*c^2*d
^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*
e^2)*x^2)*sqrt(c^2*d*e - e^2)*arctan(-sqrt(c^2*d*e - e^2)*c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2))/(c^2*d - e)) - 2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2
+ (b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e - 2*b*c^2*d
^2*e^2 + b*d*e^3)*x^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) +
2*(b*c^4*d^4 - 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 - 2*b*c^2*d*e...
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(a + b\operatorname{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^3} \, dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/8*(8*c^2*integrate(1/4*x/(c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^2*e
 + (c^2*d^2*e + 2*d*e^2)*x^2 + (c^2*e^3*x^6 + (2*c^2*d*e^2 + e^3)*x^4 + d^
2*e + (c^2*d^2*e + 2*d*e^2)*x^2)*sqrt(c^2*x^2 + 1)), x) + (2*c^2*d - e)*lo
g(e*x^2 + d)/(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2) - (2*c^4*d^4*log(c) + 2*d^2
*e^2*log(c) - d^2*e^2 - (4*d^3*e*log(c) - d^3*e)*c^2 + (c^2*d^2*e^2 - d*e^
3)*x^2 + (c^4*d^2*e^2*x^4 + 2*c^4*d^3*e*x^2 + c^4*d^4)*log(c^2*x^2 + 1) -
2*((c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 +
d*e^3)*x^2)*log(x) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2)*log(sqrt(c^2*x^2
+ 1) + 1))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3
*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2))*b -
1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2*e)
```

3.113.8 Giac [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x}{(ex^2 + d)^3} \, dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^3, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{x \left(a + b \operatorname{asinh}\left(\frac{1}{cx}\right)\right)}{(e\,x^2 + d)^3} \, dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.113. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.114 $\int \frac{a+b\,\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

3.114.1 Optimal result

Integrand size = 21, antiderivative size = 657

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^3}-\frac{b(c^2d-2e)\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{8d^3\left(c^2d-e\right)^{3/2}}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{d^3\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{2d^3}$$

output
```
1/4*e^2*(a+b*arccsch(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsch(c*x))/d^3/(e+d/x
^2)+1/2*(a+b*arccsch(c*x))^2/b/d^3-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1
+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arc
csch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+
e)^(1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1+c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*b
*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(
1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/8*b*(c^2*d-2*e)
*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*
d-e)^(3/2)+b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/
2)/d^3/(c^2*d-e)^(1/2)-1/8*b*c*e*(1+1/c^2/x^2)^(1/2)/d^2/(c^2*d-e)/(e+d/x^
2)/x
```

3.114.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 2081, normalized size of antiderivative = 3.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
a/(4*d*(d + e*x^2)^2) + a/(2*d^2*(d + e*x^2)) + (a*Log[x])/d^3 - (a*Log[d
+ e*x^2])/(2*d^3) + b*((Sqrt[e]*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sq
rt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I
)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e
)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d
- e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x))])/
(d*(c^2*d - e)^(3/2))))/(16*d^2) + (Sqrt[e]*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c
^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(
Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*
c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d]
+ Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] - I*S
qrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(16*d^2) - (((5*I)/16)*Sqrt[e]*(-(Arc
Csch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[
(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1
 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(
c^2*d) + e]))/Sqrt[d]))/d^(5/2) + (((5*I)/16)*Sqrt[e]*(-(ArcCsch[c*x]/((-I
)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]
*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^
2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e])
)/Sqrt[d]))/d^(5/2) - (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 ...
```

3.114.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^5}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3x}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2x}+\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)x}\right)d\frac{1}{x}$$

$\downarrow$ 2009

$$-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d^3}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^3}-\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d^3}+\frac{e^2\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{4d^3\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{d^3\left(\frac{d}{x^2}+e\right)}+\frac{\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)^2}{2bd^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^3}-\frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^3}+\frac{b\sqrt{e}\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{d^3\sqrt{c^2d-e}}-\frac{b\sqrt{e}(c^2d-2e)\arctan\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{8d^3\left(c^2d-e\right)^{3/2}}-\frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{8d^2x\left(c^2d-e\right)\left(\frac{d}{x^2}+e\right)}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
-1/8*(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(d^2*(c^2*d - e)*(e + d/x^2)*x) + (e^2*
(a + b*ArcSinh[1/(c*x)]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcSinh[1/(c*x
)]))/(d^3*(e + d/x^2)) + (a + b*ArcSinh[1/(c*x)])^2/(2*b*d^3) - (b*(c^2*d
- 2*e)*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)]
)/(8*d^3*(c^2*d - e)^(3/2)) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]
*Sqrt[1 + 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d - e]) - ((a + b*ArcSinh[1/(c*x
)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])
])/(2*d^3) - ((a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*
x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcSinh[1/(c*x)])*
Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(
2*d^3) - ((a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])
/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - (b*PolyLog[2, -((c*Sqrt[-d]*E^
ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^3) - (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^3)
 - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(2*d^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e
] + Sqrt[-(c^2*d) + e])])/(2*d^3)
```

3.114. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.114.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{3}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

3.114.5 Fricas [F]

$$\int \frac{a+b\mathrm{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\mathrm{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x
), x)
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.114.7 Maxima [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(
e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

3.114.8 Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^3}\,dx = \int \frac{b\operatorname{arcsch}(cx) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^3*x), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\,(d + ex^2)^3}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2 + d)^3}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.115 $\int \frac{x^4\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.115.1 Optimal result

Integrand size = 21, antiderivative size = 1106

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16(c^2d-e)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}}{16(c^2d-e)e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{-d}(a+b\text{csch}^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3(a+b\text{csch}^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}(a+b\text{csch}^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{3(a+b\text{csch}^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{3b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} + \frac{b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}(c^2d-e)^{3/2}e} - \frac{3b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2} + \frac{b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}(c^2d-e)^{3/2}e} + \frac{3(a+b\text{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\text{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3(a+b\text{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3(a+b\text{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} - \frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}} + \frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16\sqrt{-d}e^{5/2}}$$

output
```
3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccsch(c*x))*ln(1+c*
(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)
/(-d)^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccsch(c
*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/
2)))/e^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c
*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2
)/(-d)^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e
^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(1+
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2
)+1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1
+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e/d^(1/2)+1/16*b*arctanh((c^2*d+(-d)^(1
/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3
/2)/e/d^(1/2)-3/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d
-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d-e)^(1/2)-3/16*b*arctanh(
(c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2)
)/e^2/d^(1/2)/(c^2*d-e)^(1/2)+1/16*(a+b*arccsch(c*x))*(-d)^(1/2)/e^(3/2)/(
-d/x+(-d)^(1/2)*e^(1/2))^2+3/16*(a+b*arccsch(c*x))/e^2/(-d/x+(-d)^(1/2)...
```

3.115.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 2045, normalized size of antiderivative = 1.85

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*((I*c*Sqrt[
e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x
)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)
]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e]
 + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d -
 e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/e^2 - ((I/16)*Sqrt[
d]*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[
d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcS
inh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*S
qrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*
x))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/e^2
+ (5*(-(ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqr
t[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d)
+ e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x
))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*e^2) + (5*(-(ArcCsch[c*x]/((-I)*Sqr
t[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt
[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*
x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqr
t[d]))/(16*e^2) + (((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsc...
```

3.115.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 1170, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6858, 6208, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{ 6858}$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6208}$$

$$-\int \left(-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^3}-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)d^3}{8(-d)^{3/2}e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^3}-\frac{3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)d}{8e^2\left(-\frac{d^2}{x^2}-ed\right)}-\frac{3\left(a+b\text{arcsinh}\right.}{16e^2\left(\sqrt{-d}\sqrt{e}\right.}\right.$$

$\downarrow$ 2009

$$-\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16(c^2d-e)e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{-d}\sqrt{1+\frac{1}{c^2x^2}}c}{16(c^2d-e)e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{3\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{3\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{-d}\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{16e^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}(c^2d-e)^{3/2}e}-\frac{3b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2}+\frac{b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}(c^2d-e)^{3/2}e}-\frac{3b\operatorname{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16\sqrt{d}\sqrt{c^2d-e}e^2}+\frac{3\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{16\sqrt{-d}e^{5/2}}+\frac{3\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{16\sqrt{-d}e^{5/2}}-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}-\frac{3b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}+\frac{3b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16\sqrt{-d}e^{5/2}}$$

input
```
Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

3.115. $\int \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

output
```
-1/16*(b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/((c^2*d - e)*e^(3/2)*(Sqrt[-d]*
Sqrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(
3/2)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcSinh[1/(c*x)]))/(16*e
^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSinh[1/(c*x)]))/(16*e^2*
(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcSinh[1/(c*x)]))/(16*e^(3/2
)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcSinh[1/(c*x)]))/(16*e^2*(Sqrt
[-d]*Sqrt[e] + d/x)) - (3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt
[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2*d - e]*e
^2) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]
*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) - (3*b*ArcTanh[
(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*
x^2)])])/(16*Sqrt[d]*Sqrt[c^2*d - e]*e^2) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d
]*(c^2*d - e)^(3/2)*e) + (3*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E
^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
 - (3*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqr
t[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcSinh[1/(
c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) +
e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[
-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e...
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6208
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.115. $\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.115.4 Maple [F]

$$\int \frac{x^4(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^3}dx$$

input
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

output
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

3.115.5 Fricas [F]

$$\int \frac{x^4(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^4}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsch(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.115.8 Giac [F]

$$\int \frac{x^4(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^4}{(ex^2 + d)^3} dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} dx = \int \frac{x^4\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^3} dx$$

input
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.116 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

3.116.1 Optimal result

Integrand size = 21, antiderivative size = 1106

$$\int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\,(c^2d-e)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\,(c^2d-e)\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} + \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\operatorname{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(c^2d-e)^{3/2}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}(c^2d-e)^{3/2}} - \frac{b\operatorname{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e} - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{(a+b\operatorname{csch}^{-1}(cx))\log\left(1+\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} + \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}} - \frac{b\operatorname{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

$$+ \frac{b\operatorname{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{3/2}e^{3/2}}$$

output
```
-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+
1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(3/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)
*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-
e)^(3/2)-1/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsch(c*x)
)*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2))
)/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(
1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*
arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2
*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*po
lylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)
))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/
2)/e^(3/2)-1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)
^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d-e)^(1/2)-1/16*b*arctanh((c^2*
d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(
3/2)/e/(c^2*d-e)^(1/2)+1/16*(a+b*arccsch(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-
d)^(1/2)*e^(1/2))^2+1/16*(a+b*arccsch(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2...
```

3.116.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.10 (sec) , antiderivative size = 2053, normalized size of antiderivative = 1.86

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-1/16*I)*((I*c*Sqrt[e]*Sqrt[1
+ 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCs
ch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[
e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*
Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[
d] + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*
c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[
e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x
)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*
Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^
2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) -
(-(ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e]
- Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*
Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/S
qrt[-(c^2*d) + e]))/Sqrt[d])/(16*d*e) - (-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt
[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt
[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt
[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d])/(16
*d*e) + ((I/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*...
```

3.116.3 Rubi [A] (verified)

Time = 3.07 (sec) , antiderivative size = 1170, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6858, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}dx$$

$$\Big\downarrow \text{6858}$$

$$-\int \frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^2}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)}{d\left(\frac{d}{x^2}+e\right)^2}-\frac{e\left(a+b\operatorname{arcsinh}\left(\frac{1}{cx}\right)\right)}{d\left(\frac{d}{x^2}+e\right)^3}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}c}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{16de\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{16\sqrt{-d}\sqrt{e}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}-\frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e}-\frac{b\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}}-\frac{b\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\sqrt{c^2d-e}e}-\frac{b\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{3/2}\left(c^2d-e\right)^{3/2}}-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{16(-d)^{3/2}e^{3/2}}-\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{16(-d)^{3/2}e^{3/2}}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}+\frac{b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}-\frac{b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

3.116. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

output
```
-1/16*(b*c*Sqrt[1 + 1/(c^2*x^2)])/(Sqrt[-d]*(c^2*d - e)*Sqrt[e]*(Sqrt[-d]*
Sqrt[e] - d/x)) - (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqr
t[e]*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSinh[1/(c*x)])/(16*Sqrt[-d]*Sqr
t[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSinh[1/(c*x)])/(16*d*e*(Sqrt[
-d]*Sqrt[e] - d/x)) - (a + b*ArcSinh[1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[
-d]*Sqrt[e] + d/x)^2) - (a + b*ArcSinh[1/(c*x)])/(16*d*e*(Sqrt[-d]*Sqrt[e]
 + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
 - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcTanh
[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2
*x^2)])])/(16*d^(3/2)*Sqrt[c^2*d - e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*S
qrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)
*(c^2*d - e)^(3/2)) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]
*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*Sqrt[c^2*d - e]*e) -
 ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e
] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcSinh[1/(c*x
)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])
])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]
*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/
2)) + ((a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(S
qrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, ...
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6858
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

3.116. $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

3.116.4 Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^3}dx$$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

output
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

3.116.5 Fricas [F]

$$\int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
 + d^3), x)
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.116.8 Giac [F]

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{(ex^2 + d)^3} \, dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^3} \, dx = \int \frac{x^2 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^3} \, dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.117 $\int \frac{a+b\,\mathbf{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx$

3.117.1 Optimal result

Integrand size = 18, antiderivative size = 1096

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx = -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\text{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)^2} + \frac{5\left(a+b\text{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} + \frac{5b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}} + \frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d-e)^{3/2}} + \frac{5b\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}} + \frac{be\,\text{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}(c^2d-e)^{3/2}} + \frac{3\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} - \frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}} + \frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

output
```
1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1
+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(3/2)+1/16*b*e*arctanh((c^2*d+(-d)^(1
/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2
*d-e)^(3/2)+5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e
)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)+5/16*b*arctanh((c^2*d
+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5
/2)/(c^2*d-e)^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*a
rccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*
d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)
-3/16*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2
)+3/16*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2
*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*po
lylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)
))/(-d)^(5/2)/e^(1/2)+1/16*(a+b*arccsch(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d
)^(1/2)*e^(1/2))^2-5/16*(a+b*arccsch(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2)...
```

3.117.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 6.06 (sec) , antiderivative size = 2038, normalized size of antiderivative = 1.86

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

output
```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*((I*c*Sqrt[e]*Sqrt[1 +
1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch
[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]
) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sq
rt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d]
 + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*Sqrt
[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x))
 - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*
Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e
] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d -
e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - (3*(-(ArcC
sch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(
2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1
+ 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c
^2*d) + e]))/Sqrt[d]))/(16*d^2) - (3*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e]
 + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e]
 + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(
c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*d
^2) + (((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 3...
```

3.117.3 Rubi [A] (verified)

Time = 3.90 (sec) , antiderivative size = 1160, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6848, 6238, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{6848}$$

$$-\int \frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{\left(\frac{d}{x^2}+e\right)^3x^4}d\frac{1}{x}$$

$$\Big\downarrow \text{6238}$$

$$-\int \left(\frac{\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)e^2}{d^2\left(\frac{d}{x^2}+e\right)^3}-\frac{2\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)e}{d^2\left(\frac{d}{x^2}+e\right)^2}+\frac{a+b\text{arcsinh}\left(\frac{1}{cx}\right)}{d^2\left(\frac{d}{x^2}+e\right)}\right)d\frac{1}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(c^2d-e\right)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{b\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}c}{16(-d)^{3/2}\left(c^2d-e\right)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{5\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}+\frac{5\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{\sqrt{e}\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}-\frac{\sqrt{e}\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)}{16(-d)^{3/2}\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)^2}+\frac{be\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d-e\right)^{3/2}}+\frac{5b\,\text{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}}+\frac{be\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\left(c^2d-e\right)^{3/2}}+\frac{5b\,\text{arctanh}\left(\frac{dc^2+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d-e}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{16d^{5/2}\sqrt{c^2d-e}}+\frac{3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(1-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3\left(a+b\text{arcsinh}\left(\frac{1}{cx}\right)\right)\log\left(\frac{\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}c}{\sqrt{e}+\sqrt{e-c^2d}}+1\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}-\frac{3b\,\text{PolyLog}\left(2,-\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}+\frac{3b\,\text{PolyLog}\left(2,\frac{c\sqrt{-d}e^{\text{arcsinh}\left(\frac{1}{cx}\right)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{16(-d)^{5/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

3.117. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx$

output
```
-1/16*(b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/((-d)^(3/2)*(c^2*d - e)*(Sqrt[-d
]*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^
2*d - e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcSinh[1/(c*x)]))/(1
6*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcSinh[1/(c*x)]))/(1
6*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSinh[1/(c*x)]))/(16*(
-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSinh[1/(c*x)]))/(16*d
^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)
/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*
d - e]) + (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*
d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (5*b*ArcT
anh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(
c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d - e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d
]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5
/2)*(c^2*d - e)^(3/2)) + (3*(a + b*ArcSinh[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E
^ArcSinh[1/(c*x)])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]
) - (3*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(S
qrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSinh
[1/(c*x)])*Log[1 - (c*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d
) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSinh[1/(c*x)])*Log[1 + (c
*Sqrt[-d]*E^ArcSinh[1/(c*x)])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)...
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6238
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

rule 6848
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

3.117. $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx$

3.117.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(c x\right)}{\left(e\,x^{2}+d\right)^{3}}d x$$

input
```
int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

output
```
int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

3.117.5 Fricas [F]

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx=\int \frac{b\,\mathrm{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
 x)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.117.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^3}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^3, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^3}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^3}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^3, x)
```

3.118 $\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.118.1 Optimal result

Integrand size = 23, antiderivative size = 413

$$\begin{aligned}
&\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx \\
&= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} \\
&\quad -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} \\
&\quad +\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}}+\frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} \\
&\quad -\frac{2d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3} \\
&\quad +\frac{b(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{-c^2x^2}} \\
&\quad +\frac{8bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{105e^3\sqrt{-c^2x^2}}
\end{aligned}$$

output
```
1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*
arccsch(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsch(c*x))/e^3+1/1680*b*(105
*c^6*d^3+35*c^4*d^2*e+63*c^2*d*e^2-75*e^3)*x*arctan(e^(1/2)*(-c^2*x^2-1)^(
1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(-c^2*x^2)^(1/2)+8/105*b*c*d^(7/2)*x*a
rctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x^2)^(1/2)-1/8
40*b*(29*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c^3/e^2/(-c^2*x^
2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/c/e^2/(-c^2*x^2)^(1/2
)-1/1680*b*(23*c^4*d^2-12*c^2*d*e-75*e^2)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(
1/2)/c^5/e^2/(-c^2*x^2)^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.20 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.78

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$= \frac{32a(d+ex^2)^2\left(8d^2-12dex^2+15e^2x^4\right)+\frac{2be\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)\left(75e^2-2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+40e^2x^4)\right)}{c^5}+\cdots}{\cdots}$$

input
```
Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
(32*a*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4) + (2*b*e*Sqrt[1 + 1/
(c^2*x^2)]*x*(d + e*x^2)*(75*e^2 - 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^
2 + 22*d*e*x^2 + 40*e^2*x^4)))/c^5 + (b*(-128*c^4*d^4*Sqrt[1 + d/(e*x^2)]*
AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] + (e*(105*c^6*d^3 +
 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 +
(e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^
2*x^2]))/(c^5*x) + 32*b*(d + e*x^2)^2*(8*d^2 - 12*d*e*x^2 + 15*e^2*x^4)*Ar
cCsch[c*x])/(3360*e^3*Sqrt[d + e*x^2])
```

3.118.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6856, 27, 7282, 2118, 27, 171, 27, 171, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \ 6856$$

$$-\frac{bcx \int \frac{\left(ex^2+d\right)^{3/2} \left(15e^2x^4-12dex^2+8d^2\right)}{105e^3x\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2} \left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx \int \frac{\left(ex^2+d\right)^{3/2} \left(15e^2x^4-12dex^2+8d^2\right)}{x\sqrt{-c^2x^2-1}}dx}{105e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2} \left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 7282$$

$$-\frac{bcx \int \frac{\left(ex^2+d\right)^{3/2} \left(15e^2x^4-12dex^2+8d^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2} \left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 2118$$

$$-\frac{bcx \left(-\frac{\int -\frac{3e\left(ex^2+d\right)^{3/2}\left(16c^2d^2-e\left(29dc^2+25e\right)x^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{3c^2e}-\frac{5e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2} \left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\left(\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(16c^2d^2-e\left(29dc^2+25e\right)x^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{2c^2}-\frac{5e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 171$$

$$-\frac{bcx\left(\frac{\frac{e\sqrt{-c^2x^2-1}\left(29c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}-\frac{\int -\frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4-12dec^2-75e^2\right)x^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{2c^2}}{2c^2}-\frac{5e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{\sqrt{ex^2+d}\left(64c^4d^3-e\left(23d^2c^4-12dec^2-75e^2\right)x^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{4c^2}+\frac{e\sqrt{-c^2x^2-1}\left(29c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}-\frac{5e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{5/2}}{c^2}\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \ 171$$

$$-\frac{bcx\left(\frac{\frac{\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}-\frac{\int -\frac{128d^4c^6+e\left(105d^3c^6+35d^2ec^4+63de^2c^2-75e^3\right)x^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}}{4c^2}+\frac{e\sqrt{-c^2x^2-1}\left(29c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}\right.}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\left(\frac{\frac{\frac{\int \frac{128d^4c^6+e\left(105d^3c^6+35d^2ec^4+63de^2c^2-75e^3\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{e\sqrt{-c^2x^2-1}\left(29c^2d+25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{2c^2}-\cdots\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 175$$

$$-\frac{bcx\left(\frac{\frac{\frac{128c^6d^4\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+e\cdots}}{c^2}}{4c^2}\cdots}{2c^2}\cdots\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 66$$

$$-\frac{bcx\left(\frac{\frac{\frac{128c^6d^4\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+e\cdots}}{c^2}}{4c^2}\cdots}{2c^2}\cdots\right)}{210e^3\sqrt{-c^2x^2}}+\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$\Big\downarrow \; 104$$

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$-\frac{bcx\left(\frac{\frac{\frac{256c^6d^4\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e\left(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\cdots}{2c^2}\right)}{210e^3\sqrt{-c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 217

$$-\frac{bcx\left(\frac{\frac{\frac{2e\left(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}\cdots}{2c^2}\right)}{210e^3\sqrt{-c^2x^2}}$$

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$\downarrow$ 218

$$\frac{d^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}-\frac{2d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-$$

$$\frac{bcx\left(\frac{\frac{\frac{-256c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}\left(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(23c^4d^2-12c^2de-75e^2\right)\sqrt{d+ex}\cdots}{c^2}}{4c^2}\cdots}{2c^2}\right)}{210e^3\sqrt{-c^2x^2}}$$

input
```
Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

output
```
(d^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5
/2)*(a + b*ArcCsch[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]
))/(7*e^3) - (b*c*x*((-5*e*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(5/2))/c^2 + ((e
*(29*c^2*d + 25*e)*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + ((e*(23
*c^4*d^2 - 12*c^2*d*e - 75*e^2)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2 +
((-2*Sqrt[e]*(105*c^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*ArcTan[(
Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 256*c^6*d^(7/2)*ArcT
an[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(2*c^2))/(4*c^2))/(2*c^2
)))/(210*e^3*Sqrt[-(c^2*x^2)])
```

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.118. $\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

3.118.4 Maple [F]

$$\int x^5(a+b\,\text{arccsch}\,(cx))\,\sqrt{e\,x^2+d}dx$$

input
```
int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 1951, normalized size of antiderivative = 4.72

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d
^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*
e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2
*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqr
t(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 64*(15*b*c^7*e
^3*x^6 + 3*b*c^7*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 +
 d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*
x^6 + 48*a*c^7*d*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*
e^3*x^5 + 2*(11*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 - (41*b*c^6*d^2*e + 38*b*c
^4*d*e^2 - 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d)
)/(c^7*e^3), 1/3360*(64*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4
 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)
*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (105*b*c^6*d^3 + 35
*b*c^4*d^2*e + 63*b*c^2*d*e^2 - 75*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3
 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(
c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 32*(15*b*c^7*e^3*x^6 + 3*b*c^7
*d*e^2*x^4 - 4*b*c^7*d^2*e*x^2 + 8*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 48*a*c^7*d
*e^2*x^4 - 64*a*c^7*d^2*e*x^2 + 128*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(...
```

3.118.6 Sympy [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int x^5(a+b\operatorname{acsch}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**5*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.118.8 Giac [F]

$$\int x^5\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)x^5\,dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^5*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.119 $\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.119.1 Optimal result

Integrand size = 23, antiderivative size = 302

$$\begin{aligned}\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx &= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}} \\ &+ \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}} \\ &- \frac{d(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} \\ &+ \frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2} \\ &- \frac{b(15c^4d^2+10c^2de-9e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{-c^2x^2}} \\ &- \frac{2bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^2\sqrt{-c^2x^2}}\end{aligned}$$

output
```
-1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arc
csch(c*x))/e^2-1/120*b*(15*c^4*d^2+10*c^2*d*e-9*e^2)*x*arctan(e^(1/2)*(-c^
2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(-c^2*x^2)^(1/2)-2/15*b*c*d^
(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/(-c^2*x^2)^
(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/e/(-c^2*x^2)^(1/2)+1/1
20*b*(c^2*d-9*e)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(-c^2*x^2)^(1/
2)
```

3.119.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.54 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.87

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$=\frac{16a(d+ex^2)^2\left(-2d+3ex^2\right)+\frac{2be\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)\left(-9e+c^2(7d+6ex^2)\right)}{c^3}-\frac{b\left(-16c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)\right.}{}}{240e^2\sqrt{d}}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
(16*a*(d + e*x^2)^2*(-2*d + 3*e*x^2) + (2*b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
 e*x^2)*(-9*e + c^2*(7*d + 6*e*x^2)))/c^3 - (b*(-16*c^2*d^3*Sqrt[1 + d/(e*
x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] + (e*(15*c^4*
d^2 + 10*c^2*d*e - 9*e^2)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*Ap
pellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(c^3
*x) + 16*b*(d + e*x^2)^2*(-2*d + 3*e*x^2)*ArcCsch[c*x])/(240*e^2*Sqrt[d +
e*x^2])
```

3.119.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {6856, 27, 435, 171, 27, 171, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{\left(2d-3ex^2\right)\left(ex^2+d\right)^{3/2}}{15e^2x\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int\frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x\sqrt{-c^2x^2-1}}dx}{15e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx\int\frac{(2d-3ex^2)(ex^2+d)^{3/2}}{x^2\sqrt{-c^2x^2-1}}dx^2}{30e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}-\frac{\int-\frac{\sqrt{ex^2+d}\left(8c^2d^2-\left(c^2d-9e\right)ex^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{2c^2}\right)}{30e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int\frac{\sqrt{ex^2+d}\left(8c^2d^2-\left(c^2d-9e\right)ex^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}-\frac{\int-\frac{16d^3c^4+e\left(15d^2c^4+10dec^2-9e^2\right)x^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\frac{\int\frac{16d^3c^4+e\left(15d^2c^4+10dec^2-9e^2\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{30e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}$$

$$\Big\downarrow \text{175}$$

3.119. $\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$bcx\left(\frac{\frac{16c^4d^3\int\frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2+10c^2de-9e^2\right)\int\frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)}{2c^2}\right.$$
$$\overline{30e^2\sqrt{-c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\Big\downarrow$ 66

$$bcx\left(\frac{\frac{16c^4d^3\int\frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(15c^4d^2+10c^2de-9e^2\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)}{2c^2}\right.$$
$$\overline{30e^2\sqrt{-c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\Big\downarrow$ 104

$$bcx\left(\frac{\frac{32c^4d^3\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e\left(15c^4d^2+10c^2de-9e^2\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)$$
$$\overline{30e^2\sqrt{-c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\Big\downarrow$ 217

$$bcx\left(\frac{\frac{2e\left(15c^4d^2+10c^2de-9e^2\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex^2)^{3}}{2c^2}\right.$$
$$\overline{30e^2\sqrt{-c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}$$

$\Big\downarrow$ 218

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+$$
$$bcx\left(\frac{\frac{-32c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}\left(15c^4d^2+10c^2de-9e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(c^2d-9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}+\frac{3e\sqrt{-c^2x^2-1}(d+ex}{2c^2}\right.$$
$$\overline{30e^2\sqrt{-c^2x^2}}$$

3.119. $\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(
a + b*ArcCsch[c*x]))/(5*e^2) + (b*c*x*((3*e*Sqrt[-1 - c^2*x^2]*(d + e*x^2)
^(3/2))/(2*c^2) + (((c^2*d - 9*e)*e*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/c^
2 + ((-2*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*ArcTan[(Sqrt[e]*Sqrt[-1
 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 32*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^
2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(2*c^2))/(4*c^2)))/(30*e^2*Sqrt[-(c^2*x^
2)])
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.119.4 Maple [F]

$$\int x^3(a + b\,\mathrm{arccsch}\,(cx))\,\sqrt{e\,x^2 + d}dx$$

input
```
int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 1625, normalized size of antiderivative = 5.38

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e - 9
*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2
*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 -
 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e
^2*x^3 + (7*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqr
t(e*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e
+ e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(
e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4
*d^2 + 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d +
 e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4
 + (c^2*d*e + e^2)*x^2 + d*e)) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b
*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 + (6*b*c^4*e^2*x
^3 + (7*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*
x^2 + d))/(c^5*e^2), -1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*
e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)...
```

3.119.6 Sympy [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^3(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

3.119.7 Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.119.8 Giac [F]

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^3\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.120 $\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.120.1 Optimal result

Integrand size = 21, antiderivative size = 203

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{b(3c^2d-e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\sqrt{-c^2x^2}}+\frac{bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e\sqrt{-c^2x^2}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e+1/3*b*c*d^(3/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/6*b*(3*c^2*d-e)*x
*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(-c^2*x^
2)^(1/2)+1/6*b*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(-c^2*x^2)^(1/2)
```

3.120.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$=\frac{-2bd^2\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+b(3c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}x^4\sqrt{1+\frac{ex^2}{d}}\,\text{Appell}}{12cex\sqrt{1}}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
(-2*b*d^2*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -
(1/(c^2*x^2)), -(d/(e*x^2))] + b*(3*c^2*d - e)*e*Sqrt[1 + 1/(c^2*x^2)]*x^4
*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] +
2*x*Sqrt[1 + c^2*x^2]*(d + e*x^2)*(b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a*c*(d
+ e*x^2) + 2*b*c*(d + e*x^2)*ArcCsch[c*x]))/(12*c*e*x*Sqrt[1 + c^2*x^2]*Sq
rt[d + e*x^2])
```

3.120.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6854, 354, 113, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \ 6854$$
$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x\sqrt{-c^2x^2-1}}dx}{3e\sqrt{-c^2x^2}}$$
$$\Big\downarrow \ 354$$
$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x^2\sqrt{-c^2x^2-1}}dx^2}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 113

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\left(-\frac{\int -\frac{2c^2d^2+\left(3c^2d-e\right)ex^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 27

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{\int \frac{2c^2d^2+\left(3c^2d-e\right)ex^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 175

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2c^2d^2\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e(3c^2d-e)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 66

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2c^2d^2\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(3c^2d-e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 104

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e(3c^2d-e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 217

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{bcx\left(\frac{2e(3c^2d-e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

$\downarrow$ 218

3.120. $\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e} - \frac{bcx\left(\frac{-4c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}(3c^2d-e)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e\sqrt{-c^2x^2}}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) - (b*c*x*(-((e*Sqrt[-1 - c^
2*x^2]*Sqrt[d + e*x^2])/c^2) + ((-2*(3*c^2*d - e)*Sqrt[e]*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 4*c^2*d^(3/2)*ArcTan[Sqrt[d
+ e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(2*c^2)))/(6*e*Sqrt[-(c^2*x^2)])
```

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.120.4 Maple [F]

$$\int x(a+b\,\operatorname{arccsch}(cx))\,\sqrt{e\,x^{2}+d}dx$$

input
```
int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1342, normalized size of antiderivative = 6.61

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(2*b*c^3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
 d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^
4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x
^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + e^2) + 8*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x
^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e), 1/12*(b*c^3*d^(3
/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3
*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 8*d^2)/x^4) - (3*b*c^2*d - b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 +
(c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2
*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 + b*c^3*d)*sqrt(e*
x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e
*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*c^3*d)*sqrt(e*x^2 + d
))/(c^3*e), 1/24*(4*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d
*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 +
(c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d - b*e)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*
d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e...
```

3.120.6 Sympy [F]

$$\int x\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int x(a+b\operatorname{acsch}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

3.120.7 Maxima [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
1/3*((e*x^2 + d)^(3/2)*log(sqrt(c^2*x^2 + 1) + 1)/e + 3*integrate(1/3*(c^2
*e*x^3 + c^2*d*x)*sqrt(e*x^2 + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^
2 + 1) + e), x) - 3*integrate(1/3*((3*e*log(c) + e)*c^2*x^3 + (c^2*d + 3*e
*log(c))*x + 3*(c^2*e*x^3 + e*x)*log(x))*sqrt(e*x^2 + d)/(c^2*e*x^2 + e),
x))*b + 1/3*(e*x^2 + d)^(3/2)*a/e
```

3.120.8 Giac [F]

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.121 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.121.2 Mathematica [N/A]

Not integrable

Time = 8.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

3.121.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x} \, dx$$
$$\Big\downarrow \; 6866$$
$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
$Aborted
```

3.121.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.121.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b \,\text{arccsch}\,(cx)) \sqrt{e\,x^2+d}}{x}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

3.121.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

3.121.6 Sympy [N/A]

Not integrable

Time = 9.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x, x)
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.121.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

3.121.9 Mupad [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.122.2 Mathematica [N/A]

Not integrable

Time = 12.77 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

3.122.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x^3} \, dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x^3}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.122.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.122.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\ \text{arccsch}\,(cx))\sqrt{e\,x^2+d}}{x^3}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

3.122.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

3.122.6 Sympy [N/A]

Not integrable

Time = 14.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**3, x)
```

3.122.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.122.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

3.122.9 Mupad [N/A]

Not integrable

Time = 5.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.123 $\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.123.2 Mathematica [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

3.123.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$
$$\Big\downarrow \text{6866}$$
$$\int x^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)dx$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.123.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.123.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2(a+b\operatorname{arccsch}(cx))\sqrt{e\,x^2+d}dx$$

input
```
int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.123.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

3.123.6 Sympy [N/A]

Not integrable

Time = 25.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

3.123.7 Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.123.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

3.123.9 Mupad [N/A]

Not integrable

Time = 5.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.124 $\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.124.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Int}\left(\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.124.2 Mathematica [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

3.124.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.124.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.124.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(a+b\,\operatorname{arccsch}\left(c x\right)\right)\sqrt{e\,x^{2}+d}\,dx$$

input
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.124.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

3.124.6 Sympy [N/A]

Not integrable

Time = 6.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \int (a+b\,\text{acsch}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.124.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

3.124.9 Mupad [N/A]

Not integrable

Time = 5.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

3.125.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.125.2 Mathematica [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

3.125.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x^2} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{x^2}dx$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.125.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.125.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\,\operatorname{arccsch}(cx))\,\sqrt{e\,x^2+d}}{x^2}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

3.125.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.125.6 Sympy [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**2, x)
```

3.125.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{csch}^{-1}(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.125.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

3.125.9 Mupad [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 389

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx &= -\frac{2bc^3(c^2d-2e)\,x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{2bc(c^2d-2e)\,\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}}\\&\quad+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}\\&\quad+\frac{2bc^2(c^2d-2e)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\left|1-\frac{e}{c^2d}\right.\right)}{9d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\\&\quad-\frac{b(c^2d-3e)\,ex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

output
```
-1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/d/x^3-2/9*b*c^3*(c^2*d-2*e)*x^2*(e
*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-2/9*b*c*(c^2*d-2*e)*(-
c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)+1/9*b*c*(-c^2*x^2-1)^(
1/2)*(e*x^2+d)^(1/2)/x^2/(-c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d-2*e)*x*(1/(c^2*
x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d
)^(1/2))*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/
d/(c^2*x^2+1))^(1/2)-1/9*b*(c^2*d-3*e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+
1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2
)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

3.126.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.03 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{csch}^{-1}(cx))}{x^4}\,dx =$$
$$-\frac{\sqrt{d+ex^2}\left(bc\sqrt{1+\frac{1}{c^2x^2}}x(-d+2c^2dx^2-4ex^2)+3a(d+ex^2)+3b(d+ex^2)\operatorname{csch}^{-1}(cx)\right)}{9dx^3}$$
$$-\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d-2e)\,E\left(i\text{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+(-2c^4d^2+5c^2de-3e^2)\,\text{EllipticF}\,(\right.}{9\sqrt{c^2}d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
-1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d + 2*c^2*d*x^2 - 4*e
*x^2) + 3*a*(d + e*x^2) + 3*b*(d + e*x^2)*ArcCsch[c*x]))/(d*x^3) - ((I/9)*
b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 2*e)*Ell
ipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-2*c^4*d^2 + 5*c^2*d*e - 3*e^
2)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sqrt[1 + c^
2*x^2]*Sqrt[d + e*x^2])
```

3.126.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6856, 27, 376, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{csch}^{-1}(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{ 6856}$$
$$-\frac{bcx\int -\frac{(ex^2+d)^{3/2}}{3dx^4\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{(d+ex^2)^{3/2}(a+b\operatorname{csch}^{-1}(cx))}{3dx^3}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{bcx\int \frac{\left(ex^2+d\right)^{3/2}}{x^4\sqrt{-c^2x^2-1}}dx}{3d\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 376

$$\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int \frac{\left(c^2d-3e\right)ex^2+2d\left(c^2d-2e\right)}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 445

$$\frac{bcx\left(\frac{1}{3}\left(-\frac{\int \frac{de\left(2\left(c^2d-2e\right)x^2c^2+dc^2-3e\right)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d}-\frac{2\sqrt{-c^2x^2-1}\left(c^2d-2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 27

$$\frac{bcx\left(\frac{1}{3}\left(-e\int \frac{2\left(c^2d-2e\right)x^2c^2+dc^2-3e}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx-\frac{2\sqrt{-c^2x^2-1}\left(c^2d-2e\right)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d\sqrt{-c^2x^2}}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 406

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(2c^2\left(c^2d-2e\right)\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\left(c^2d-3e\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)-\frac{2\sqrt{-c^2x^2-1}\left(c^2d-2e\right)\sqrt{d+ex^2}}{x}\right)+\right.}{3d\sqrt{-c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 320

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(2c^2\left(c^2d-2e\right)\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\left(c^2d-3e\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)-\frac{2\sqrt{-c^2x^2-1}\left(c^2d-2e\right)\sqrt{}}{x}\right.\right.}{3d\sqrt{-c^2x^2}}$$
$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\Big\downarrow$ 388

3.126. $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(2c^2(c^2d-2e)\left(\frac{\int\frac{\sqrt{ex^2+d}}{(-c^2x^2-1)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)+\frac{(c^2d-3e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right)-\frac{2\sqrt{-c^2x}\cdots}{}\right.\right.}{3d\sqrt{-c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3dx^3}$$

$$\Big\downarrow \text{313}$$

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(\frac{(c^2d-3e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+2c^2(c^2d-2e)\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d+ex^2}E\left(\arctan(cx)\middle|1-\frac{e}{c^2d}\right)}{ce\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right)\right)\right)\cdots}{3d\sqrt{-c^2x^2}}$$

$$\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3dx^3}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(d*x^3) + (b*c*x*((d*Sqrt[-1
 - c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + ((-2*(c^2*d - 2*e)*Sqrt[-1 - c^2*x^
2]*Sqrt[d + e*x^2])/x - e*(2*c^2*(c^2*d - 2*e)*((x*Sqrt[d + e*x^2])/(e*Sqr
t[-1 - c^2*x^2]) - (Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])
/(c*e*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])) + ((c^2*d -
 3*e)*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1
- c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])))/3))/(3*d*Sqrt[-(c^2*x^2)
])
```

3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

3.126. $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.126. $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$

3.126.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsch}\,(cx))\,\sqrt{e\,x^2+d}}{x^4}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{d+ex^2}(a+b\mathrm{csch}^{-1}(cx))}{x^4}\,dx$$
$$= \frac{2\,(bc^6d^2-2\,bc^4de)\sqrt{-c^2}\sqrt{d}x^3E(\arcsin\left(\sqrt{-c^2}x\right)\mid\frac{e}{c^2d})-(2\,bc^6d^2-(4\,bc^4-bc^2)de-3\,be^2)\sqrt{-c^2}\sqrt{d}x^3F(}{}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/9*(2*(b*c^6*d^2 - 2*b*c^4*d*e)*sqrt(-c^2)*sqrt(d)*x^3*elliptic_e(arcsin(
sqrt(-c^2)*x), e/(c^2*d)) - (2*b*c^6*d^2 - (4*b*c^4 - b*c^2)*d*e - 3*b*e^2
)*sqrt(-c^2)*sqrt(d)*x^3*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - 3*(
b*c^2*d*e*x^2 + b*c^2*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) + 1)/(c*x)) - (3*a*c^2*d*e*x^2 + 3*a*c^2*d^2 - (b*c^3*d^2*x - 2*(b
*c^5*d^2 - 2*b*c^3*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d
))/(c^2*d^2*x^3)
```

3.126.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\mathrm{csch}^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{acsch}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**4, x)
```

3.126.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.126.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\operatorname{arcsch}(cx)+a)}{x^4}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 527

$$
\begin{aligned}
&\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx \\
&= \frac{bc^3(24c^4d^2-19c^2de-31e^2)\,x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&\quad + \frac{bc(24c^4d^2-19c^2de-31e^2)\,\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} \\
&\quad - \frac{bc(12c^2d+e)\,\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}} \\
&\quad - \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3} \\
&\quad - \frac{bc^2(24c^4d^2-19c^2de-31e^2)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&\quad + \frac{2be(6c^4d^2-4c^2de-15e^2)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}
$$

output
```
-1/5*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*
arccsch(c*x))/d^2/x^3-1/45*b*c^3*(2*c^2*d-e)*e*x^2*(e*x^2+d)^(1/2)/d^2/(-c
^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-2/15*b*c^3*e^2*x^2*(e*x^2+d)^(1/2)/d^2/(-
c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+1/75*b*c^3*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x
^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/45*b*c*(2*c^2
*d-e)*e*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)-2/15*b*c*e
^2*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/75*b*c*(8*c^4
*d^2-3*c^2*d*e-2*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1
/2)+1/25*b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4/(-c^2*x^2)^(1/2)-1/75*
b*c*(4*c^2*d-e)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+
1/45*b*c*e*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/45*
b*c^2*(2*c^2*d-e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*
x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2
)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+2/15*b*c^2*e^2*x*(1/(
c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c
^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x
^2+d)/d/(c^2*x^2+1))^(1/2)-1/75*b*c^2*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x*(1/(c^
2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2
/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2
+d)/d/(c^2*x^2+1))^(1/2)+1/75*b*c^2*(4*c^2*d-e)*e*x*(1/(c^2*x^2+1))^(1/...
```

3.127.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.99 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d+ex^2}\left(-15a(3d^2+dex^2-2e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(-31e^2x^4+dex^2(8-19c^2x^2)+3d^2(3-4c^2x^2+8\ldots\right.}{225d^2x^5} + \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(24c^4d^2-19c^2de-31e^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)\mid\frac{e}{c^2d}\right)+(-24c^6d^3+31c^4d^2e\ldots\right.}{225\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
(Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 + 1/(c^
2*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 - 19*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 +
 8*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsch[c*x]))/(225*d^2*
x^5) + ((I/225)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24
*c^4*d^2 - 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d
)] + (-24*c^6*d^3 + 31*c^4*d^2*e + 23*c^2*d*e^2 - 30*e^3)*EllipticF[I*ArcS
inh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e
*x^2])
```

3.127.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6856, 27, 442, 442, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{15d^2x^6\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{x^6\sqrt{-c^2x^2-1}}dx}{15d^2\sqrt{-c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{3d\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{5x^5}-\frac{1}{5}\int \frac{\sqrt{ex^2+d}\left(e\left(3dc^2+10e\right)x^2+d\left(12dc^2+e\right)\right)}{x^4\sqrt{-c^2x^2-1}}dx\right)}{15d^2\sqrt{-c^2x^2}}+\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int\frac{2e(6d^2c^4-4dec^2-15e^2)x^2+d(24d^2c^4-19dec^2-31e^2)}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx-\frac{d\sqrt{-c^2x^2-1}(12c^2d+e)\sqrt{d+ex^2}}{3x^3}\right)+\frac{3d\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{15d^2\sqrt{-c^2x^2}}+$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 445

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int\frac{de\left(c^2\left(24d^2c^4-19dec^2-31e^2\right)x^2+2\left(6d^2c^4-4dec^2-15e^2\right)\right)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{-c^2x^2-1}(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{-c^2x^2-1}(12}{3}\right.\right.}{15d^2\sqrt{-c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 27

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{c^2(24d^2c^4-19dec^2-31e^2)x^2+2(6d^2c^4-4dec^2-15e^2)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{-c^2x^2-1}(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{x}\right)-\frac{d\sqrt{-c^2x^2-1}(12}{3x}\right.\right.}{15d^2\sqrt{-c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 406

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(c^2(24c^4d^2-19c^2de-31e^2)\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+2(6c^4d^2-4c^2de-15e^2)\int\frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\right.\right.\right.}{15d^2\sqrt{-c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 320

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(c^2(24c^4d^2-19c^2de-31e^2)\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{2(6c^4d^2-4c^2de-15e^2)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2}\right.}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right.\right.\right.\right.}{15d^2\sqrt{-c^2x^2}}$$
$$\frac{2e(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$\downarrow$ 388

3.127. $\int\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}dx$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(c^2\left(24c^4d^2-19c^2de-31e^2\right)\left(\frac{\int\frac{\sqrt{ex^2+d}}{\left(-c^2x^2-1\right)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)+\frac{2\left(6c^4d^2-4c^2de-15e^2\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(a\right.}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right.\right.\right.\right.}{15d^2\sqrt{\cdot}}$$

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}$$

$\downarrow$ 313

$$\frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}+$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{2\left(6c^4d^2-4c^2de-15e^2\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+c^2\left(24c^4d^2-19c^2de-31e^2\right)\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d\cdot}}{c}\right.\right.\right.\right.\right.}{15}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(
3/2)*(a + b*ArcCsch[c*x]))/(15*d^2*x^3) + (b*c*x*((3*d*Sqrt[-1 - c^2*x^2]*
(d + e*x^2)^(3/2))/(5*x^5) + (-1/3*(d*(12*c^2*d + e)*Sqrt[-1 - c^2*x^2]*Sq
rt[d + e*x^2])/x^3 + (((24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*Sqrt[-1 - c^2*x^
2]*Sqrt[d + e*x^2])/x + e*(c^2*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*((x*Sqrt
[d + e*x^2])/(e*Sqrt[-1 - c^2*x^2]) - (Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(c*e*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x
^2))])) + (2*(6*c^4*d^2 - 4*c^2*d*e - 15*e^2)*Sqrt[d + e*x^2]*EllipticF[Ar
cTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1
+ c^2*x^2))])))/3)/5))/(15*d^2*Sqrt[-(c^2*x^2)])
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&& !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.127. $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$

3.127.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arccsch}\,(cx))\,\sqrt{e\,x^2+d}}{x^6}dx$$

input
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

output
```
int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{d+ex^2}(a+b\operatorname{csch}^{-1}(cx))}{x^6}\,dx =$$
$$-\frac{(24\,bc^8d^3 - 19\,bc^6d^2e - 31\,bc^4de^2)\sqrt{-c^2}\sqrt{d}x^5E(\arcsin\left(\sqrt{-c^2}x\right) \mid \frac{e}{c^2d}) - (24\,bc^8d^3 - (19\,bc^6 - 12\,bc^4)d^2}{}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/225*((24*b*c^8*d^3 - 19*b*c^6*d^2*e - 31*b*c^4*d*e^2)*sqrt(-c^2)*sqrt(d
)*x^5*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - (24*b*c^8*d^3 - (19*b*
c^6 - 12*b*c^4)*d^2*e - (31*b*c^4 + 8*b*c^2)*d*e^2 - 30*b*e^3)*sqrt(-c^2)*
sqrt(d)*x^5*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - 15*(2*b*c^2*d*e^
2*x^4 - b*c^2*d^2*e*x^2 - 3*b*c^2*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (30*a*c^2*d*e^2*x^4 - 15*a*c^2*d^2*e*x^2
 - 45*a*c^2*d^3 + (9*b*c^3*d^3*x + (24*b*c^7*d^3 - 19*b*c^5*d^2*e - 31*b*c
^3*d*e^2)*x^5 - 4*(3*b*c^5*d^3 - 2*b*c^3*d^2*e)*x^3)*sqrt((c^2*x^2 + 1)/(c
^2*x^2)))*sqrt(e*x^2 + d))/(c^2*d^3*x^5)
```

3.127.6 Sympy [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

output
```
Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**6, x)
```

3.127.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.127.8 Giac [F]

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = \int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.128 $\int x^3(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 384

$$\begin{aligned}\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=&-\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}\\&+\frac{b(13c^2d-25e)\,x\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}}\\&-\frac{d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}\\&-\frac{b(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{-c^2x^2}}\\&-\frac{2bcd^{7/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{35e^2\sqrt{-c^2x^2}}\end{aligned}$$

output
```
-1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arc
csch(c*x))/e^2-1/560*b*(35*c^6*d^3+35*c^4*d^2*e-63*c^2*d*e^2+25*e^3)*x*arc
tan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(-c^2*x^2)^(
1/2)-2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))
/e^2/(-c^2*x^2)^(1/2)+1/840*b*(13*c^2*d-25*e)*x*(e*x^2+d)^(3/2)*(-c^2*x^2-
1)^(1/2)/c^3/e/(-c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2
)/c/e/(-c^2*x^2)^(1/2)-1/560*b*(3*c^4*d^2+38*c^2*d*e-25*e^2)*x*(-c^2*x^2-1
)^(1/2)*(e*x^2+d)^(1/2)/c^5/e/(-c^2*x^2)^(1/2)
```

3.128.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.54 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.79

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{96a\left(d+ex^2\right)^3\left(-2d+5ex^2\right)+\frac{2be\sqrt{1+\frac{1}{c^2x^2}}x\left(d+ex^2\right)\left(75e^2-2c^2e\left(82d+25ex^2\right)+c^4\left(57d^2+106dex^2+40e^2x\right.\right.}{c^5}}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(96*a*(d + e*x^2)^3*(-2*d + 5*e*x^2) + (2*b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
 e*x^2)*(75*e^2 - 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 +
40*e^2*x^4)))/c^5 - (3*b*(-32*c^4*d^4*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2,
 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))] + (e*(35*c^6*d^3 + 35*c^4*d^2*e - 6
3*c^2*d*e^2 + 25*e^3)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*Appell
F1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(c^5*x)
+ 96*b*(d + e*x^2)^3*(-2*d + 5*e*x^2)*ArcCsch[c*x])/(3360*e^2*Sqrt[d + e*x
^2])
```

3.128.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.609, Rules used = {6856, 27, 435, 171, 27, 171, 27, 171, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{\left(2d-5ex^2\right)\left(ex^2+d\right)^{5/2}}{35e^2x\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x\sqrt{-c^2x^2-1}}dx}{35e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\text{csch}^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx\int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{x^2\sqrt{-c^2x^2-1}}dx^2}{70e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\text{csch}^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{5e\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}-\frac{\int -\frac{(ex^2+d)^{3/2}\left(12c^2d^2-\left(13c^2d-25e\right)ex^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{3c^2}\right)}{70e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\text{csch}^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{(ex^2+d)^{3/2}\left(12c^2d^2-\left(13c^2d-25e\right)ex^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{6c^2}+\frac{5e\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\text{csch}^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{\frac{e\sqrt{-c^2x^2-1}\left(13c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}-\frac{\int -\frac{3\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4+38dec^2-25e^2\right)x^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{2c^2}}{6c^2}+\frac{5e\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\text{csch}^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(16d^3c^4+e\left(3d^2c^4+38dec^2-25e^2\right)x^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{4c^2}+\frac{e\sqrt{-c^2x^2-1}\left(13c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}+\frac{5e\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{3c^2}\right)}{70e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{7/2}(a+b\text{csch}^{-1}(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e^2}$$

3.128. $\int x^3(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$\Big\downarrow$ 171

$$\frac{bcx\left(\frac{\frac{3\left(-\frac{\int -\frac{32d^4c^6+e\left(35d^3c^6+35d^2ec^4-63de^2c^2+25e^3\right)x^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}+\frac{e\sqrt{-c^2x^2-1}\left(13c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}\right.}{70e^2\sqrt{-c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 27

$$\frac{bcx\left(\frac{\frac{3\left(\frac{\int \frac{32d^4c^6+e\left(35d^3c^6+35d^2ec^4-63de^2c^2+25e^3\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}+\frac{e\sqrt{-c^2x^2-1}\left(13c^2d-25e\right)\left(d+ex^2\right)^{3/2}}{2c^2}}{6c^2}+\right.}{70e^2\sqrt{-c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 175

$$\frac{bcx\left(\frac{\frac{3\left(\frac{32c^6d^4\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right.}{4c^2}}{6c^2}\right.}{70e^2\sqrt{-c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$\Big\downarrow$ 66

3.128. $\int x^3(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$bcx\left(\frac{\frac{3\left(\frac{32c^6d^4\int\frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}}{4c^2}}{6c^2}\right.$$

$$\overline{70e^2\sqrt{-c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{104}$$

$$bcx\left(\frac{\frac{3\left(\frac{64c^6d^4\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}+\ldots}{6c^2}\right.$$

$$\overline{70e^2\sqrt{-c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{217}$$

$$bcx\left(\frac{\frac{3\left(\frac{2e\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\right)}{4c^2}}{6c^2}\right.$$

$$\overline{70e^2\sqrt{-c^2x^2}}$$

$$\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$\Big\downarrow \text{218}$$

3.128. $\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{(d+ex^2)^{7/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7e^2}-\frac{d(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{bcx\left(\frac{\frac{3\left(\frac{-64c^6d^{7/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}\left(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(3c^4d^2+38c^2de-25e^2\right)\sqrt{d+ex^2}}{c^2}\cdots\right)}{4c^2}}{6c^2}\cdots\right)}{70e^2\sqrt{-c^2x^2}}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(
a + b*ArcCsch[c*x]))/(7*e^2) + (b*c*x*((5*e*Sqrt[-1 - c^2*x^2]*(d + e*x^2)
^(5/2))/(3*c^2) + (((13*c^2*d - 25*e)*e*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/
2))/(2*c^2) + (3*(-((e*(3*c^4*d^2 + 38*c^2*d*e - 25*e^2)*Sqrt[-1 - c^2*x^2
]*Sqrt[d + e*x^2])/c^2) + ((-2*Sqrt[e]*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2
*d*e^2 + 25*e^3)*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])
/c - 64*c^6*d^(7/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/
(2*c^2)))/(4*c^2))/(6*c^2)))/(70*e^2*Sqrt[-(c^2*x^2)])
```

3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.128. $\int x^3 (d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$

3.128.4 Maple [F]

$$\int x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccsch}\left(c x \right)\right)d x$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.128.5 Fricas [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 1943, normalized size of antiderivative = 5.06

$$\int x^3 \left(d+e x^2\right)^{3/2} \left(a+b \text{csch}^{-1}(c x)\right) \, dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
[1/6720*(96*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^
2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*
e - 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2
*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqr
t(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 192*(5*b*c^7*e
^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d
)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^
6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^
3*x^5 + 2*(53*b*c^6*d*e^2 - 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e - 164*b*c^
4*d*e^2 + 75*b*c^2*e^3)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))
/(c^7*e^2), 1/3360*(48*b*c^7*d^(7/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4
+ 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*
sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 + 35
*b*c^4*d^2*e - 63*b*c^2*d*e^2 + 25*b*e^3)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3
 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(
c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*
d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e
^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 + (40*b*c^6*e^3*x^5 + 2*(53*...
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \left(d + e x^2\right)^{3/2} \left(a + b \operatorname{csch}^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

output
```
Timed out
```

3.128.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(d + e x^2\right)^{3/2} \left(a + b \operatorname{csch}^{-1}(cx)\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.128.8 Giac [F]

$$\int x^3 \left(d + e x^2\right)^{3/2} \left(a + b \operatorname{csch}^{-1}(cx)\right) \, dx = \int \left(e x^2 + d\right)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a) x^3 \, dx$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^3, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (d+ex^2)^{3/2} \left(a+b\operatorname{csch}^{-1}(cx)\right) dx = \int x^3 \left(e\,x^2+d\right)^{3/2} \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right) dx$$

input
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^3*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.129 $\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.129.1 Optimal result

Integrand size = 21, antiderivative size = 270

$$\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}+\frac{b(15c^4d^2-10c^2de+3e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\sqrt{-c^2x^2}}+\frac{bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{5e\sqrt{-c^2x^2}}$$

output
```
1/5*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/e+1/5*b*c*d^(5/2)*x*arctan((e*x^2+d
)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2-
10*c^2*d*e+3*e^2)*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c
^4/e^(1/2)/(-c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/
(-c^2*x^2)^(1/2)+1/40*b*(7*c^2*d-3*e)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)
/c^3/(-c^2*x^2)^(1/2)
```

3.129.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.50 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{\frac{16a\left(d+ex^2\right)^3}{e}+\frac{2b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(d+ex^2\right)\left(-3e+c^2\left(9d+2ex^2\right)\right)}{c^3}+\frac{b\left(-\frac{8c^2d^3\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{e}+\cdots\right.}{\cdots}}{80\sqrt{d+ex\cdots}}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
((16*a*(d + e*x^2)^3)/e + (2*b*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2)*(-3*e +
 c^2*(9*d + 2*e*x^2)))/c^3 + (b*((-8*c^2*d^3*Sqrt[1 + d/(e*x^2)]*AppellF1[
1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))])/e + ((15*c^4*d^2 - 10*c^2*d
*e + 3*e^2)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2,
 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/(c^3*x) + (16*b*(d
 + e*x^2)^3*ArcCsch[c*x])/e)/(80*Sqrt[d + e*x^2])
```

3.129.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {6854, 354, 113, 27, 171, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$$

$$\Big\downarrow \text{6854}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{5/2}}{x\sqrt{-c^2x^2-1}}dx}{5e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{\left(d+ex^2\right)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\int\frac{\left(ex^2+d\right)^{5/2}}{x^2\sqrt{-c^2x^2-1}}dx^2}{10e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \; 113$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\left(-\frac{\int -\frac{\sqrt{ex^2+d}\left(4c^2d^2+\left(7c^2d-3e\right)ex^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(4c^2d^2+\left(7c^2d-3e\right)ex^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \; 171$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{-\frac{\int -\frac{8d^3c^4+e\left(15d^2c^4-10dec^2+3e^2\right)x^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}-\frac{e\sqrt{-c^2x^2-1}\left(7c^2d-3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\frac{\int \frac{8d^3c^4+e\left(15d^2c^4-10dec^2+3e^2\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(7c^2d-3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \; 175$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{5e}-\frac{bcx\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(15c^4d^2-10c^2de+3e^2\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}-\frac{e\sqrt{-c^2x^2-1}\left(7c^2d-3e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3}}{2c^2}\right)}{10e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \; 66$$

3.129. $\int x(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)\,dx$

$$\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{8c^4d^3\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(15c^4d^2-10c^2de+3e^2)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}(7c^2d-3e)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3}}{2c^2}\right.}{10e\sqrt{-c^2x^2}}$$

$\downarrow$ 104

$$\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{16c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e(15c^4d^2-10c^2de+3e^2)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}(7c^2d-3e)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{2c^2}\right)}{10e\sqrt{-c^2x^2}}$$

$\downarrow$ 217

$$\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{2e(15c^4d^2-10c^2de+3e^2)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}-\frac{e\sqrt{-c^2x^2-1}(7c^2d-3e)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+ex^2)^{3}}{2c^2}\right.}{10e\sqrt{-c^2x^2}}$$

$\downarrow$ 218

$$\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5e}-\frac{bcx\left(\frac{\frac{-16c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}(15c^4d^2-10c^2de+3e^2)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}-\frac{e\sqrt{-c^2x^2-1}(7c^2d-3e)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{e\sqrt{-c^2x^2-1}(d+e}{2c^2}\right.}{10e\sqrt{-c^2x^2}}$$

input
```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) - (b*c*x*(-1/2*(e*Sqrt[-1 -
 c^2*x^2]*(d + e*x^2)^(3/2))/c^2 + (-(((7*c^2*d - 3*e)*e*Sqrt[-1 - c^2*x^2
]*Sqrt[d + e*x^2])/c^2) + ((-2*Sqrt[e]*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*A
rcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 16*c^4*d^(5/2
)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(2*c^2))/(4*c^2)))
/(10*e*Sqrt[-(c^2*x^2)])
```

3.129. $\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 113
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

3.129. $\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.129.4 Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsch}\left(cx\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 1625, normalized size of antiderivative = 6.02

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
[1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
+ d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e + 3*
b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*
e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 +
b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*
x)) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x
^3 + 3*(3*b*c^4*d*e - b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*
x^2 + d))/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*
x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 +
 d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 -
10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*
sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2
*d*e + e^2)*x^2 + d*e)) + 16*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)
*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(8
*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 + (2*b*c^4*e^2*x^3 + 3*(3*
b*c^4*d*e - b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/
(c^5*e), 1/160*(16*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*
d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^...
```

3.129.6 Sympy [F]

$$\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x(a+b\,\text{acsch}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

output
```
Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)
```

3.129.7 Maxima [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
1/5*(e*x^2 + d)^(5/2)*a/e + 1/5*((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 +
d)*log(sqrt(c^2*x^2 + 1) + 1)/e + 5*integrate(1/5*(c^2*e^2*x^5 + 2*c^2*d*e
*x^3 + c^2*d^2*x)*sqrt(e*x^2 + d)/(c^2*e*x^2 + (c^2*e*x^2 + e)*sqrt(c^2*x^
2 + 1) + e), x) - 5*integrate(1/5*((5*e^2*log(c) + e^2)*c^2*x^5 + ((5*d*e*
log(c) + 2*d*e)*c^2 + 5*e^2*log(c))*x^3 + (c^2*d^2 + 5*d*e*log(c))*x + 5*(
c^2*e^2*x^5 + (c^2*d*e + e^2)*x^3 + d*e*x)*log(x))*sqrt(e*x^2 + d)/(c^2*e*
x^2 + e), x))*b
```

3.129.8 Giac [F]

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \left(ex^2+d\right)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)x\,dx$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.130 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$

3.130.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

3.130.2 Mathematica [N/A]

Not integrable

Time = 9.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

3.130.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
```

output
```
$Aborted
```

3.130.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.130.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\operatorname{arccsch}\left(c x\right)\right)}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

3.130.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x,
 x)
```

3.130.6 Sympy [N/A]

Not integrable

Time = 65.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x, x)
```

3.130.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.130.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x, x)
```

3.130.9 Mupad [N/A]

Not integrable

Time = 5.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.131 $\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$

3.131.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

3.131.2 Mathematica [N/A]

Not integrable

Time = 13.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

3.131.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx))}{x^3} \, dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{(d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx))}{x^3}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

output
```
$Aborted
```

3.131.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.131.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}} (a+b\,\text{arccsch}\,(cx))}{x^3}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

3.131.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\operatorname{arcsch}(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^
3, x)
```

3.131.6 Sympy [N/A]

Not integrable

Time = 62.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{acsch}(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**3,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

3.131.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.131.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^3, x)
```

3.131.9 Mupad [N/A]

Not integrable

Time = 5.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^3,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.132 $\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.132.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.132.2 Mathematica [N/A]

Not integrable

Time = 10.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

3.132.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6866}$$

$$\int x^2 \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.132.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.132.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2 \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\ \text{arccsch}\left(cx\right)\right)dx$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.132.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 (d + ex^2)^{3/2} \left(a + b\text{csch}^{-1}(cx)\right) dx = \int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arcsch}(cx) + a)x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arccsch(c*x))*sqrt(e*x^2
 + d), x)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (d + ex^2)^{3/2} \left(a + b\text{csch}^{-1}(cx)\right) dx = \text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

output
```
Timed out
```

3.132.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 (d + ex^2)^{3/2} \left(a + b\text{csch}^{-1}(cx)\right) dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.132.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^2, x)
```

3.132.9 Mupad [N/A]

Not integrable

Time = 5.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\,(e\,x^2+d)^{3/2}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.133 $\int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx$

3.133.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx = \text{Int}\left((d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)), x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.133.2 Mathematica [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx = \int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

3.133.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.133.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.133.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\operatorname{arccsch}\left(c x\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.133.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx = \int (ex^2+d)^{\frac{3}{2}} (b\,\text{arcsch}\,(cx)+a)\, dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d), x
)
```

3.133.6 Sympy [N/A]

Not integrable

Time = 62.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx = \int (a+b\,\text{acsch}\,(cx)) \left(d+ex^2\right)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)
```

3.133.7 Maxima [F(-2)]

Exception generated.

$$\int (d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx)) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.133.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcsch}\left(cx\right)+a\right)dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

3.133.9 Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)\, dx = \int \left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

input
```
int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.134 $\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

3.134.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

3.134.2 Mathematica [N/A]

Not integrable

Time = 8.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]
```

3.134.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}dx$$
$$\Big\downarrow \text{6866}$$
$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

output
```
$Aborted
```

3.134.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.134.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\text{arccsch}\,(cx)\right)}{x^2}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

3.134.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^
2, x)
```

3.134.6 Sympy [N/A]

Not integrable

Time = 57.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**2,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

3.134.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.134.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^2, x)
```

3.134.9 Mupad [N/A]

Not integrable

Time = 6.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^2}\,dx = \int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^2,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.135 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$

3.135.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \operatorname{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4},x\right)$$

output
```
Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

3.135.2 Mathematica [N/A]

Not integrable

Time = 19.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

3.135.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx))}{x^4} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{(d+ex^2)^{3/2} (a+b\text{csch}^{-1}(cx))}{x^4}dx$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

output
```
$Aborted
```

3.135.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.135.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}} (a+b \operatorname{arccsch}(cx))}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

3.135.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)/x^
4, x)
```

3.135.6 Sympy [N/A]

Not integrable

Time = 66.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^4}\,dx = \int \frac{(a+b\,\text{acsch}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**4,x)
```

output
```
Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

3.135.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.135.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^4, x)
```

3.135.9 Mupad [N/A]

Not integrable

Time = 6.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx=\int \frac{(e\,x^2+d)^{3/2}\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^4,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.136 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx$

3.136.1 Optimal result

Integrand size = 23, antiderivative size = 492

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx &= \frac{bc^3(8c^4d^2-23c^2de+23e^2)\,x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\ &+ \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} \\ &- \frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} \\ &+ \frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} \\ &- \frac{bc^2(8c^4d^2-23c^2de+23e^2)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{75d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\ &+ \frac{be(4c^4d^2-11c^2de+15e^2)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

output
```
-1/5*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^(3/2)*(-c
^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)+1/75*b*c^3*(8*c^4*d^2-23*c^2*d*e+23*e
^2)*x^2*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+1/75*b*c*(8*
c^4*d^2-23*c^2*d*e+23*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2*x^2)
^(1/2)-4/75*b*c*(c^2*d-2*e)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(-c^2*x
^2)^(1/2)-1/75*b*c^2*(8*c^4*d^2-23*c^2*d*e+23*e^2)*x*(1/(c^2*x^2+1))^(1/2)
*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x
^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1)
)^(1/2)+1/75*b*e*(4*c^4*d^2-11*c^2*d*e+15*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(c^
2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d
)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^
(1/2)
```

3.136.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.59

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d+ex^2}\left(-15a(d+ex^2)^2+bc\sqrt{1+\frac{1}{c^2x^2}}x(23e^2x^4+dex^2(11-23c\right.}{75dx^5}$$
$$+\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2-23c^2de+23e^2)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)\mid\frac{e}{c^2d}\right)+(-8c^6d^3+27c^4d^2e-34c\right.}{75\sqrt{c^2}d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
(Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(23*e^
2*x^4 + d*e*x^2*(11 - 23*c^2*x^2) + d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*
b*(d + e*x^2)^2*ArcCsch[c*x]))/(75*d*x^5) + ((I/75)*b*c*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*Ellipti
cE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-8*c^6*d^3 + 27*c^4*d^2*e - 34*c^
2*d*e^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2
]*d*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.136.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6856, 27, 376, 442, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{x^6}dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{(ex^2+d)^{5/2}}{5dx^6\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(ex^2+d)^{5/2}}{x^6\sqrt{-c^2x^2-1}}dx}{5d\sqrt{-c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{376}$$

$$\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}-\frac{1}{5}\int \frac{\sqrt{ex^2+d}((c^2d-5e)ex^2+4d(c^2d-2e))}{x^4\sqrt{-c^2x^2-1}}dx\right)}{5d\sqrt{-c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{e(4d^2c^4-11dec^2+15e^2)x^2+d(8d^2c^4-23dec^2+23e^2)}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx-\frac{4d\sqrt{-c^2x^2-1}(c^2d-2e)\sqrt{d+ex^2}}{3x^3}\right)+\frac{d\sqrt{-c^2x^2-1}(d+ex^2)^{3/2}}{5x^5}\right)}{5d\sqrt{-c^2x^2}}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow \text{445}$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\int \frac{de\left(4d^2c^4+\left(8d^2c^4-23dec^2+23e^2\right)x^2c^2-11dec^2+15e^2\right)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{-c^2x^2-1}(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{x}\right)-\frac{4d\sqrt{-c^2x^2-1}(c^2d-}{3x^3}\right.\right.}{5d\sqrt{-c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{5dx^5}$$

$$\Big\downarrow 27$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\int\frac{4d^2c^4+(8d^2c^4-23dec^2+23e^2)x^2c^2-11dec^2+15e^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{-c^2x^2-1}(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{x}\right)-\frac{4d\sqrt{-c^2x^2-1}(c^2d-}{3x^3}\right.\right.}{5d\sqrt{-c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow 406$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(c^2(8c^4d^2-23c^2de+23e^2)\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+(4c^4d^2-11c^2de+15e^2)\int\frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\right.\right.\right.}{5d\sqrt{-c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow 320$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(c^2(8c^4d^2-23c^2de+23e^2)\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{(4c^4d^2-11c^2de+15e^2)\sqrt{d+ex^2}\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right.\right.\right.\right.}{5d\sqrt{-c^2x^2}}$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow 388$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(c^2(8c^4d^2-23c^2de+23e^2)\left(\frac{\int\frac{\sqrt{ex^2+d}}{(-c^2x^2-1)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)+\frac{(4c^4d^2-11c^2de+15e^2)\sqrt{d+ex^2}\operatorname{EllipticF}(\operatorname{arc}}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right.\right.\right.\right.}{5d\sqrt{-c}}$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5}$$

$$\Big\downarrow 313$$

$$\frac{bcx\left(\frac{1}{5}\left(\frac{1}{3}\left(e\left(\frac{(4c^4d^2-11c^2de+15e^2)\sqrt{d+ex^2}\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+c^2(8c^4d^2-23c^2de+23e^2)\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d+}}{ce}\right.\right.\right.\right.\right.}{5d}$$

$$\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5}$$

3.136. $\int\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}dx$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(d*x^5) + (b*c*x*((d*Sqrt[-1
 - c^2*x^2]*(d + e*x^2)^(3/2))/(5*x^5) + ((-4*d*(c^2*d - 2*e)*Sqrt[-1 - c^
2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (((8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*Sqrt
[-1 - c^2*x^2]*Sqrt[d + e*x^2])/x + e*(c^2*(8*c^4*d^2 - 23*c^2*d*e + 23*e^
2)*((x*Sqrt[d + e*x^2])/(e*Sqrt[-1 - c^2*x^2]) - (Sqrt[d + e*x^2]*Elliptic
E[ArcTan[c*x], 1 - e/(c^2*d)])/(c*e*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d
*(1 + c^2*x^2))])) + ((4*c^4*d^2 - 11*c^2*d*e + 15*e^2)*Sqrt[d + e*x^2]*El
lipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x
^2)/(d*(1 + c^2*x^2))])))/3)/5))/(5*d*Sqrt[-(c^2*x^2)])
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.136. $\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$

3.136.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\operatorname{arccsch}(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^6}\,dx =$$

$$-\frac{(8\,bc^8d^3-23\,bc^6d^2e+23\,bc^4de^2)\sqrt{-c^2}\sqrt{d}x^5E(\arcsin\left(\sqrt{-c^2}x\right)\mid\frac{e}{c^2d})-(8\,bc^8d^3-(23\,bc^6-4\,bc^4)d^2e+(2\cdots}{\cdots}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/75*((8*b*c^8*d^3 - 23*b*c^6*d^2*e + 23*b*c^4*d*e^2)*sqrt(-c^2)*sqrt(d)*
x^5*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - (8*b*c^8*d^3 - (23*b*c^6
 - 4*b*c^4)*d^2*e + (23*b*c^4 - 11*b*c^2)*d*e^2 + 15*b*e^3)*sqrt(-c^2)*sqr
t(d)*x^5*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + 15*(b*c^2*d*e^2*x^4
 + 2*b*c^2*d^2*e*x^2 + b*c^2*d^3)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) + (15*a*c^2*d*e^2*x^4 + 30*a*c^2*d^2*e*x^2 + 15
*a*c^2*d^3 - (3*b*c^3*d^3*x + (8*b*c^7*d^3 - 23*b*c^5*d^2*e + 23*b*c^3*d*e
^2)*x^5 - (4*b*c^5*d^3 - 11*b*c^3*d^2*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
))*sqrt(e*x^2 + d))/(c^2*d^2*x^5)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx=\text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**6,x)
```

output
```
Timed out
```

3.136.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.136.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^6, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx=\int \frac{(e\,x^2+d)^{3/2}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^6,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.137 $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx$

3.137.1 Optimal result

Integrand size = 23, antiderivative size = 643

$$\begin{aligned}
&\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx = \\
&-\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} \\
&-\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}} \\
&+\frac{bc(120c^4d^2-159c^2de-37e^2)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} \\
&-\frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} \\
&-\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{7dx^7}+\frac{2e(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{35d^2x^5} \\
&+\frac{bc^2(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{3675d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\
&-\frac{be(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3)\,x\sqrt{d+ex^2}\,\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}$$

output
```
-1/7*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*
arccsch(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d-11*e)*(e*x^2+d)^(3/2)*(-c^2*x^2
-1)^(1/2)/d/x^4/(-c^2*x^2)^(1/2)+1/49*b*c*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/
2)/d/x^6/(-c^2*x^2)^(1/2)-1/3675*b*c^3*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*
d*e^2+247*e^3)*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
-1/3675*b*c*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*(-c^2*x^2-1)
^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/3675*b*c*(120*c^4*d^2-159*c^
2*d*e-37*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/
3675*b*c^2*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*x*(1/(c^2*x^2
+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(
1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d
/(c^2*x^2+1))^(1/2)-1/3675*b*e*(120*c^6*d^3-249*c^4*d^2*e+71*c^2*d*e^2+210
*e^3)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^
(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)
^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

3.137.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.58

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx =$$

$$-\frac{\sqrt{d+ex^2}\left(105a(5d-2ex^2)\left(d+ex^2\right)^2+bc\sqrt{1+\frac{1}{c^2x^2}}x(247e^3x^6+de^2x^4(-71+193c^2x^2)-3d^2ex^2(61-8\cdots\right.}{3675d^2\cdots}$$

$$-\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,E\left(i\text{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)-2(120c\cdots\right.}{3675\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+\cdots}}$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

output
```
-1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
 + 1/(c^2*x^2)]*x*(247*e^3*x^6 + d*e^2*x^4*(-71 + 193*c^2*x^2) - 3*d^2*e*x
^2*(61 - 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(-5 + 6*c^2*x^2 - 8*c^4*x^4 +
16*c^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/(d^2*x^7
) - ((I/3675)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*
c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*EllipticE[I*ArcSinh[Sqr
t[c^2]*x], e/(c^2*d)] - 2*(120*c^8*d^4 - 324*c^6*d^3*e + 221*c^4*d^2*e^2 +
 88*c^2*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(S
qrt[c^2]*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.137.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 542, normalized size of antiderivative = 0.84, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {6856, 27, 442, 442, 442, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{x^8}dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{(5d-2ex^2)(ex^2+d)^{5/2}}{35d^2x^8\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{2e(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{7dx^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{(5d-2ex^2)(ex^2+d)^{5/2}}{x^8\sqrt{-c^2x^2-1}}dx}{35d^2\sqrt{-c^2x^2}}+\frac{2e(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{7dx^7}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{5d\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{7x^7}-\frac{1}{7}\int \frac{(ex^2+d)^{3/2}(e(5dc^2+14e)x^2+d(30c^2d-11e))}{x^6\sqrt{-c^2x^2-1}}dx\right)}{35d^2\sqrt{-c^2x^2}}+\frac{2e(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{35d^2x^5}-\frac{(d+ex^2)^{5/2}(a+b\text{csch}^{-1}(cx))}{7dx^7}$$

$$\Big\downarrow \text{442}$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\int\frac{\sqrt{ex^2+d}\left(2e\left(15d^2c^4-18dec^2-35e^2\right)x^2+d\left(120d^2c^4-159dec^2-37e^2\right)\right)}{x^4\sqrt{-c^2x^2-1}}dx-\frac{d\sqrt{-c^2x^2-1}\left(30c^2d-11e\right)\left(d+ex^2\right)^{3/2}}{5x^5}\right)+\frac{5d\sqrt{-c^2x^2}}{}\right.}{35d^2\sqrt{-c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 442

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{d\sqrt{-c^2x^2-1}\left(120c^4d^2-159c^2de-37e^2\right)\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int\frac{e\left(120d^3c^6-249d^2ec^4+71de^2c^2+210e^3\right)x^2+d\left(240d^3c^6-528d^2ec^4+193de^2\right.}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}\right.\right.\right.}{35d^2\sqrt{-c^2x^2}}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 445

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-\frac{\int\frac{de\left(120d^3c^6-249d^2ec^4+71de^2c^2+\left(240d^3c^6-528d^2ec^4+193de^2c^2+247e^3\right)x^2c^2+210e^3\right)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{-c^2x^2-1}\left(240c^6d^3-528c^4d^2e+1\right.}{x}\right.\right.\right.\right.}{3}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 27

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-e\int\frac{120d^3c^6-249d^2ec^4+71de^2c^2+\left(240d^3c^6-528d^2ec^4+193de^2c^2+247e^3\right)x^2c^2+210e^3}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx-\frac{\sqrt{-c^2x^2-1}\left(240c^6d^3-528c^4d^2\right.}{x}\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 406

$$bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-e\left(c^2\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\left(120c^6d^3-249c^4d^2e+71c\right.\right.\right.\right.\right.\right.$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 320

3.137. $\int\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-e\left(c^2\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\left(120c^6d^3-249c^4d^2e+71c^2de^2+\cdots\right.}{cd\sqrt{-\cdots}}\right.\right.\right.\right.\right.}{}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 388

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-e\left(c^2\left(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3\right)\left(\frac{\int\frac{\sqrt{ex^2+d}}{\left(-c^2x^2-1\right)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)+\frac{\left(120c^6d^3-249c\cdots\right.}{}\right.\right.\right.\right.\right.}{}$$

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}$$

$\downarrow$ 313

$$\frac{2e\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7}+$$

$$\frac{bcx\left(\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(-e\left(\frac{\left(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+c^2\left(240c^6d^3-528c^4d^2e+193\cdots\right.\right.\right.\right.\right.\right.}{}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

output
```
-1/7*((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(d*x^7) + (2*e*(d + e*x^2)^(
5/2)*(a + b*ArcCsch[c*x]))/(35*d^2*x^5) + (b*c*x*((5*d*Sqrt[-1 - c^2*x^2]*
(d + e*x^2)^(5/2))/(7*x^7) + (-1/5*(d*(30*c^2*d - 11*e)*Sqrt[-1 - c^2*x^2]
*(d + e*x^2)^(3/2))/x^5 + ((d*(120*c^4*d^2 - 159*c^2*d*e - 37*e^2)*Sqrt[-1
 - c^2*x^2]*Sqrt[d + e*x^2])/(3*x^3) + (-(((240*c^6*d^3 - 528*c^4*d^2*e +
193*c^2*d*e^2 + 247*e^3)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/x) - e*(c^2*(
240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*((x*Sqrt[d + e*x^2]
)/(e*Sqrt[-1 - c^2*x^2]) - (Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(
c^2*d)])/(c*e*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])) + (
(120*c^6*d^3 - 249*c^4*d^2*e + 71*c^2*d*e^2 + 210*e^3)*Sqrt[d + e*x^2]*Ell
ipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^
2)/(d*(1 + c^2*x^2))]))))/3)/5)/7))/(35*d^2*Sqrt[-(c^2*x^2)])
```

3.137. $\int \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx$

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

3.137. $\int \frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^8}\,dx$

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.137.4 Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsch}\,(cx))}{x^8}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\text{csch}^{-1}(cx))}{x^8}\,dx = \frac{(240\,bc^{10}d^4 - 528\,bc^8d^3e + 193\,bc^6d^2e^2 + 247\,bc^4de^3)\sqrt{-c^2}\sqrt{d}x^7E(\mathrm{ar}}{}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/3675*((240*b*c^10*d^4 - 528*b*c^8*d^3*e + 193*b*c^6*d^2*e^2 + 247*b*c^4*
d*e^3)*sqrt(-c^2)*sqrt(d)*x^7*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d))
- (240*b*c^10*d^4 - 24*(22*b*c^8 - 5*b*c^6)*d^3*e + (193*b*c^6 - 249*b*c^4
)*d^2*e^2 + (247*b*c^4 + 71*b*c^2)*d*e^3 + 210*b*e^4)*sqrt(-c^2)*sqrt(d)*x
^7*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + 105*(2*b*c^2*d*e^3*x^6 -
b*c^2*d^2*e^2*x^4 - 8*b*c^2*d^3*e*x^2 - 5*b*c^2*d^4)*sqrt(e*x^2 + d)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (210*a*c^2*d*e^3*x^6 - 105
*a*c^2*d^2*e^2*x^4 - 840*a*c^2*d^3*e*x^2 - 525*a*c^2*d^4 + (75*b*c^3*d^4*x
 - (240*b*c^9*d^4 - 528*b*c^7*d^3*e + 193*b*c^5*d^2*e^2 + 247*b*c^3*d*e^3)
*x^7 + (120*b*c^7*d^4 - 249*b*c^5*d^3*e + 71*b*c^3*d^2*e^2)*x^5 - 3*(30*b*
c^5*d^4 - 61*b*c^3*d^3*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 +
 d))/(c^2*d^3*x^7)
```

3.137.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**8,x)
```

output
```
Timed out
```

3.137.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{x^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.137.8 Giac [F]

$$\int \frac{(d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)}{x^8} \, dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)}{x^8} \, dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^8, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right)}{x^8} \, dx = \int \frac{(e\,x^2+d)^{3/2} \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8} \, dx$$

input
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^8,x)
```

output
```
int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.138 $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.138.1 Optimal result

Integrand size = 23, antiderivative size = 329

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} + \frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3} + \frac{b(45c^4d^2+10c^2de+9e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{-c^2x^2}} + \frac{8bcd^{5/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{15e^3\sqrt{-c^2x^2}}$$

output
```
-2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arc
csch(c*x))/e^3+1/120*b*(45*c^4*d^2+10*c^2*d*e+9*e^2)*x*arctan(e^(1/2)*(-c^
2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(-c^2*x^2)^(1/2)+8/15*b*c*d^
(5/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x^2)^
(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/e^2/(-c^2*x^2)^(1/2)+d
^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d+9*e)*x*(-c^2*x
^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e^2/(-c^2*x^2)^(1/2)
```

3.138.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 3.03 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.85

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{16a(d+ex^2)\left(8d^2-4dex^2+3e^2x^4\right)+\frac{2be\sqrt{1+\frac{1}{c^2x^2}}(d+ex^2)\left(-9ex+c^2\left(-13dx+6ex^3\right)\right)}{c^3}+\frac{b\left(-64c^2d^3\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\right.\right.}{}}{}$$

input
```
Integrate[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(16*a*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + (2*b*e*Sqrt[1 + 1/(c^2
*x^2)]*(d + e*x^2)*(-9*e*x + c^2*(-13*d*x + 6*e*x^3)))/c^3 + (b*(-64*c^2*d
^3*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2
))] + (e*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)]*x^4*Sqrt[
1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)])/Sqrt[1
+ c^2*x^2]))/(c^3*x) + 16*b*(d + e*x^2)*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*Ar
cCsch[c*x])/(240*e^3*Sqrt[d + e*x^2])
```

3.138.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {6856, 27, 7282, 2118, 27, 171, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \; 6856$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{15e^3x\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{x\sqrt{-c^2x^2-1}}dx}{15e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 7282$$

$$-\frac{bcx\int \frac{\sqrt{ex^2+d}\left(3e^2x^4-4dex^2+8d^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 2118$$

$$-\frac{bcx\left(-\frac{\int -\frac{e\sqrt{ex^2+d}\left(32c^2d^2-e\left(19dc^2+9e\right)x^2\right)}{2x^2\sqrt{-c^2x^2-1}}dx^2}{2c^2e}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\left(\frac{\int \frac{\sqrt{ex^2+d}\left(32c^2d^2-e\left(19dc^2+9e\right)x^2\right)}{x^2\sqrt{-c^2x^2-1}}dx^2}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 171$$

$$-\frac{bcx\left(\frac{\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}-\frac{\int -\frac{64d^3c^4+e\left(45d^2c^4+10dec^2+9e^2\right)x^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 27$$

3.138. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

$$-\frac{bcx\left(\frac{\frac{\int \frac{64d^3c^4+e\left(45d^2c^4+10dec^2+9e^2\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 175

$$-\frac{bcx\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(45c^4d^2+10c^2de+9e^2\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 66

$$-\frac{bcx\left(\frac{\frac{64c^4d^3\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(45c^4d^2+10c^2de+9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 104

$$-\frac{bcx\left(\frac{\frac{128c^4d^3\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e\left(45c^4d^2+10c^2de+9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}\left(d+ex^2\right)^{3/2}}{2c^2}\right)}{30e^3\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$\downarrow$ 217

3.138. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

$$-\frac{bcx\left(\frac{\frac{2e\left(45c^4d^2+10c^2de+9e^2\right)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}(d+e}{2c^2}\right.}{30e^3\sqrt{-c^2x^2}}$$

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{218}$$

$$\frac{d^2\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{2d\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-$$

$$\frac{bcx\left(\frac{\frac{-128c^4d^{5/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}\left(45c^4d^2+10c^2de+9e^2\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\left(19c^2d+9e\right)\sqrt{d+ex^2}}{c^2}}{4c^2}-\frac{3e\sqrt{-c^2x^2-1}(d+}{2c^2}\right.}{30e^3\sqrt{-c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(d^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a
 + b*ArcCsch[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(5*
e^3) - (b*c*x*((-3*e*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(2*c^2) + ((e*(
19*c^2*d + 9*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2 + ((-2*Sqrt[e]*(45
*c^4*d^2 + 10*c^2*d*e + 9*e^2)*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt
[d + e*x^2])])/c - 128*c^4*d^(5/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1
 - c^2*x^2])])/(2*c^2))/(4*c^2)))/(30*e^3*Sqrt[-(c^2*x^2)])
```

3.138. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

3.138. $\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.138.4 Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 1633, normalized size of antiderivative = 4.96

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2
 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (45*b*c^4*d^2 + 10*b*c^2*d*e + 9
*b*e^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2
*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2
 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c^
4*e^2*x^3 - (13*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))
*sqrt(e*x^2 + d))/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 + 6*c^2
*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*
sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*
b*c^4*d^2 + 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^
2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^
2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^
2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 + (6*b*c
^4*e^2*x^3 - (13*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
)*sqrt(e*x^2 + d))/(c^5*e^3), 1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(1/2*((c
^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/...
```

3.138.6 Sympy [F]

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{x^5 (a+b\,\text{acsch}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**5*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

3.138.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 (a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.138.8 Giac [F]

$$\int \frac{x^5 (a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^5}{\sqrt{ex^2 + d}}\, dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 (a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}}\, dx = \int \frac{x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{\sqrt{e\, x^2 + d}}\, dx$$

input
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.139 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.139.1 Optimal result

Integrand size = 23, antiderivative size = 229

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{b(3c^2d+e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{-c^2x^2}}-\frac{2bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^2\sqrt{-c^2x^2}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^2-1/6*b*(3*c^2*d+e)*x*arctan(e^(1
/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(-c^2*x^2)^(1/2)-2/3
*b*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/(-c^
2*x^2)^(1/2)-d*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(-c^2*x^2-1)
^(1/2)*(e*x^2+d)^(1/2)/c/e/(-c^2*x^2)^(1/2)
```

3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.74 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.03

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{4bd^2\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-be(3c^2d+e)\sqrt{1+\frac{1}{c^2x^2}}x^4\sqrt{1+\frac{ex^2}{d}}\,\text{AppellF1}}{12ce^2x\sqrt{}}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(4*b*d^2*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -(
1/(c^2*x^2)), -(d/(e*x^2))] - b*e*(3*c^2*d + e)*Sqrt[1 + 1/(c^2*x^2)]*x^4*
Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] + 2
*x*Sqrt[1 + c^2*x^2]*(d + e*x^2)*(-4*a*c*d + b*e*Sqrt[1 + 1/(c^2*x^2)]*x +
 2*a*c*e*x^2 + 2*b*c*(-2*d + e*x^2)*ArcCsch[c*x]))/(12*c*e^2*x*Sqrt[1 + c^
2*x^2]*Sqrt[d + e*x^2])
```

3.139.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {6856, 27, 435, 171, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 6856$$
$$-\frac{bcx\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2x\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{e^2}$$
$$\Big\downarrow \; 27$$
$$\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x\sqrt{-c^2x^2-1}}dx}{3e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}(a+b\text{csch}^{-1}(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{e^2}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{x^2\sqrt{-c^2x^2-1}}dx^2}{6e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{171}$$

$$\frac{bcx\left(\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}-\frac{\int -\frac{4c^2d^2+e\left(3dc^2+e\right)x^2}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int \frac{4c^2d^2+e\left(3dc^2+e\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e(3c^2d+e)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{66}$$

$$\frac{bcx\left(\frac{4c^2d^2\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e(3c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(\frac{8c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e(3c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{217}$$

3.139. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

$$\frac{bcx\left(\frac{2e(3c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}+$$
$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+$$
$$\frac{bcx\left(\frac{-8c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}\left(3c^2d+e\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^2\sqrt{-c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a +
b*ArcCsch[c*x]))/(3*e^2) + (b*c*x*((e*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/
c^2 + ((-2*Sqrt[e]*(3*c^2*d + e)*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sq
rt[d + e*x^2])])/c - 8*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1
 - c^2*x^2])])/(2*c^2)))/(6*e^2*Sqrt[-(c^2*x^2)])
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.139. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.139.4 Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arccsch}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1341, normalized size of antiderivative = 5.86

$$\int \frac{x^3(a+b\mathrm{csch}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
 d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(e)*log(8*c^
4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x
^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^
3*d^(3/2)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4
*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d + b*e)*sqrt(-e)*arctan(1/2*(2*c^2*e*
x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)
)/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)
*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2
*a*c^3*e*x^2 + b*c^2*e*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e
*x^2 + d))/(c^3*e^2), -1/24*(8*b*c^3*sqrt(-d)*d*arctan(1/2*((c^3*d + c*e)*
x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2
*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) - (3*b*c^2*d + b*e)*sqrt(e)*log(8*c
^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*
x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^...
```

3.139.6 Sympy [F]

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3(a+b\,\text{acsch}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

3.139.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.139.8 Giac [F]

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.140 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.140.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}}+\frac{bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}$$

output
```
b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e/(-c^2*x
^2)^(1/2)+b*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)
/(-c^2*x^2)^(1/2)+(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e
```

3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.80

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\frac{\sqrt{d+ex^2}\left(a-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\,\text{AppellF1}\left(\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e\left(1+c^2x^2\right)}{-c^2d+e},1+c^2x^2\right)}{\sqrt{\frac{c^2\left(d+ex^2\right)}{c^2d-e}}}+b\text{csch}^{-1}(cx)\right)}{e}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a - (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*AppellF1[1/2, -1/2, 1,
3/2, (e*(1 + c^2*x^2))/(-(c^2*d) + e), 1 + c^2*x^2])/Sqrt[(c^2*(d + e*x^2)
)/(c^2*d - e)] + b*ArcCsch[c*x]))/e
```

3.140.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {6854, 354, 140, 27, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{6854}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\int \frac{\sqrt{ex^2+d}}{x\sqrt{-c^2x^2-1}}dx}{e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{-c^2x^2-1}}dx^2}{2e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{140}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\left(e\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+\int \frac{d}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\left(e\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+d\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{66}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\left(d\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\left(2d\int\frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\left(2e\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)\right)}{2e\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{bcx\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-2\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)\right)}{2e\sqrt{-c^2x^2}}$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e - (b*c*x*((-2*Sqrt[e]*ArcTan[(Sqr
t[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 2*Sqrt[d]*ArcTan[Sqrt[d
 + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])]))/(2*e*Sqrt[-(c^2*x^2)])
```

3.140.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 140
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*d^(m + n)*f^p   Int[(a + b*x)^(m - 1)/(c + d*x)^m, x]
, x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(a + b*x
)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (GtQ[m,
0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.140.4 Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccsch}\,(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(113) = 226$.

Time = 0.34 (sec) , antiderivative size = 1064, normalized size of antiderivative = 7.88

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{4\sqrt{ex^2+d}\,bc\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+bc\sqrt{d}\log\left(\frac{(c^4d^2+6c^2de+e^2)x^4+8(c^2d^2+de)x^2-4((c^3d+ce)x^3+2cdx)\sqrt{ex^2+d}\sqrt{d}\sqrt{\frac{c^2}{c}}}{x^4}\right.}{}\right.$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) + b*c*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e
)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*log(
8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4
*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*
x^4 + 8*(c^2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 +
 d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d
)*a*c - 2*b*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 +
 d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*
x^2 + d*e)))/(c*e), 1/4*(2*b*c*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4
 + (c^2*d^2 + d*e)*x^2 + d^2)) + 4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*log(8*
c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e
*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) + e^2))/(c*e), 1/2*(b*c*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c
*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x...
```

3.140.6 Sympy [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{x(a + b\,\text{acsch}\,(cx))}{\sqrt{d + ex^2}} \, dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

3.140.7 Maxima [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{\sqrt{ex^2 + d}} \, dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
b*(sqrt(e*x^2 + d)*log(sqrt(c^2*x^2 + 1) + 1)/e + integrate((c^2*e*x^3 + c
^2*d*x)/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^2*e*x^2 +
e)*sqrt(e*x^2 + d)), x) - integrate(((e*log(c) + e)*c^2*x^3 + (c^2*d + e*l
og(c))*x + (c^2*e*x^3 + e*x)*log(x))/((c^2*e*x^2 + e)*sqrt(e*x^2 + d)), x)
) + sqrt(e*x^2 + d)*a/e
```

3.140.8 Giac [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x}{\sqrt{ex^2 + d}} \, dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} dx = \int \frac{x\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}} dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.141 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

3.141.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.141.2 Mathematica [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]
```

3.141.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.141.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.141.4 Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

3.141.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d + ex^2}}\, dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{ex^2 + dx}}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^3 + d*x), x)
```

3.141.6 Sympy [N/A]

Not integrable

Time = 7.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d + ex^2}}\, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x\sqrt{d + ex^2}}\, dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x**2)), x)
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.141.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{b\operatorname{arcsch}(cx)+a}{\sqrt{ex^2+dx}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

3.141.9 Mupad [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.142 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

3.142.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.142.2 Mathematica [N/A]

Not integrable

Time = 7.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

3.142.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

3.142.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.142.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b \,\text{arccsch}\left(c x \right)}{x^{3} \sqrt{e \,x^{2}+d}}d x$$

input
```
int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

3.142.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\, dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{ex^2 + dx^3}}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e*x^5 + d*x^3), x)
```

3.142.6 Sympy [N/A]

Not integrable

Time = 28.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x^3\sqrt{d + ex^2}}\, dx$$

input
```
integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3\sqrt{d + ex^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.142.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

3.142.9 Mupad [N/A]

Not integrable

Time = 5.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.143 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.143.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\text{Int}\left(\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.143.2 Mathematica [N/A]

Not integrable

Time = 10.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

3.143.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \; 6866$$

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.143.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.143.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2 (a+b\,\operatorname{arccsch}(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.143.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2 (a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^2}{\sqrt{ex^2 + d}} \, dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

3.143.6 Sympy [N/A]

Not integrable

Time = 13.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2 (a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \int \frac{x^2 (a + b \operatorname{acsch}(cx))}{\sqrt{d + ex^2}} \, dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 (a + b\text{csch}^{-1}(cx))}{\sqrt{d + ex^2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.143.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

3.143.9 Mupad [N/A]

Not integrable

Time = 5.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.144 $\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}} dx$

3.144.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}} dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}, x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.144.2 Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}} dx = \int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

3.144.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}} \, dx$$
$$\big\downarrow \text{ 6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.144.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.144.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.144.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \int \frac{b\,\operatorname{arcsch}\,(cx) + a}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.144.6 Sympy [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\operatorname{acsch}\,(cx)}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.144.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

3.144.9 Mupad [N/A]

Not integrable

Time = 5.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.145 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

3.145.1 Optimal result

Integrand size = 23, antiderivative size = 294

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = {} & \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}} \\ & -\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx} \\ & -\frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\ & +\frac{bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

output
```
-(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/d/x+b*c^3*x^2*(e*x^2+d)^(1/2)/d/(-c^2*
x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c
^2*x^2)^(1/2)-b*c^2*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*
x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/
(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+b*e*x*(1/(c^2*x^2+1))^(
1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*
(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*
x^2+1))^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.47

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx=\frac{\sqrt{d+ex^2}\left(-a+bc\sqrt{1+\frac{1}{c^2x^2}}x-b\text{csch}^{-1}(cx)\right)}{dx}-\frac{bce\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
(Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x - b*ArcCsch[c*x]))/(d*x
) - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sq
rt[-(e/d)]*x], (c^2*d)/e])/(d*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^
2])
```

3.145.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {6856, 25, 27, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{\sqrt{ex^2+d}}{dx^2\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{25}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{dx^2\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{\sqrt{ex^2+d}}{x^2\sqrt{-c^2x^2-1}}dx}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow 377$$

$$\frac{bcx\left(\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}-\int \frac{e\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow 27$$

$$\frac{bcx\left(\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}-e\int \frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}dx\right)}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow 324$$

$$\frac{bcx\left(\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(c^2\left(-\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)-\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)\right)}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow 320$$

$$\frac{bcx\left(\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(c^2\left(-\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)-\frac{\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)\right)}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow 388$$

$$\frac{bcx\left(\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(-\left(c^2\left(\frac{\int \frac{\sqrt{ex^2+d}}{\left(-c^2x^2-1\right)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)\right)-\frac{\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)\right)}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

$$\Big\downarrow 313$$

$$\frac{bcx\left(\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}-e\left(-\frac{\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\left(c^2\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d+ex^2}E\left(\arctan(cx)\middle|1-\frac{e}{c^2d}\right)}{ce\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)\right)\right)\right)}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}$$

3.145. $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(d*x)) + (b*c*x*((Sqrt[-1 - c^2*x
^2]*Sqrt[d + e*x^2])/x - e*(-(c^2*((x*Sqrt[d + e*x^2])/(e*Sqrt[-1 - c^2*x^
2]) - (Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(c*e*Sqrt[-1
 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]))) - (Sqrt[d + e*x^2]*Elli
pticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2
)/(d*(1 + c^2*x^2))]))))/(d*Sqrt[-(c^2*x^2)])
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.145.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arccsch}\,(cx)}{x^2\sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.61

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx =$$
$$-\frac{\sqrt{-c^2}bc^4d^{\frac{3}{2}}xE(\arcsin\left(\sqrt{-c^2}x\right) \mid \frac{e}{c^2d}) + \sqrt{ex^2 + d}bc^2d\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (bc^4d + be)\sqrt{-c^2}\sqrt{d}xF(\text{arc}}{c^2d^2x}$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-c^2)*b*c^4*d^(3/2)*x*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) +
 sqrt(e*x^2 + d)*b*c^2*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) - (b*c^4*d + b*e)*sqrt(-c^2)*sqrt(d)*x*elliptic_f(arcsin(sqrt(-c^2)*x),
e/(c^2*d)) - (b*c^3*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - a*c^2*d)*sqrt(e*x^
2 + d))/(c^2*d^2*x)
```

3.145.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.145.8 Giac [F]

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.146 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.146.1 Optimal result

Integrand size = 23, antiderivative size = 425

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = & -\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{bc(2c^2d+5e)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}} \\ & +\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3} \\ & +\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x} \\ & +\frac{bc^2(2c^2d+5e)\,x\sqrt{d+ex^2}\,E\left(\arctan(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\ & -\frac{be(c^2d+6e)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

output
```
-1/3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsch(c*x))*(e*
x^2+d)^(1/2)/d^2/x-1/9*b*c^3*(2*c^2*d+5*e)*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x
^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/9*b*c*(2*c^2*d+5*e)*(-c^2*x^2-1)^(1/2)*(e*x
^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/9*b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2
)/d/x^2/(-c^2*x^2)^(1/2)+1/9*b*c^2*(2*c^2*d+5*e)*x*(1/(c^2*x^2+1))^(1/2)*(
c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2
+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1)
)^(1/2)-1/9*b*e*(c^2*d+6*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*Elli
pticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x
^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

3.146.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.56

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx =$$

$$-\frac{\sqrt{d+ex^2}\left(3a(d-2ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}x(-d+2c^2dx^2+5ex^2)+3b(d-2ex^2)\operatorname{csch}^{-1}(cx)\right)}{9d^2x^3}$$

$$-\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d+5e)\,E\left(i\operatorname{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)-2(c^4d^2+2c^2de-3e^2)\operatorname{EllipticF}\left(i\right.\right.}{9\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/9*(Sqrt[d + e*x^2]*(3*a*(d - 2*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d
 + 2*c^2*d*x^2 + 5*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsch[c*x]))/(d^2*x^3) - (
(I/9)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d + 5*
e)*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] - 2*(c^4*d^2 + 2*c^2*d*e -
 3*e^2)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt
[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.146.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6856, 27, 442, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 6856}$$

$$-\frac{bcx\int -\frac{(d-2ex^2)\sqrt{ex^2+d}}{3d^2x^4\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bcx\int \frac{(d-2ex^2)\sqrt{ex^2+d}}{x^4\sqrt{-c^2x^2-1}}dx}{3d^2\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 442

$$\frac{bcx\left(\frac{d\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{3x^3}-\frac{1}{3}\int \frac{e(dc^2+6e)x^2+d(2dc^2+5e)}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)}{3d^2\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 445

$$\frac{bcx\left(\frac{1}{3}\left(-\frac{\int \frac{de\left(\left(2dc^2+5e\right)x^2c^2+dc^2+6e\right)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d}-\frac{\sqrt{-c^2x^2-1}(2c^2d+5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 27

$$\frac{bcx\left(\frac{1}{3}\left(-e\int \frac{(2dc^2+5e)x^2c^2+dc^2+6e}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx-\frac{\sqrt{-c^2x^2-1}(2c^2d+5e)\sqrt{d+ex^2}}{x}\right)+\frac{d\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{3x^3}\right)}{3d^2\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 406

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(c^2\left(2c^2d+5e\right)\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\left(c^2d+6e\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)-\frac{\sqrt{-c^2x^2-1}(2c^2d+5e)\sqrt{d+ex^2}}{x}\right)+\cdots}{3d^2\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 320

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(c^2\left(2c^2d+5e\right)\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{(c^2d+6e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)-\frac{\sqrt{-c^2x^2-1}(2c^2d+5e)\sqrt{\cdots}}{x}\cdots}{3d^2\sqrt{-c^2x^2}}+\frac{2e\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}$$

$\big\downarrow$ 388

3.146. $\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

$$\frac{bcx\left(\frac{1}{3}\left(-e\left(c^2(2c^2d+5e)\left(\frac{\int \frac{\sqrt{ex^2+d}}{(-c^2x^2-1)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)+\frac{(c^2d+6e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right)-\sqrt{-c^2x^2\cdots}\right.\right.}{3d^2\sqrt{-c^2x^2}}$$
$$\frac{2e\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{3dx^3}$$

$\downarrow$ 313

$$\frac{2e\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{3dx^3}+$$
$$\frac{bcx\left(\frac{1}{3}\left(-e\left(\frac{(c^2d+6e)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+c^2(2c^2d+5e)\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d+ex^2}E\left(\arctan(cx)\middle|1-\frac{e}{c^2d}\right)}{ce\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right)\right)\right)\cdots}{3d^2\sqrt{-c^2x^2}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]
*(a + b*ArcCsch[c*x]))/(3*d^2*x) + (b*c*x*((d*Sqrt[-1 - c^2*x^2]*Sqrt[d +
e*x^2])/(3*x^3) + (-(((2*c^2*d + 5*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/
x) - e*(c^2*(2*c^2*d + 5*e)*((x*Sqrt[d + e*x^2])/(e*Sqrt[-1 - c^2*x^2]) -
(Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(c*e*Sqrt[-1 - c^2
*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])) + ((c^2*d + 6*e)*Sqrt[d + e*x^
2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^2]*Sqrt[(d
+ e*x^2)/(d*(1 + c^2*x^2))])))/3))/(3*d^2*Sqrt[-(c^2*x^2)])
```

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 442
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^q/(a*g*(m + 1))), x] - Simp[1/(a*g^2*(m + 1))   Int[(g*x)
^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*2
*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*2*(p + q + 1))*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && GtQ[q, 0] && LtQ[m, -1]
&&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^2])
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.146. $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

3.146.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x^4\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.64

$$\int \frac{a+b\mathrm{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$
$$= \frac{(2\,bc^6d^2+5\,bc^4de)\sqrt{-c^2}\sqrt{d}x^3E(\arcsin\left(\sqrt{-c^2}x\right) \mid \frac{e}{c^2d}) - (2\,bc^6d^2+(5\,bc^4+bc^2)de+6\,be^2)\sqrt{-c^2}\sqrt{d}x^3F(}{}$$

input
```
integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
1/9*((2*b*c^6*d^2 + 5*b*c^4*d*e)*sqrt(-c^2)*sqrt(d)*x^3*elliptic_e(arcsin(
sqrt(-c^2)*x), e/(c^2*d)) - (2*b*c^6*d^2 + (5*b*c^4 + b*c^2)*d*e + 6*b*e^2
)*sqrt(-c^2)*sqrt(d)*x^3*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + 3*(
2*b*c^2*d*e*x^2 - b*c^2*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x)) + (6*a*c^2*d*e*x^2 - 3*a*c^2*d^2 + (b*c^3*d^2*x - (2
*b*c^5*d^2 + 5*b*c^3*d*e)*x^3)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 +
 d))/(c^2*d^3*x^3)
```

3.146.6 Sympy [F]

$$\int \frac{a+b\mathrm{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{acsch}\left(cx\right)}{x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.146.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{ex^2 + dx^4}}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2 + d}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.147 $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.147.1 Optimal result

Integrand size = 23, antiderivative size = 256

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{b(9c^2d+e)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{-c^2x^2}}-\frac{8bcd^{3/2}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$

output
```
1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3-1/6*b*(9*c^2*d+e)*x*arctan(e^(1
/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(-c^2*x^2)^(1/2)-8/3
*b*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^
2*x^2)^(1/2)-d^2*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsch(c
*x))*(e*x^2+d)^(1/2)/e^3+1/6*b*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/
(-c^2*x^2)^(1/2)
```

3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.84 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.02

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \frac{16bd^2\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-be(9c^2d+e)\sqrt{1}$$

input
```
Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(16*b*d^2*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -
(1/(c^2*x^2)), -(d/(e*x^2))] - b*e*(9*c^2*d + e)*Sqrt[1 + 1/(c^2*x^2)]*x^4
*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] +
2*x*Sqrt[1 + c^2*x^2]*(b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*(8*
d^2 + 4*d*e*x^2 - e^2*x^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsch[c
*x]))/(12*c*e^3*x*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.147.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6856, 27, 7282, 2118, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6856$$

$$-\frac{bcx\int -\frac{-e^2x^4+4dex^2+8d^2}{3e^3x\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \; 27$$

$$\frac{bcx\int\frac{-e^2x^4+4dex^2+8d^2}{x\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{3e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{7282}$$

$$\frac{bcx\int\frac{-e^2x^4+4dex^2+8d^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{2118}$$

$$\frac{bcx\left(\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}-\frac{\int-\frac{e\left(16c^2d^2+e\left(9dc^2+e\right)x^2\right)}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{c^2e}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(\frac{\int\frac{16c^2d^2+e\left(9dc^2+e\right)x^2}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{175}$$

$$\frac{bcx\left(\frac{16c^2d^2\int\frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+e\left(9c^2d+e\right)\int\frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{66}$$

$$\frac{bcx\left(\frac{16c^2d^2\int\frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\left(9c^2d+e\right)\int\frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{104}$$

3.147. $\int\frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$

$$\frac{bcx\left(\frac{32c^2d^2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e(9c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\left(\frac{2e(9c^2d+e)\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}$$

$$\Big\downarrow \text{218}$$

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{bcx\left(\frac{-32c^2d^{3/2}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2\sqrt{e}\left(9c^2d+e\right)\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}}{2c^2}+\frac{e\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{c^2}\right)}{6e^3\sqrt{-c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2])) - (2*d*Sqrt[d + e*x^2]
*(a + b*ArcCsch[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e
^3) + (b*c*x*((e*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/c^2 + ((-2*Sqrt[e]*(9
*c^2*d + e)*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c -
32*c^2*d^(3/2)*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(2*c^
2)))/(6*e^3*Sqrt[-(c^2*x^2)])
```

3.147. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

3.147. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.147.4 Maple [F]

$$\int \frac{x^5(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1719, normalized size of antiderivative = 6.71

$$\int \frac{x^5\left(a+b\mathrm{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/24*((9*b*c^2*d^2 + b*d*e + (9*b*c^2*d*e + b*e^2)*x^2)*sqrt(e)*log(8*c^4
*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2*e^2)*x^2 - 4*(2*c^4*e*x^
3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2
)) + e^2) + 8*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sqrt(e*x^2 +
 d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(b*c^3*d*e*x^2
 + b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 +
d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^
2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x
^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), 1/12*((9*b*c^2*d^2 + b*d
*e + (9*b*c^2*d*e + b*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d
+ e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^
4 + (c^2*d*e + e^2)*x^2 + d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b
*c^3*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) + 8*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2
)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2
 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^3*e^2
*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 + (b*c^2*e^2*x^3 + b*c^2*d*e*x)*sqrt
((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3), -1/
24*(32*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x...
```

3.147.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

3.147.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.147.8 Giac [F]

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{\left(b\,\text{arcsch}\,(cx)+a\right)x^5}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^5\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.148.1 Optimal result

Integrand size = 23, antiderivative size = 160

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}}+\frac{2bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}}$$

output
```
b*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(-c^2*x^2
)^(1/2)+2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)
/e^2/(-c^2*x^2)^(1/2)+d*(a+b*arccsch(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsc
h(c*x))*(e*x^2+d)^(1/2)/e^2
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 1.47 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\frac{-2bd\sqrt{1+\frac{d}{ex^2}}\sqrt{1+c^2x^2}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)+cx\left(bce\sqrt{1+\frac{1}{c^2x}}\right.}{2ce^2x}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(-2*b*d*Sqrt[1 + d/(e*x^2)]*Sqrt[1 + c^2*x^2]*AppellF1[1, 1/2, 1/2, 2, -(1
/(c^2*x^2)), -(d/(e*x^2))] + c*x*(b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x^3*Sqrt[1 +
 (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2), -((e*x^2)/d)] + 2*Sqrt[1
+ c^2*x^2]*(2*d + e*x^2)*(a + b*ArcCsch[c*x])))/(2*c*e^2*x*Sqrt[1 + c^2*x^
2]*Sqrt[d + e*x^2])
```

3.148.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6856, 27, 435, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int \frac{ex^2+2d}{e^2x\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bcx\int \frac{ex^2+2d}{x\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{e^2\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bcx\int \frac{ex^2+2d}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e^2\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{175}$$

$$-\frac{bcx\left(e\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2d\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2\right)}{2e^2\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}(a+b\text{csch}^{-1}(cx))}{e^2}+\frac{d(a+b\text{csch}^{-1}(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{66}$$

$$-\frac{bcx\left(2d\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$\Big\downarrow$ 104

$$-\frac{bcx\left(4d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}+2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}\right)}{2e^2\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$\Big\downarrow$ 217

$$-\frac{bcx\left(2e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)\right)}{2e^2\sqrt{-c^2x^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}$$

$\Big\downarrow$ 218

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{bcx\left(-\frac{2\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-4\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)\right)}{2e^2\sqrt{-c^2x^2}}$$

input

```
Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output

```
(d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*A
rcCsch[c*x]))/e^2 - (b*c*x*((-2*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2]
)/(c*Sqrt[d + e*x^2])])/c - 4*Sqrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt
[-1 - c^2*x^2])]))/(2*e^2*Sqrt[-(c^2*x^2)])
```

3.148. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

3.148. $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.148.4 Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(136) = 272$.

Time $= 0.35$ (sec) , antiderivative size $= 1274$, normalized size of antiderivative $= 7.96$

$$\int \frac{x^3(a+b\mathrm{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*
(c^4*d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 +
 d)*sqrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)
*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(b
*c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^
2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c*e*x^2 + 2*a*c*d)*sqrt(e*
x^2 + d))/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*sqrt(-e)*arctan(1/2
*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)
/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)) - 2*(b*c*e*x^2 + 2*
b*c*d)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
- (b*c*e*x^2 + b*c*d)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^
2*d^2 + d*e)*x^2 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*sqr
t(e*x^2 + d))/(c*e^3*x^2 + c*d*e^2), 1/4*(4*(b*c*e*x^2 + b*c*d)*sqrt(-d)*a
rctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2
*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + (b*e*x^2
 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*d*e + c^2
*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sq
rt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(b*c*e*x^2 + 2*b*c*d)*sqrt(e*x^2...
```

3.148.6 Sympy [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}\, dx = \int \frac{x^3(a + b\,\text{acsch}\,(cx))}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

3.148.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.148.8 Giac [F]

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}} dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^3 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{(e x^2 + d)^{3/2}} dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx$

3.149.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=-\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

output
```
-b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/d^(1/2)/(-c^2*
x^2)^(1/2)+(-a-b*arccsch(c*x))/e/(e*x^2+d)^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\frac{b\sqrt{1+\frac{d}{ex^2}}\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)-2cx\left(a+b\text{csch}^{-1}(cx)\right)}{2cex\sqrt{d+ex^2}}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(b*Sqrt[1 + d/(e*x^2)]*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2
))] - 2*c*x*(a + b*ArcCsch[c*x]))/(2*c*e*x*Sqrt[d + e*x^2])
```

3.149.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6854, 354, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 6854$$

$$\frac{bcx\int \frac{1}{x\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 354$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{2e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 104$$

$$\frac{bcx\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}}{e\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 217$$

$$-\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}e\sqrt{-c^2x^2}}$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2
]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(Sqrt[d]*e*Sqrt[-(c^2*x^2)])
```

3.149.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.149.4 Maple [F]

$$\int \frac{x(a+b\ \mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(70) = 140$.

Time = 0.29 (sec) , antiderivative size = 368, normalized size of antiderivative = 4.49

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{ex^2+d}bd\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 4\sqrt{ex^2+d}ad - (bex^2+bd)\sqrt{d}\log\left(\frac{(c^4d}{}\right.}{4(de^2x^2+d^2}\right.$$

$$-\frac{2\sqrt{ex^2+d}bd\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 2\sqrt{ex^2+d}ad + (bex^2+bd)\sqrt{-d}\arctan\left(\frac{((c^3d+ce)x^3+2cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{c}}{2(c^2dex^4+(c^2d^2+de)x^2+d^2)}\right.}{2(de^2x^2+d^2e)}$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 + 6*
c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*
x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4))/(d
*e^2*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d + (b*e*x^2 + b*d)*sqrt(-d)
*arctan(1/2*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c
^2*x^2 + 1)/(c^2*x^2))/(c^2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)))/(d*e^2*
x^2 + d^2*e)]
```

3.149.6 Sympy [F]

$$\int \frac{x(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x(a + b\,\text{acsch}\,(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

3.149.7 Maxima [F]

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
-(c^2*integrate(x/((c^2*e*x^2 + e)*sqrt(c^2*x^2 + 1)*sqrt(e*x^2 + d) + (c^
2*e*x^2 + e)*sqrt(e*x^2 + d)), x) + log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(e*x^2
 + d)*e) + integrate(((e*log(c) - e)*c^2*x^3 - (c^2*d - e*log(c))*x + (c^2
*e*x^3 + e*x)*log(x))/((c^2*e^2*x^4 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(e*x^
2 + d)), x))*b - a/(sqrt(e*x^2 + d)*e)
```

3.149.8 Giac [F]

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx=\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.150 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

3.150.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.150.2 Mathematica [N/A]

Not integrable

Time = 11.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

3.150.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.150.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.150.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsch}\left(c x\right)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}d x$$

input
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

3.150.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\operatorname{arcsch}(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x
), x)
```

3.150.6 Sympy [N/A]

Not integrable

Time = 70.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\operatorname{acsch}(cx)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.150.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\,(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

3.150.9 Mupad [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\,(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

3.151.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.151.2 Mathematica [N/A]

Not integrable

Time = 14.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

3.151.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3 (d+ex^2)^{3/2}} \, dx$$
$$\Big\downarrow \; 6866$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3 (d+ex^2)^{3/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

3.151.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.151.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsch}\left(c x\right)}{x^{3} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

3.151.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x
^3), x)
```

3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.151.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} \, dx = \int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

3.151.9 Mupad [N/A]

Not integrable

Time = 5.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} \, dx = \int \frac{a + b \operatorname{asinh}\left(\frac{1}{c x}\right)}{x^3 (e x^2 + d)^{3/2}} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.152 $\int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.152.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.152.2 Mathematica [N/A]

Not integrable

Time = 15.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

3.152.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 6866$$

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}dx$$

input
```
Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.152.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.152.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\,\operatorname{arccsch}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.152.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^4(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

3.152.6 Sympy [N/A]

Not integrable

Time = 134.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^4(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^4(a+b\operatorname{acsch}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**4*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

3.152.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.152.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^4}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

3.152.9 Mupad [N/A]

Not integrable

Time = 6.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.153 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.153.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.153.2 Mathematica [N/A]

Not integrable

Time = 7.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

3.153.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.153.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.153.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^{2} \left(a +b \,\operatorname{arccsch}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.153.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

3.153.6 Sympy [N/A]

Not integrable

Time = 33.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2(a+b\,\text{acsch}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.153.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}\, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

3.153.9 Mupad [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{3/2}}\, dx = \int \frac{x^2\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2 + d)^{3/2}}\, dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.154 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

3.154.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

output
```
x*(a+b*arccsch(c*x))/d/(e*x^2+d)^(1/2)-b*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+
1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2
)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

3.154.2 Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-c^2}x\right),\frac{e}{c^2d}\right)}{\sqrt{-c^2}d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*
x*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[Sqrt[-c^2]*x], e/(c^2*d)])/(Sqrt[-c
^2]*d*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.154.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {6846, 27, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6846}$$

$$\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bcx\int \frac{1}{d\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bcx\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[d + e*x^2]*Ellipt
icF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*
Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 6846
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsch[c*x])   u
, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[-1 -
 c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ
[p + 1/2, 0])
```

3.154.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arccsch}\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{ex^2+d}bc^2dx\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\sqrt{ex^2+d}ac^2dx-(bex^2+bd)\sqrt{-c^2}\sqrt{d}F(\text{arcs}}{c^2d^2ex^2+c^2d^3}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(e*x^2 + d)*b*c^2*d*x*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*
x)) + sqrt(e*x^2 + d)*a*c^2*d*x - (b*e*x^2 + b*d)*sqrt(-c^2)*sqrt(d)*ellip
tic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)))/(c^2*d^2*e*x^2 + c^2*d^3)
```

3.154.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

3.154.7 Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x) + a
*x/(sqrt(e*x^2 + d)*d)
```

3.154.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^{3/2}}\, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.155 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

3.155.1 Optimal result

Integrand size = 23, antiderivative size = 321

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}} - \frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}} - \frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{bc^2x\sqrt{d+ex^2}E\left(\arctan(cx)\,|1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} + \frac{2bex\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{d^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

output
```
(-a-b*arccsch(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsch(c*x))/d^2/(e*x^
2+d)^(1/2)+b*c^3*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/
2)+b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)-b*c^2*x*(1/
(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/
c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*
x^2+d)/d/(c^2*x^2+1))^(1/2)+2*b*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2
)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(
-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

3.155.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.63

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\,\text{csch}^{-1}(cx)}{d^2x\sqrt{d+ex^2}} + \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+(-c^2d+2e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{c^2}x\right),\frac{e}{c^2d}\right)\right)}{\sqrt{c^2}d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^
2)*ArcCsch[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] +
(-(c^2*d) + 2*e)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.155.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {6856, 25, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 6856$$

$$-\frac{bcx\int -\frac{2ex^2+d}{d^2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{-c^2x^2}} - \frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \ 25$$

$$\frac{bcx\int \frac{2ex^2+d}{d^2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{\sqrt{-c^2x^2}} - \frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}} - \frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{2ex^2+d}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d^2\sqrt{-c^2x^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{bcx\left(\frac{\int \frac{de\left(c^2x^2+2\right)}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{d}+\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{-c^2x^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(e\int \frac{c^2x^2+2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{-c^2x^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{406}$$

$$\frac{bcx\left(e\left(c^2\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+2\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)+\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{-c^2x^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{320}$$

$$\frac{bcx\left(e\left(c^2\int \frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx+\frac{2\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)+\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{-c^2x^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{bcx\left(e\left(c^2\left(\frac{\int \frac{\sqrt{ex^2+d}}{\left(-c^2x^2-1\right)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)+\frac{2\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)+\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{-c^2x^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{313}$$

3.155. $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx$

$$-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{bcx\left(e\left(\frac{2\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+c^2\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d+ex^2}\,E\left(\arctan(cx)\middle|1-\frac{e}{c^2d}\right)}{ce\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}\right)\right)+\frac{\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{x}\right)}{d^2\sqrt{-c^2x^2}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcCsch[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcCsch[c*x]
))/(d^2*Sqrt[d + e*x^2]) + (b*c*x*((Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/x
+ e*(c^2*((x*Sqrt[d + e*x^2])/(e*Sqrt[-1 - c^2*x^2]) - (Sqrt[d + e*x^2]*El
lipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(c*e*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x
^2)/(d*(1 + c^2*x^2))])) + (2*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e
/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]))))
/(d^2*Sqrt[-(c^2*x^2)])
```

3.155.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.155.4 Maple [F]

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x^2 \left(e\,x^2 + d\right)^{\frac{3}{2}}} dx$$

input
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.85

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}}\, dx =$$

$$-\frac{(bc^4dex^3 + bc^4d^2x)\sqrt{-c^2}\sqrt{d}E(\arcsin\left(\sqrt{-c^2}x\right) \mid \frac{e}{c^2d}) - ((bc^4de + 2\,be^2)x^3 + (bc^4d^2 + 2\,bde)x)\sqrt{-c^2}\sqrt{d}F(}{}$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
-((b*c^4*d*e*x^3 + b*c^4*d^2*x)*sqrt(-c^2)*sqrt(d)*elliptic_e(arcsin(sqrt(
-c^2)*x), e/(c^2*d)) - ((b*c^4*d*e + 2*b*e^2)*x^3 + (b*c^4*d^2 + 2*b*d*e)*
x)*sqrt(-c^2)*sqrt(d)*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + (2*b*c
^2*d*e*x^2 + b*c^2*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) + (2*a*c^2*d*e*x^2 + a*c^2*d^2 - (b*c^3*d*e*x^3 + b*c^3*d
^2*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(e*x^2 + d))/(c^2*d^3*e*x^3 + c^2
*d^4*x)
```

3.155.6 Sympy [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}}\, dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x^2 (d + ex^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.155.8 Giac [F]

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}}\, dx = \int \frac{b \operatorname{arcsch}(cx) + a}{(ex^2 + d)^{\frac{3}{2}} x^2}\, dx$$

input
```
integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}}\, dx = \int \frac{a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)}{x^2 (e\,x^2 + d)^{3/2}}\, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.156 $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.156.1 Optimal result

Integrand size = 23, antiderivative size = 251

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{bx\arctan\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{-c^2x^2}} + \frac{8bc\sqrt{d}x\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3e^3\sqrt{-c^2x^2}}$$

output
```
-1/3*d^2*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctan(e^(1/2)*(-c^2*x
^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(-c^2*x^2)^(1/2)+8/3*b*c*x*arctan((
e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(-c^2*x^2)^(1/2)+2*
d*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(-c^2*x^2-1)^(1/2)/(c
^2*d-e)/e^2/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsch(c*x))*(e*x^2+d)^
(1/2)/e^3
```

3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 2.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{\frac{2bcde\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d-e}+2a(8d^2+12dex^2+3e^2x^4)+\frac{bc(d+ex^2)\left(-\frac{8d\sqrt{1+\frac{d}{ex^2}}\operatorname{AppellF1}\left(1\right.}{c^2}\right.}{}}{}$$

input
```
Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((2*b*c*d*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d - e) + 2*a*(8*d^2
+ 12*d*e*x^2 + 3*e^2*x^4) + (b*c*(d + e*x^2)*((-8*d*Sqrt[1 + d/(e*x^2)]*Ap
pellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2)), -(d/(e*x^2))])/c^2 + (3*e*Sqrt[1 +
1/(c^2*x^2)]*x^4*Sqrt[1 + (e*x^2)/d]*AppellF1[1, 1/2, 1/2, 2, -(c^2*x^2),
-((e*x^2)/d)])/Sqrt[1 + c^2*x^2]))/x + 2*b*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4
)*ArcCsch[c*x])/(6*e^3*(d + e*x^2)^(3/2))
```

3.156.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.90, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {6856, 27, 7282, 2117, 27, 175, 66, 104, 217, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 6856$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{3e^3x\sqrt{-c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{x\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 7282}$$

$$-\frac{bcx\int \frac{3e^2x^4+12dex^2+8d^2}{x^2\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 2117}$$

$$-\frac{bcx\left(\frac{2\int \frac{d\left(c^2d-e\right)\left(3ex^2+8d\right)}{2x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d-e)}-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{bcx\left(\int \frac{3ex^2+8d}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 175}$$

$$-\frac{bcx\left(3e\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+8d\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 66}$$

$$-\frac{bcx\left(8d\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2+6e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{ 104}$$

3.156. $\displaystyle\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

$$-\frac{bcx\left(6e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}+16d\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{217}$$

$$-\frac{bcx\left(6e\int \frac{1}{-ex^4-c^2}d\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$\Big\downarrow \text{218}$$

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{bcx\left(-\frac{6\sqrt{e}\arctan\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{c}-16\sqrt{d}\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)-\frac{2de\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^3\sqrt{-c^2x^2}}$$

input
```
Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(d^2*(a + b*ArcCsch[c*x]))/(e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b*ArcC
sch[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/
e^3 - (b*c*x*((-2*d*e*Sqrt[-1 - c^2*x^2])/((c^2*d - e)*Sqrt[d + e*x^2]) -
(6*Sqrt[e]*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/c - 1
6*Sqrt[d]*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])]))/(6*e^3*Sq
rt[-(c^2*x^2)])
```

3.156.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 66
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
2   Subst[Int[1/(b - d*x^2), x], x, Sqrt[a + b*x]/Sqrt[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] &&  !GtQ[c - a*(d/b), 0]
```

3.156. $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2117
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Si
mp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*
(e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1
) - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x]
, x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n, 2*p]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.156. $\int \frac{x^5 \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{5/2}} \, dx$

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

3.156.4 Maple [F]

$$\int \frac{x^5(a + b\ \mathrm{arccsch}\,(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(213) = 426$.

Time = 0.48 (sec) , antiderivative size = 2421, normalized size of antiderivative = 9.65

$$\int \frac{x^5\left(a + b\mathrm{csch}^{-1}(cx)\right)}{(d + ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e
 - b*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 + 6*c^2*d*e + 8*(c^4*
d*e + c^2*e^2)*x^2 + 4*(2*c^4*e*x^3 + (c^4*d + c^2*e)*x)*sqrt(e*x^2 + d)*s
qrt(e)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + e^2) + 4*(8*b*c^3*d^3 - 8*b*c*d^2*e
 + 3*(b*c^3*d*e^2 - b*c*e^3)*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(
e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(b*c^3*d
^3 - b*c*d^2*e + (b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)
*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2
 - 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(8*a*c^3*d^3 - 8*a*c*d^2*e + 3*(a*c^3*d*e
^2 - a*c*e^3)*x^4 + 12*(a*c^3*d^2*e - a*c*d*e^2)*x^2 + (b*c^2*d*e^2*x^3 +
b*c^2*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*d^3*e^
3 - c*d^2*e^4 + (c^3*d*e^5 - c*e^6)*x^4 + 2*(c^3*d^2*e^4 - c*d*e^5)*x^2),
-1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e
- b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d + e)*x)*sqrt(e*x
^2 + d)*sqrt(-e)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*e^2*x^4 + (c^2*d*e + e
^2)*x^2 + d*e)) - 2*(8*b*c^3*d^3 - 8*b*c*d^2*e + 3*(b*c^3*d*e^2 - b*c*e^3)
*x^4 + 12*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^3*d*e^
2 - b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d...
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.156.8 Giac [F]

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{\left(b \operatorname{arcsch}(cx) + a\right)x^5}{\left(ex^2 + d\right)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \left(a + b\text{csch}^{-1}(cx)\right)}{\left(d + ex^2\right)^{5/2}} \, dx = \int \frac{x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{\left(e x^2 + d\right)^{5/2}} \, dx$$

input
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.157.1 Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{2bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3\sqrt{d}e^2\sqrt{-c^2x^2}}$$

output
```
1/3*d*(a+b*arccsch(c*x))/e^2/(e*x^2+d)^(3/2)-2/3*b*c*x*arctan((e*x^2+d)^(1
/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(-c^2*x^2)^(1/2)+(-a-b*arccsch
(c*x))/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(-c^2*x^2-1)^(1/2)/(c^2*d-e)/e/(-c^2*
x^2)^(1/2)/(e*x^2+d)^(1/2)
```

3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{\frac{bce\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)}{c^2d-e} + a(2d+3ex^2) - \frac{b\sqrt{1+\frac{d}{ex^2}}(d+ex^2)\,\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cx} + b(2d+3ex^2)\text{csch}^{-1}(cx)}{3e^2\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*((b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2))/(c^2*d - e) + a*(2*d + 3
*e*x^2) - (b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, -(1/
(c^2*x^2)), -(d/(e*x^2))])/(c*x) + b*(2*d + 3*e*x^2)*ArcCsch[c*x])/(e^2*(d
 + e*x^2)^(3/2))
```

3.157.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6856, 27, 435, 169, 27, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int -\frac{3ex^2+2d}{3e^2x\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\int \frac{3ex^2+2d}{x\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx}{3e^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bcx\int \frac{3ex^2+2d}{x^2\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx^2}{6e^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{169}$$

$$\frac{bcx\left(\frac{2\int \frac{d\left(c^2d-e\right)}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d(c^2d-e)}-\frac{2e\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bcx\left(2\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2-\frac{2e\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 104}$$

$$\frac{bcx\left(4\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}-\frac{2e\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}+\frac{bcx\left(-\frac{4\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}}-\frac{2e\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6e^2\sqrt{-c^2x^2}}$$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsch[c*x])/
(e^2*Sqrt[d + e*x^2]) + (b*c*x*((-2*e*Sqrt[-1 - c^2*x^2])/((c^2*d - e)*Sqr
t[d + e*x^2]) - (4*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/S
qrt[d]))/(6*e^2*Sqrt[-(c^2*x^2)])
```

3.157.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.157.4 Maple [F]

$$\int \frac{x^3(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(143) = 286$.

Time = 0.37 (sec) , antiderivative size = 786, normalized size of antiderivative = 4.65

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{2\left(2\,bc^2d^3-2\,bd^2e+3\,(bc^2d^2e-bde^2)x^2\right)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(b\right.$$

$$-\frac{(bc^2d^3+(bc^2de^2-be^3)x^4-bd^2e+2\,(bc^2d^2e-bde^2)x^2)\sqrt{-d}\arctan\left(\frac{((c^3d+ce)x^3+2\,cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,(c^2dex^4+(c^2d^2+de)x^2+d^2)}\right)}{3\,(c^2}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(2*(2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(e*
x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 +
 (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt
(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3
*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2
)*x^2 + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(
e*x^2 + d))/(c^2*d^4*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^
3*e^3 - d^2*e^4)*x^2), -1/3*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^
2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)*x^
3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d
*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + (2*b*c^2*d^3 - 2*b*d^2*e + 3*(b*c^2
*d^2*e - b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 1)/(c*x)) + (2*a*c^2*d^3 - 2*a*d^2*e + 3*(a*c^2*d^2*e - a*d*e^2)*x^2
 + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2
 + d))/(c^2*d^4*e^2 - d^3*e^3 + (c^2*d^2*e^4 - d*e^5)*x^4 + 2*(c^2*d^3*e^3
 - d^2*e^4)*x^2)]
```

3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.157.7 Maxima [F]

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*int
egrate(x^3*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

3.157.8 Giac [F]

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.158.1 Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} - \frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}} - \frac{bcx\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}$$

output
```
1/3*(-a-b*arccsch(c*x))/e/(e*x^2+d)^(3/2)-1/3*b*c*x*arctan((e*x^2+d)^(1/2)
/d^(1/2)/(-c^2*x^2-1)^(1/2))/d^(3/2)/e/(-c^2*x^2)^(1/2)+1/3*b*c*x*(-c^2*x^
2-1)^(1/2)/d/(c^2*d-e)/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{-\frac{2a}{e} + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}x\left(d+ex^2\right)}{d(c^2d-e)} + \frac{b\sqrt{1+\frac{d}{ex^2}}\left(d+ex^2\right)\text{AppellF1}\left(1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2x^2},-\frac{d}{ex^2}\right)}{cdex} - \frac{2b\text{csch}^{-1}(}{e}}{6\left(d+ex^2\right)^{3/2}}$$

input
```
Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

3.158. $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

output
```
((-2*a)/e + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2))/(d*(c^2*d - e)) +
(b*Sqrt[1 + d/(e*x^2)]*(d + e*x^2)*AppellF1[1, 1/2, 1/2, 2, -(1/(c^2*x^2))
, -(d/(e*x^2))])/(c*d*e*x) - (2*b*ArcCsch[c*x])/e)/(6*(d + e*x^2)^(3/2))
```

3.158.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6854, 354, 107, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{6854}$$

$$\frac{bcx\int \frac{1}{x\sqrt{-c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{354}$$

$$\frac{bcx\int \frac{1}{x^2\sqrt{-c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx^2}{6e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{107}$$

$$\frac{bcx\left(\frac{\int \frac{1}{x^2\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx^2}{d}+\frac{2e\sqrt{-c^2x^2-1}}{d\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{6e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{104}$$

$$\frac{bcx\left(\frac{2\int \frac{1}{-x^4-d}d\frac{\sqrt{ex^2+d}}{\sqrt{-c^2x^2-1}}}{d}+\frac{2e\sqrt{-c^2x^2-1}}{d\left(c^2d-e\right)\sqrt{d+ex^2}}\right)}{6e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{bcx\left(\frac{2e\sqrt{-c^2x^2-1}}{d\left(c^2d-e\right)\sqrt{d+ex^2}}-\frac{2\arctan\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{d^{3/2}}\right)}{6e\sqrt{-c^2x^2}}-\frac{a+b\operatorname{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

input
```
Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcCsch[c*x])/(e*(d + e*x^2)^(3/2)) + (b*c*x*((2*e*Sqrt[-1 - c
^2*x^2])/(d*(c^2*d - e)*Sqrt[d + e*x^2]) - (2*ArcTan[Sqrt[d + e*x^2]/(Sqrt
[d]*Sqrt[-1 - c^2*x^2])])/d^(3/2)))/(6*e*Sqrt[-(c^2*x^2)])
```

3.158.3.1 Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 107
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[(a*d*f*(m + 1) + b*c*f*(n +
 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m +
 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x
] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 6854
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Simp[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2]))   Int[(d + e*x^2)^(p + 1)
/(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -
1]
```

3.158. $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}\,dx$

3.158.4 Maple [F]

$$\int \frac{x(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(120) = 240$.

Time = 0.35 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.85

$$\int \frac{x\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \Bigg[-\frac{4\,(bc^2d^3-bd^2e)\sqrt{ex^2+d}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(bc^2d^3+(bc^2de^2-be^3)x^4-}{}$$

$$-\frac{(bc^2d^3+(bc^2de^2-be^3)x^4-bd^2e+2\,(bc^2d^2e-bde^2)x^2)\sqrt{-d}\arctan\left(\frac{((c^3d+ce)x^3+2\,cdx)\sqrt{ex^2+d}\sqrt{-d}\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,(c^2dex^4+(c^2d^2+de)x^2+d^2)}\right)}{6\,(c^2d^5e-d^4e^2+(c^2d^3e^3-}$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(4*(b*c^2*d^3 - b*d^2*e)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1
)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*
e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 + 6*c^2*d*e + e^2
)*x^4 + 8*(c^2*d^2 + d*e)*x^2 + 4*((c^3*d + c*e)*x^3 + 2*c*d*x)*sqrt(e*x^2
 + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*(a*c^2*d^3 -
 a*d^2*e - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sq
rt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^2
*d^4*e^2 - d^3*e^3)*x^2), -1/6*((b*c^2*d^3 + (b*c^2*d*e^2 - b*e^3)*x^4 - b
*d^2*e + 2*(b*c^2*d^2*e - b*d*e^2)*x^2)*sqrt(-d)*arctan(1/2*((c^3*d + c*e)
*x^3 + 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^
2*d*e*x^4 + (c^2*d^2 + d*e)*x^2 + d^2)) + 2*(b*c^2*d^3 - b*d^2*e)*sqrt(e*x
^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(a*c^2*d^3
- a*d^2*e - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*s
qrt(e*x^2 + d))/(c^2*d^5*e - d^4*e^2 + (c^2*d^3*e^3 - d^2*e^4)*x^4 + 2*(c^
2*d^4*e^2 - d^3*e^3)*x^2)]
```

3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.158.7 Maxima [F]

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x) -
 1/3*a/((e*x^2 + d)^(3/2)*e)
```

3.158.8 Giac [F]

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.159.2 Mathematica [N/A]

Not integrable

Time = 19.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

3.159.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.159.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.159.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsch}\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

3.159.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{b\operatorname{arcsch}\left(cx\right) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d
^2*e*x^3 + d^3*x), x)
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\operatorname{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.159.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

3.159.9 Mupad [N/A]

Not integrable

Time = 5.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\left(d + ex^2\right)^{5/2}}\,dx = \int \frac{a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\left(e\,x^2 + d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.160 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$

3.160.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{5/2}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.160.2 Mathematica [N/A]

Not integrable

Time = 23.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

3.160.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

3.160.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.160.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\,\mathrm{arccsch}\left(c x\right)}{x^{3}\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

3.160.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d
^2*e*x^5 + d^3*x^3), x)
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.160.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.160.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\,\text{arcsch}\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

3.160.9 Mupad [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^3\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.161 $\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.161.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.161.2 Mathematica [N/A]

Not integrable

Time = 17.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

3.161.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{x^6 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

input
```
Int[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.161.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.161.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^6 (a + b \operatorname{arccsch}(cx))}{(e\,x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.161.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^6(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \operatorname{arcsch}(cx) + a)x^6}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^6*arccsch(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x
^4 + 3*d^2*e*x^2 + d^3), x)
```

3.161.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**6*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.161.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6(a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.161.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^6 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{\left(b\operatorname{arcsch}\left(cx\right)+a\right)x^6}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

3.161.9 Mupad [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^6 \left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^6\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((x^6*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^6*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.162 $\int \frac{x^4\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.162.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\text{Int}\left(\frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
Unintegrable(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.162.2 Mathematica [N/A]

Not integrable

Time = 13.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

3.162.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} \, dx$$
$$\Big\downarrow \text{6866}$$
$$\int \frac{x^4 (a + b\text{csch}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

input
```
Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

3.162.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.162.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4 (a + b \operatorname{arccsch}(cx))}{(e\, x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.162.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x
^4 + 3*d^2*e*x^2 + d^3), x)
```

3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.162.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

3.162.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

3.162.9 Mupad [N/A]

Not integrable

Time = 5.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^4(a+b\text{csch}^{-1}(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.163 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.163.1 Optimal result

Integrand size = 23, antiderivative size = 359

$$\begin{aligned}\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx &= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}} \\ &+ \frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} \\ &- \frac{bc^2x\sqrt{d+ex^2}\,E\left(\arctan(cx)\left|1-\frac{e}{c^2d}\right.\right)}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}} \\ &+ \frac{bx\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}\end{aligned}$$

output
```
1/3*x^3*(a+b*arccsch(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(-c^2*x^2-1)^(1/2
)/d/(c^2*d-e)/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+1/3*b*c^3*x^2*(e*x^2+d)^(1/
2)/d/(c^2*d-e)/e/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/3*b*c^2*x*(1/(c^2*x
^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)
^(1/2))*(e*x^2+d)^(1/2)/d/(c^2*d-e)/e/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/
((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+1/3*b*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d
^2/(c^2*d-e)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))
^(1/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.53

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x^2\left(a(c^2d-e)\,x+bc\sqrt{1+\frac{1}{c^2x^2}}(d+ex^2)+b(c^2d-e)\,x\text{csch}^{-1}(cx)\right)}{3d\,(c^2d-e)\,(d+ex^2)^{3/2}} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}E\left(\arcsin\left(\sqrt{-\frac{e}{d}}x\right)|\frac{c^2d}{e}\right)}{3d\,(c^2d-e)\,\sqrt{-\frac{e}{d}}\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^2*(a*(c^2*d - e)*x + b*c*Sqrt[1 + 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d -
 e)*x*ArcCsch[c*x]))/(3*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 + 1
/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], (c^2*d
)/e])/(3*d*(c^2*d - e)*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.163.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {6856, 27, 373, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 6856}$$

$$\frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bcx\int \frac{x^2}{3d\sqrt{-c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{bcx\int \frac{x^2}{\sqrt{-c^2x^2-1}\left(ex^2+d\right)^{3/2}}dx}{3d\sqrt{-c^2x^2}}$$

$$\Big\downarrow \text{ 373}$$

$$\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{\int\frac{\sqrt{-c^2x^2-1}}{\sqrt{ex^2+d}}dx}{c^2d-e}-\frac{x\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3d\sqrt{-c^2x^2}}$$

$\downarrow$ 324

$$\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{c^2\left(-\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)-\int\frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d-e}-\frac{x\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3d\sqrt{-c^2x^2}}$$

$\downarrow$ 320

$$\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{c^2\left(-\int\frac{x^2}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx\right)-\frac{\sqrt{d+ex^2}\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}}{c^2d-e}-\frac{x\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3d\sqrt{-c^2x^2}}$$

$\downarrow$ 388

$$\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{-\left(c^2\left(\frac{\int\frac{\sqrt{ex^2+d}}{\left(-c^2x^2-1\right)^{3/2}}dx}{e}+\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}\right)\right)-\frac{\sqrt{d+ex^2}\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}}{c^2d-e}-\frac{x\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3d\sqrt{-c^2x^2}}$$

$\downarrow$ 313

$$\frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{-\frac{\sqrt{d+ex^2}\operatorname{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}-\left(c^2\left(\frac{x\sqrt{d+ex^2}}{e\sqrt{-c^2x^2-1}}-\frac{\sqrt{d+ex^2}E\left(\arctan(cx)\middle|1-\frac{e}{c^2d}\right)}{ce\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}\right)\right)}{c^2d-e}-\frac{x\sqrt{-c^2x^2-1}}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3d\sqrt{-c^2x^2}}$$

3.163. $\int\frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^{5/2}}dx$

input
```
Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c*x*(-((x*Sqrt[-1
- c^2*x^2])/((c^2*d - e)*Sqrt[d + e*x^2])) + (-(c^2*((x*Sqrt[d + e*x^2])/(
e*Sqrt[-1 - c^2*x^2]) - (Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2
*d)])/(c*e*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]))) - (Sq
rt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*Sqrt[-1 - c^2*x^
2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]))/(c^2*d - e)))/(3*d*Sqrt[-(c^2*x^2
)])
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.163.4 Maple [F]

$$\int \frac{x^2(a+b\,\mathrm{arccsch}\,(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.03

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{(bc^4d^2e-bc^2de^2)\sqrt{ex^2+d}x^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(bc^4de^2x^4+2\,bc^4d^2ex^2+bc}{}$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((b*c^4*d^2*e - b*c^2*d*e^2)*sqrt(e*x^2 + d)*x^3*log((c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c
^4*d^3)*sqrt(-c^2)*sqrt(d)*elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + (
b*c^4*d^3 + (b*c^4*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^4*d^2*e + b*d*e^2
)*x^2)*sqrt(-c^2)*sqrt(d)*elliptic_f(arcsin(sqrt(-c^2)*x), e/(c^2*d)) + ((
a*c^4*d^2*e - a*c^2*d*e^2)*x^3 + (b*c^3*d*e^2*x^4 + b*c^3*d^2*e*x^2)*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^4*d^5*e - c^2*d^4*e^2 + (c^4
*d^3*e^3 - c^2*d^2*e^4)*x^4 + 2*(c^4*d^4*e^2 - c^2*d^3*e^3)*x^2)
```

3.163.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.163.7 Maxima [F]

$$\int \frac{x^2 \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x
^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

3.163.8 Giac [F]

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{(b\,\operatorname{arcsch}\,(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.164 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$

3.164.1 Optimal result

Integrand size = 20, antiderivative size = 278

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{bc\sqrt{e}x\sqrt{-1-c^2x^2}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{\frac{d(1+c^2x^2)}{d+ex^2}}\sqrt{d+ex^2}} - \frac{b(3c^2d-2e)\,x\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{3d^3\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

output
```
1/3*x*(a+b*arccsch(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsch(c*x))/d^2/(e
*x^2+d)^(1/2)-1/3*b*c*x*(1/(1+e*x^2/d))^(1/2)*(1+e*x^2/d)^(1/2)*EllipticE(
x*e^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2),(1-c^2*d/e)^(1/2))*e^(1/2)*(-c^2*x^2-1
)^(1/2)/d^(3/2)/(c^2*d-e)/(-c^2*x^2)^(1/2)/(d*(c^2*x^2+1)/(e*x^2+d))^(1/2)
/(e*x^2+d)^(1/2)-1/3*b*(3*c^2*d-2*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(
1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^
3/(c^2*d-e)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^
(1/2)
```

3.164.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.87 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.89

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(-bce\sqrt{1+\frac{1}{c^2x^2}}x(d+ex^2)+a(c^2d-e)\,(3d+2ex^2)+b(c^2d-e)\,(3d+2ex^2)\,\text{csc}\right.}{3d^2\,(c^2d-e)\,(d+ex^2)^{3/2}}$$
$$-\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\text{arcsinh}\left(\sqrt{c^2}x\right)|\frac{e}{c^2d}\right)+2(c^2d-e)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{c^2}x\right),\frac{e}{c^2d}\right)\right)}{3\sqrt{c^2}d^2\,(c^2d-e)\,\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(-(b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d - e)*(3*d + 2*
e*x^2) + b*(c^2*d - e)*(3*d + 2*e*x^2)*ArcCsch[c*x]))/(3*d^2*(c^2*d - e)*(
d + e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]
*(c^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + 2*(c^2*d - e)*Ellip
ticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*(c^2*d - e)*Sqrt[
1 + c^2*x^2]*Sqrt[d + e*x^2])
```

3.164.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6846, 27, 400, 313, 320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 6846}$$
$$-\frac{bcx\int \frac{2ex^2+3d}{3d^2\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx}{\sqrt{-c^2x^2}}+\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{bcx\int \frac{2ex^2+3d}{\sqrt{-c^2x^2-1}(ex^2+d)^{3/2}}dx}{3d^2\sqrt{-c^2x^2}}+\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \; 400$$

$$-\frac{bcx\left(\frac{de\int \frac{\sqrt{-c^2x^2-1}}{\left(ex^2+d\right)^{3/2}}dx}{c^2d-e}+\frac{\left(3c^2d-2e\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d-e}\right)}{3d^2\sqrt{-c^2x^2}}+\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 313$$

$$-\frac{bcx\left(\frac{\left(3c^2d-2e\right)\int \frac{1}{\sqrt{-c^2x^2-1}\sqrt{ex^2+d}}dx}{c^2d-e}+\frac{\sqrt{d}\sqrt{e}\sqrt{-c^2x^2-1}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}\sqrt{\frac{d\left(c^2x^2+1\right)}{d+ex^2}}}\right)}{3d^2\sqrt{-c^2x^2}}+\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \; 320$$

$$\frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bcx\left(\frac{\left(3c^2d-2e\right)\sqrt{d+ex^2}\,\text{EllipticF}\left(\arctan(cx),1-\frac{e}{c^2d}\right)}{cd\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{\sqrt{d}\sqrt{e}\sqrt{-c^2x^2-1}E\left(\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{\left(c^2d-e\right)\sqrt{d+ex^2}\sqrt{\frac{d\left(c^2x^2+1\right)}{d+ex^2}}}\right)}{3d^2\sqrt{-c^2x^2}}$$

input
```
Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsch[c*x
]))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*x*((Sqrt[d]*Sqrt[e]*Sqrt[-1 - c^2*x^2]*
EllipticE[ArcTan[(Sqrt[e]*x)/Sqrt[d]], 1 - (c^2*d)/e])/((c^2*d - e)*Sqrt[(
d*(1 + c^2*x^2))/(d + e*x^2)]*Sqrt[d + e*x^2]) + ((3*c^2*d - 2*e)*Sqrt[d +
 e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(c*d*(c^2*d - e)*Sqrt[-1 -
c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])))/(3*d^2*Sqrt[-(c^2*x^2)])
```

3.164. $\int \frac{a+b\text{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 400
```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2]), x], x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[Sqrt[a + b*x^
2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &
& PosQ[d/c]
```

rule 6846
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Simp[(a + b*ArcCsch[c*x])   u
, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[SimplifyIntegrand[u/(x*Sqrt[-1 -
 c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ
[p + 1/2, 0])
```

3.164.4 Maple [F]

$$\int \frac{a + b\,\mathrm{arccsch}\left(c x\right)}{\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.59

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{(bc^4de^2x^4+2\,bc^4d^2ex^2+bc^4d^3)\sqrt{-c^2}\sqrt{d}E\left(\arcsin\left(\sqrt{-c^2}x\right)\mid\frac{e}{c^2d}\right)-(((bc^4+3\,bc^2)c}{}$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
1/3*((b*c^4*d*e^2*x^4 + 2*b*c^4*d^2*e*x^2 + b*c^4*d^3)*sqrt(-c^2)*sqrt(d)*
elliptic_e(arcsin(sqrt(-c^2)*x), e/(c^2*d)) - (((b*c^4 + 3*b*c^2)*d*e^2 -
2*b*e^3)*x^4 + (b*c^4 + 3*b*c^2)*d^3 - 2*b*d^2*e + 2*((b*c^4 + 3*b*c^2)*d^
2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2)*sqrt(d)*elliptic_f(arcsin(sqrt(-c^2)*x),
e/(c^2*d)) + (2*(b*c^4*d^2*e - b*c^2*d*e^2)*x^3 + 3*(b*c^4*d^3 - b*c^2*d^2
*e)*x)*sqrt(e*x^2 + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
+ (2*(a*c^4*d^2*e - a*c^2*d*e^2)*x^3 + 3*(a*c^4*d^3 - a*c^2*d^2*e)*x - (b*
c^3*d*e^2*x^4 + b*c^3*d^2*e*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(e*x^2
 + d))/(c^4*d^6 - c^2*d^5*e + (c^4*d^4*e^2 - c^2*d^3*e^3)*x^4 + 2*(c^4*d^5
*e - c^2*d^4*e^2)*x^2)
```

3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

3.164.7 Maxima [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b \, \text{arcsch}\,(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(
log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)
```

3.164.8 Giac [F]

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b \, \text{arcsch}\,(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{csch}^{-1}(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{a + b \, \text{asinh}\left(\frac{1}{c\,x}\right)}{(e\,x^2 + d)^{5/2}} \, dx$$

input
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.165 $\int (fx)^m (d+ex^2)^3 (a+b\text{csch}^{-1}(cx))\,dx$

3.165.1 Optimal result

Integrand size = 23, antiderivative size = 596

$$\int (fx)^m (d+ex^2)^3 (a+b\text{csch}^{-1}(cx))\,dx$$
$$= \frac{be\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}}$$
$$-\frac{be^2(e(5+m)^2-3c^2d(42+13m+m^2))\,x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}}$$
$$+\frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}}+\frac{d^3(fx)^{1+m}(a+b\text{csch}^{-1}(cx))}{f(1+m)}$$
$$+\frac{3d^2e(fx)^{3+m}(a+b\text{csch}^{-1}(cx))}{f^3(3+m)}$$
$$+\frac{3de^2(fx)^{5+m}(a+b\text{csch}^{-1}(cx))}{f^5(5+m)}+\frac{e^3(fx)^{7+m}(a+b\text{csch}^{-1}(cx))}{f^7(7+m)}$$
$$-\frac{b\left(\frac{c^6d^3(2+m)(4+m)(6+m)}{1+m}-\frac{e(1+m)\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2(42+13m+m^2)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)}{(3+m)(5+m)(7+m)}\right)x}{c^5f(1+m)(2+m)(4+m)(6+m)\sqrt{-c^2x^2}\sqrt{-1}}$$

output
```
d^3*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsc
h(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsch(c*x))/f^5/(5+m)+e^3*(f*
x)^(7+m)*(a+b*arccsch(c*x))/f^7/(7+m)+b*e*(e^2*(m^2+8*m+15)^2-3*c^2*d*e*(3
+m)^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m
)*(-c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(-c^2*x
^2)^(1/2)-b*e^2*(e*(5+m)^2-3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(-c^2*x^2-
1)^(1/2)/c^3/f^3/(4+m)/(6+m)/(m^2+12*m+35)/(-c^2*x^2)^(1/2)+b*e^3*x*(f*x)^
(5+m)*(-c^2*x^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(-c^2*x^2)^(1/2)-b*(c^6*d^3*(2+
m)*(4+m)*(6+m)/(1+m)-e*(1+m)*(e^2*(m^2+8*m+15)^2-3*c^2*d*e*(3+m)^2*(m^2+13
*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*
(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(
1/2)/c^5/f/(1+m)/(2+m)/(4+m)/(6+m)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

3.165.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.66

$$\begin{aligned}\int (fx)^m\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx &= x(fx)^m\left(\frac{ad^3}{1+m}+\frac{3ad^2ex^2}{3+m}+\frac{3ade^2x^4}{5+m}+\frac{ae^3x^6}{7+m}+\frac{bd^3\text{csch}^{-1}(cx)}{1+m}+\frac{3bd^2ex^2\text{csch}^{-1}(cx)}{3+m}\right.\\ &\quad+\frac{3bde^2x^4\text{csch}^{-1}(cx)}{5+m}+\frac{be^3x^6\text{csch}^{-1}(cx)}{7+m}\\ &\quad+\frac{bcd^3\sqrt{1+\frac{1}{c^2x^2}}\,x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)^2\sqrt{1+c^2x^2}}\\ &\quad+\frac{3bcd^2e\sqrt{1+\frac{1}{c^2x^2}}\,x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(3+m)^2\sqrt{1+c^2x^2}}\\ &\quad+\frac{3bcde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^5\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},-c^2x^2\right)}{(5+m)^2\sqrt{1+c^2x^2}}\\ &\quad\left.+\frac{bce^3\sqrt{1+\frac{1}{c^2x^2}}\,x^7\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7+m}{2},\frac{9+m}{2},-c^2x^2\right)}{(7+m)^2\sqrt{1+c^2x^2}}\right)\end{aligned}$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^3)/(1 + m) + (3*a*d^2*e*x^2)/(3 + m) + (3*a*d*e^2*x^4)/(5
+ m) + (a*e^3*x^6)/(7 + m) + (b*d^3*ArcCsch[c*x])/(1 + m) + (3*b*d^2*e*x^2
*ArcCsch[c*x])/(3 + m) + (3*b*d*e^2*x^4*ArcCsch[c*x])/(5 + m) + (b*e^3*x^6
*ArcCsch[c*x])/(7 + m) + (b*c*d^3*Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)^2*Sqrt[1 + c^2*x^2]) +
(3*b*c*d^2*e*Sqrt[1 + 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (
5 + m)/2, -(c^2*x^2)])/((3 + m)^2*Sqrt[1 + c^2*x^2]) + (3*b*c*d*e^2*Sqrt[1
 + 1/(c^2*x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, -(c^2*x^2
)])/((5 + m)^2*Sqrt[1 + c^2*x^2]) + (b*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x^7*Hyp
ergeometric2F1[1/2, (7 + m)/2, (9 + m)/2, -(c^2*x^2)])/((7 + m)^2*Sqrt[1 +
 c^2*x^2]))
```

3.165.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {6856, 2340, 25, 1590, 25, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^3 (fx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6856}$$

$$-\frac{bcx\int \frac{(fx)^m\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \text{2340}$$

$$-\frac{bcx\left(-\frac{\int -\frac{(fx)^m\left(-\frac{e^2\left(e(m+5)^2-3c^2d\left(m^2+13m+42\right)\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{-c^2x^2-1}}dx}{c^2(m+6)}-\frac{e^3\sqrt{-c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)}{\sqrt{-c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{\int \frac{(fx)^m\left(-\frac{e^2\left(e(m+5)^2-3c^2d\left(m^2+13m+42\right)\right)x^4}{(m+5)(m+7)}+\frac{3c^2d^2e(m+6)x^2}{m+3}+\frac{c^2d^3(m+6)}{m+1}\right)}{\sqrt{-c^2x^2-1}}dx}{c^2(m+6)}-\frac{e^3\sqrt{-c^2x^2-1}(fx)^{m+5}}{c^2f^5(m+6)(m+7)}\right)}{\sqrt{-c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \; 1590$$

$$-\frac{bcx\left(\frac{\frac{e^2\sqrt{-c^2x^2-1}(fx)^{m+3}\left(e(m+5)^2-3c^2d\left(m^2+13m+42\right)\right)}{c^2f^3(m+4)(m+5)(m+7)}-\frac{\int -\frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4-3de(m+3)^2\left(m\right.\right.}{(m+3)(m+5)(m+7}\right.}{\sqrt{-c^2x^2-1}}}{c^2(m+4)}}{c^2(m+6)}\right)}{\sqrt{-c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{\frac{\int \frac{(fx)^m\left(\frac{d^3(m+4)(m+6)c^4}{m+1}+\frac{e\left(3d^2\left(m^4+22m^3+179m^2+638m+840\right)c^4-3de(m+3)^2\left(m^2+13m+42\right)c^2+e^2\left(m^2+8m+15\right)^2\right)x^2}{(m+3)(m+5)(m+7)}\right)}{\sqrt{-c^2x^2-1}}dx}{c^2(m+4)}+\frac{e^2\sqrt{-c^2x^2-1}}{}}{c^2(m+6)}\right)}{\sqrt{-c^2x^2}}+\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$$\Big\downarrow \; 363$$

3.165. $\int (fx)^m\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$-\frac{bcx\left(\frac{\left(\frac{c^4d^3(m+4)(m+6)}{m+1}-\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)-3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{-c^2x^2-1}}dx-\frac{e\sqrt{-}}{}}{c^2(m+4)}\right.}{}$$

$$\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$\downarrow$ 279

$$-\frac{bcx\left(\frac{\frac{\sqrt{c^2x^2+1}\left(\frac{c^4d^3(m+4)(m+6)}{m+1}-\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)-3c^2de(m+3)^2\left(m^2+13m+42\right)+e^2\left(m^2+8m+15\right)^2\right)}{c^2(m+2)(m+3)(m+5)(m+7)}\right)\int\frac{(fx)^m}{\sqrt{c^2x^2+1}}d}{\sqrt{-c^2x^2-1}}}{c^2(m+4)}\right.}{}$$

$$\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}$$

$\downarrow$ 278

$$\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}-$$

$$\frac{bcx\left(\frac{e^2\sqrt{-c^2x^2-1}(fx)^{m+3}\left(e(m+5)^2-3c^2d\left(m^2+13m+42\right)\right)}{c^2f^3(m+4)(m+5)(m+7)}+\frac{\frac{\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{c^4d^3(m+4)(m+6)}{m+1}-\frac{e(m+1)\left(3c^4d^2\left(m^4+22m^3+179m^2+638m+840\right)-3\right.}{c^2(m+2)(m+3}\right.}{f(m+1)}}{}\right.}{}$$

input `Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]`

3.165. $\int(fx)^m\left(d+ex^2\right)^3\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

output
```
(d^3*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 +
 m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*Ar
cCsch[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsch[c*x]))/(f^7
*(7 + m)) - (b*c*x*(-((e^3*(f*x)^(5 + m)*Sqrt[-1 - c^2*x^2])/(c^2*f^5*(6 +
 m)*(7 + m))) + ((e^2*(e*(5 + m)^2 - 3*c^2*d*(42 + 13*m + m^2))*(f*x)^(3 +
 m)*Sqrt[-1 - c^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)*(7 + m)) + (-((e*(e^2*(15
 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 +
 638*m + 179*m^2 + 22*m^3 + m^4))*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c^2*f
*(2 + m)*(3 + m)*(5 + m)*(7 + m))) + (((c^4*d^3*(4 + m)*(6 + m))/(1 + m) -
 (e*(1 + m)*(e^2*(15 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2
) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4)))/(c^2*(2 + m)*(3 + m
)*(5 + m)*(7 + m)))*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2,
 (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(f*(1 + m)*Sqrt[-1 - c^2*x^2]))/(c^2*(
4 + m)))/(c^2*(6 + m))))/Sqrt[-(c^2*x^2)]
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.165.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^3 \left(a+b\,\operatorname{arccsch}\left(cx\right)\right)dx$$

input
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

3.165.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\operatorname{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccsch(c*x))*(f*x)^m, x)
```

3.165.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\operatorname{csch}^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsch(c*x)),x)
```

output
```
Timed out
```

3.165.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\operatorname{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*
x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) - (((m^3 + 9*m^2 + 23*m
+ 15)*b*e^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5 + 3*(m^
3 + 13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^
3*f^m*x)*x^m*log(x) - ((m^3 + 9*m^2 + 23*m + 15)*b*e^3*f^m*x^7 + 3*(m^3 +
11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*e
*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^m*x)*x^m*log(sqrt(c^2*x^2 +
 1) + 1))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + integrate(((m^3 + 9*m^2
+ 23*m + 15)*b*c^2*e^3*f^m*x^8 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^2*d*e^2*
f^m*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*e*f^m*x^4 + (m^3 + 15*m^2
 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105
)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*x^2 + 1) + 176*m
 + 105), x) - integrate(((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*b*c^2*e^3*f
^m*x^8*log(c) + (3*(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*d*e^2*f^m*log
(c) + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*e^3*f^m*log(c) - (m^3 + 9*m^2
+ 23*m + 15)*e^3*f^m)*b*x^6 + 3*((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2
*d^2*e*f^m*log(c) + (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*d*e^2*f^m*log(c)
 - (m^3 + 11*m^2 + 31*m + 21)*d*e^2*f^m)*b*x^4 + ((m^4 + 16*m^3 + 86*m^2 +
 176*m + 105)*c^2*d^3*f^m*log(c) + 3*(m^4 + 16*m^3 + 86*m^2 + 176*m + 1...
```

3.165.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^3 (b\,\text{arcsch}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\left(e\,x^2+d\right)^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^3*(a + b*asinh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^3*(a + b*asinh(1/(c*x))), x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.166.1 Optimal result

Integrand size = 23, antiderivative size = 379

$$\begin{aligned}
&\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx \\
&= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{-c^2x^2}} \\
&\quad+\frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}}+\frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} \\
&\quad+\frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}+\frac{e^2(fx)^{5+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(5+m)} \\
&\quad-\frac{b(c^4d^2(2+m)(3+m)(4+m)(5+m)+e(1+m)^2\left(e(3+m)^2-2c^2d(20+9m+m^2)\right))\,x(fx)^{1+m}\sqrt{1+}}{c^3f(1+m)^2(2+m)(3+m)(4+m)(5+m)\sqrt{-c^2x^2}\sqrt{-1-}}
\end{aligned}$$

output
```
d^2*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsch(
c*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsch(c*x))/f^5/(5+m)-b*e*(e*(3+m)^
2-2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)
/(m^2+5*m+6)/(-c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(-c^2*x^2-1)^(1/2)/c/f^3
/(4+m)/(5+m)/(-c^2*x^2)^(1/2)-b*(c^4*d^2*(2+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2
*(e*(3+m)^2-2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m
],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c^3/f/(1+m)^2/(2+m)/(3+m)/(4+m)/
(5+m)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.76

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(\frac{ad^2}{1+m} + \frac{2adex^2}{3+m} + \frac{ae^2x^4}{5+m} + \frac{bd^2\text{csch}^{-1}(cx)}{1+m} + \frac{2bdex^2\text{csch}^{-1}(cx)}{3+m} + \frac{be^2x^4\text{csch}^{-1}(cx)}{5+m} + \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}\,x \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)^2\sqrt{1+c^2x^2}} + \frac{2bcde\sqrt{1+\frac{1}{c^2x^2}}\,x^3 \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{(3+m)^2\sqrt{1+c^2x^2}} + \frac{bce^2\sqrt{1+\frac{1}{c^2x^2}}\,x^5 \,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5+m}{2},\frac{7+m}{2},-c^2x^2\right)}{(5+m)^2\sqrt{1+c^2x^2}}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
x*(f*x)^m*((a*d^2)/(1 + m) + (2*a*d*e*x^2)/(3 + m) + (a*e^2*x^4)/(5 + m) +
 (b*d^2*ArcCsch[c*x])/(1 + m) + (2*b*d*e*x^2*ArcCsch[c*x])/(3 + m) + (b*e^
2*x^4*ArcCsch[c*x])/(5 + m) + (b*c*d^2*Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeomet
ric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/((1 + m)^2*Sqrt[1 + c^2*x^2
]) + (2*b*c*d*e*Sqrt[1 + 1/(c^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2
, (5 + m)/2, -(c^2*x^2)])/((3 + m)^2*Sqrt[1 + c^2*x^2]) + (b*c*e^2*Sqrt[1
+ 1/(c^2*x^2)]*x^5*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, -(c^2*x^2)
])/((5 + m)^2*Sqrt[1 + c^2*x^2]))
```

3.166.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {6856, 27, 1590, 25, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (fx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 6856$$

$$-\frac{bcx\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{(m^3+9m^2+23m+15)\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 27$$

$$-\frac{bcx\int \frac{(fx)^m\left(e^2(m+1)(m+3)x^4+2de(m+1)(m+5)x^2+d^2(m+3)(m+5)\right)}{\sqrt{-c^2x^2-1}}dx}{(m^3+9m^2+23m+15)\sqrt{-c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 1590$$

$$-\frac{bcx\left(-\frac{\int -\frac{(fx)^m\left(c^2d^2(m+3)(m+4)(m+5)-e(m+1)\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)x^2\right)}{\sqrt{-c^2x^2-1}}dx}{c^2(m+4)}-\frac{e^2(m+1)(m+3)\sqrt{-c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{(m^3+9m^2+23m+15)\sqrt{-c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 25$$

$$-\frac{bcx\left(\frac{\int \frac{(fx)^m\left(c^2d^2(m+3)(m+4)(m+5)-e(m+1)\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)x^2\right)}{\sqrt{-c^2x^2-1}}dx}{c^2(m+4)}-\frac{e^2(m+1)(m+3)\sqrt{-c^2x^2-1}(fx)^{m+3}}{c^2f^3(m+4)}\right)}{(m^3+9m^2+23m+15)\sqrt{-c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 363$$

$$-\frac{bcx\left(\frac{\frac{\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{m+2}\right)\int \frac{(fx)^m}{\sqrt{-c^2x^2-1}}dx}{c^2}+\frac{e(m+1)\sqrt{-c^2x^2-1}(fx)^{m+1}\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{c^2f(m+2)}}{c^2(m+4)}\right)}{(m^3+9m^2+23m+15)\sqrt{-c^2x^2}}+\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 279$$

3.166. $\int (fx)^m\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

$$-\frac{bcx \left(\frac{\frac{\sqrt{c^2x^2+1}\left(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{m+2}\right)\int \frac{(fx)^m}{\sqrt{c^2x^2+1}}dx}{c^2\sqrt{-c^2x^2-1}}+\frac{e(m+1)\sqrt{-c^2x^2-1}(fx)^{m+1}(e(m+3)^2-2c^2d(m^2 \cdots}{c^2f(m+2)} \cdots}{c^2(m+4)} \cdots \right)}{\left(m^3+9m^2+23m+15\right)\sqrt{-c^2x^2}}$$

$$\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}$$

$$\Big\downarrow \; 278$$

$$\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}-$$

$$\frac{bcx \left(\frac{\frac{e(m+1)\sqrt{-c^2x^2-1}(fx)^{m+1}\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{c^2f(m+2)}+\frac{\sqrt{c^2x^2+1}(fx)^{m+1}(c^4d^2(m+3)(m+4)(m+5)+\frac{e(m+1)^2\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{m+2} \cdots}{c^2f(m+1)\sqrt{-c^2x^2-1}} \cdots}{c^2(m+4)} \cdots \right)}{\left(m^3+9m^2+23m+15\right)\sqrt{-c^2x^2}}$$

input
```
Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

output
```
(d^2*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m
)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCsch[
c*x]))/(f^5*(5 + m)) - (b*c*x*(-((e^2*(1 + m)*(3 + m)*(f*x)^(3 + m)*Sqrt[-
1 - c^2*x^2])/(c^2*f^3*(4 + m))) + ((e*(1 + m)*(e*(3 + m)^2 - 2*c^2*d*(20
+ 9*m + m^2))*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c^2*f*(2 + m)) + ((c^4*d^
2*(3 + m)*(4 + m)*(5 + m) + (e*(1 + m)^2*(e*(3 + m)^2 - 2*c^2*d*(20 + 9*m
+ m^2)))/(2 + m))*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (
1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(c^2*f*(1 + m)*Sqrt[-1 - c^2*x^2]))/(c^2
*(4 + m))))/((15 + 23*m + 9*m^2 + m^3)*Sqrt[-(c^2*x^2)])
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

3.166. $\int (fx)^m (d+ex^2)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right) dx$

3.166.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right)^2 (a+b \operatorname{arccsch}(cx))\,dx$$

input
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

3.166.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int \left(ex^2+d\right)^2 (b\operatorname{arcsch}(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arccsch(c*x))*(f*x)^m, x)
```

3.166.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = \int (fx)^m (a+b\operatorname{acsch}(cx)) \left(d+ex^2\right)^2\,dx$$

input
```
integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

3.166.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsch}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*
d^2/(f*(m + 1)) - (((m^2 + 4*m + 3)*b*e^2*f^m*x^5 + 2*(m^2 + 6*m + 5)*b*d*
e*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(x) - ((m^2 + 4*m + 3)*b*
e^2*f^m*x^5 + 2*(m^2 + 6*m + 5)*b*d*e*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m
*x)*x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(
((m^2 + 4*m + 3)*b*c^2*e^2*f^m*x^6 + 2*(m^2 + 6*m + 5)*b*c^2*d*e*f^m*x^4 +
 (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^
2 + m^3 + 9*m^2 + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m
+ 15)*sqrt(c^2*x^2 + 1) + 23*m + 15), x) - integrate(((m^3 + 9*m^2 + 23*m
+ 15)*b*c^2*e^2*f^m*x^6*log(c) + (2*(m^3 + 9*m^2 + 23*m + 15)*c^2*d*e*f^m*
log(c) + (m^3 + 9*m^2 + 23*m + 15)*e^2*f^m*log(c) - (m^2 + 4*m + 3)*e^2*f^
m)*b*x^4 + ((m^3 + 9*m^2 + 23*m + 15)*c^2*d^2*f^m*log(c) + 2*(m^3 + 9*m^2
+ 23*m + 15)*d*e*f^m*log(c) - 2*(m^2 + 6*m + 5)*d*e*f^m)*b*x^2 + ((m^3 + 9
*m^2 + 23*m + 15)*d^2*f^m*log(c) - (m^2 + 8*m + 15)*d^2*f^m)*b)*x^m/((m^3
+ 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15), x)
```

3.166.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^2 (b\,\text{arcsch}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.166.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx=\int (f\,x)^m\,{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right) \,dx$$

input
```
int((f*x)^m*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.167 $\int (fx)^m (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx$

3.167.1 Optimal result

Integrand size = 21, antiderivative size = 220

$$\int (fx)^m (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf(6+5m+m^2)\sqrt{-c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} +\frac{b(e(1+m)^2-c^2d(2+m)(3+m))\,x(fx)^{1+m}\sqrt{1+c^2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{cf(1+m)^2(2+m)(3+m)\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}$$

output
```
d*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsch(c*x))/
f^3/(3+m)+b*e*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c/f/(m^2+5*m+6)/(-c^2*x^2)^
(1/2)+b*(e*(1+m)^2-c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/
2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(-c^2
*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$= x(fx)^m \left(\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}x \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-c^2x^2\right)}{(1+m)^2\sqrt{1+c^2x^2}} + \frac{\frac{(3+m)(d(3+m)+e(1+m)x^2)\left(a+b\text{csch}^{-1}(cx)\right)}{1+m}+\frac{bce\sqrt{1+\frac{1}{c^2x^2}}x^3 \text{ Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-c^2x^2\right)}{\sqrt{1+c^2x^2}}}{(3+m)^2}\right)$$

input
```
Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
x*(f*x)^m*((b*c*d*Sqrt[1 + 1/(c^2*x^2)]*x*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, -(c^2*x^2)])/((1 + m)^2*Sqrt[1 + c^2*x^2]) + (((3 + m)*(d*(3
+ m) + e*(1 + m)*x^2)*(a + b*ArcCsch[c*x]))/(1 + m) + (b*c*e*Sqrt[1 + 1/(c
^2*x^2)]*x^3*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, -(c^2*x^2)])/Sqr
t[1 + c^2*x^2])/(3 + m)^2)
```

3.167.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6856, 27, 363, 279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right) (fx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$
$$\Big\downarrow \ 6856$$
$$-\frac{bcx\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{(m^2+4m+3)\sqrt{-c^2x^2-1}}dx}{\sqrt{-c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}$$
$$\Big\downarrow \ 27$$

$$-\frac{bcx\int \frac{(fx)^m\left(e(m+1)x^2+d(m+3)\right)}{\sqrt{-c^2x^2-1}}dx}{(m^2+4m+3)\sqrt{-c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \ 363$$

$$-\frac{bcx\left(-\left(\frac{e(m+1)^2}{c^2(m+2)}-d(m+3)\right)\int \frac{(fx)^m}{\sqrt{-c^2x^2-1}}dx-\frac{e(m+1)\sqrt{-c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{-c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \ 279$$

$$-\frac{bcx\left(-\frac{\sqrt{c^2x^2+1}\left(\frac{e(m+1)^2}{c^2(m+2)}-d(m+3)\right)\int \frac{(fx)^m}{\sqrt{c^2x^2+1}}dx}{\sqrt{-c^2x^2-1}}-\frac{e(m+1)\sqrt{-c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{-c^2x^2}}+\frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}$$

$$\Big\downarrow \ 278$$

$$\frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}-\frac{bcx\left(-\frac{\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{e(m+1)^2}{c^2(m+2)}-d(m+3)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-c^2x^2\right)}{f(m+1)\sqrt{-c^2x^2-1}}-\frac{e(m+1)\sqrt{-c^2x^2-1}(fx)^{m+1}}{c^2f(m+2)}\right)}{(m^2+4m+3)\sqrt{-c^2x^2}}$$

input
```
Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

output
```
(d*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a +
 b*ArcCsch[c*x]))/(f^3*(3 + m)) - (b*c*x*(-((e*(1 + m)*(f*x)^(1 + m)*Sqrt[
-1 - c^2*x^2])/(c^2*f*(2 + m))) - (((e*(1 + m)^2)/(c^2*(2 + m)) - d*(3 + m
))*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -(c^2*x^2)])/(f*(1 + m)*Sqrt[-1 - c^2*x^2])))/((3 + 4*m + m^2)*Sqrt[
-(c^2*x^2)])
```

3.167. $\int (fx)^m\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 6856
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Si
mp[(a + b*ArcCsch[c*x])   u, x] - Simp[b*c*(x/Sqrt[(-c^2)*x^2])   Int[Simpl
ifyIntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e,
 f, m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3,
0])) || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (I
LtQ[(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

3.167.4 Maple [F]

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b \,\mathrm{arccsch}\left(cx\right)\right) dx$$

input
```
int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

3.167.5 Fricas [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx=\int (ex^2+d)(b\,\text{arcsch}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*(f*x)^m, x)
```

3.167.6 Sympy [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx=\int (fx)^m\,(a+b\,\text{acsch}\,(cx)) \left(d+ex^2\right) \, dx$$

input
```
integrate((f*x)**m*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

output
```
Integral((f*x)**m*(a + b*acsch(c*x))*(d + e*x**2), x)
```

3.167.7 Maxima [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx=\int (ex^2+d)(b\,\text{arcsch}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

output
```
a*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) - ((b*e*f^m*(m + 1
)*x^3 + b*d*f^m*(m + 3)*x)*x^m*log(x) - (b*e*f^m*(m + 1)*x^3 + b*d*f^m*(m
+ 3)*x)*x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^2 + 4*m + 3) + integrate((b*c^2
*e*f^m*(m + 1)*x^4 + b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2
 + m^2 + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x^2 + 1) + 4*m
 + 3), x) - integrate(((m^2 + 4*m + 3)*b*c^2*e*f^m*x^4*log(c) + ((m^2 + 4*
m + 3)*c^2*d*f^m*log(c) + (m^2 + 4*m + 3)*e*f^m*log(c) - e*f^m*(m + 1))*b*
x^2 + ((m^2 + 4*m + 3)*d*f^m*log(c) - d*f^m*(m + 3))*b)*x^m/((m^2 + 4*m +
3)*c^2*x^2 + m^2 + 4*m + 3), x)
```

3.167.8 Giac [F]

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (ex^2+d)(b \operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.167.9 Mupad [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (f\,x)^m \, \left(e\,x^2+d\right) \, \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

3.168.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

3.168.2 Mathematica [N/A]

Not integrable

Time = 3.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

3.168.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

3.168.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.168.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\left(f x\right)^{m} \left(a+b \,\operatorname{arccsch}\left(c x\right)\right)}{e \,x^{2}+d}d x$$

input
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

output
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

3.168.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.168.6 Sympy [N/A]

Not integrable

Time = 41.89 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\,(a+b\,\text{acsch}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d),x)
```

output
```
Integral((f*x)**m*(a + b*acsch(c*x))/(d + e*x**2), x)
```

3.168.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)(fx)^m}{ex^2+d}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.168.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2} \, dx = \int \frac{(b \operatorname{arcsch}(cx)+a)(fx)^m}{ex^2+d} \, dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

3.168.9 Mupad [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a+b\operatorname{csch}^{-1}(cx)\right)}{d+ex^2} \, dx = \int \frac{(f\,x)^m \left(a+b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d} \, dx$$

input
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
```

output
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.169 $\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

3.169.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\operatorname{Int}\left(\frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.169.2 Mathematica [N/A]

Not integrable

Time = 6.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

3.169.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^2} \, dx$$
$$\big\downarrow \text{ 6866}$$
$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^2}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

3.169.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.169.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b \,\operatorname{arccsch}\left(cx\right)\right)}{\left(e\,x^2+d\right)^2}dx$$

input
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

output
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

3.169.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{(d + ex^2)^2} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{(ex^2 + d)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arccsch(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

3.169.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{(d + ex^2)^2} \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

3.169.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{(d + ex^2)^2} \, dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{(ex^2 + d)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.169.8 Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx = \int \frac{(b \operatorname{arcsch}(cx)+a)(fx)^m}{\left(ex^2+d\right)^2} \, dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

3.169.9 Mupad [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2} \, dx = \int \frac{(f\,x)^m \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2+d\right)^2} \, dx$$

input
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

output
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.170 $\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

3.170.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \text{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.170.2 Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

input
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

3.170.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \text{6866}$$

$$\int \left(d+ex^2\right)^{3/2} (fx)^m \left(a+b\text{csch}^{-1}(cx)\right)dx$$

input
```
Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.170.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.170.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \left(f x \right)^{m} \left(e \,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \,\operatorname{arccsch}\left(c x \right)\right)d x$$

input
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

output
```
int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

3.170.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas"
)
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccsch(c*x))*sqrt(e*x^2 + d)*(f
*x)^m, x)
```

3.170.6 Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

output
```
Timed out
```

3.170.7 Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b \operatorname{arcsch}(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima"
)
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.170.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx=\int (ex^2+d)^{\frac{3}{2}}(b\,\text{arcsch}\,(cx)+a)(fx)^m \, dx$$

input
```
integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.170.9 Mupad [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx=\int (f\,x)^m \left(e\,x^2+d\right)^{3/2} \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right) \, dx$$

input
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.171 $\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right) dx$

3.171.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right) dx = \text{Int}\left((fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.171.2 Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right) dx = \int (fx)^m \sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right) dx$$

input
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

3.171.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

$$\Big\downarrow \; 6866$$

$$\int \sqrt{d+ex^2}(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)dx$$

input
```
Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

output
```
$Aborted
```

3.171.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.171.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (fx)^m \left(a+b\,\text{arccsch}\left(cx\right)\right)\sqrt{e\,x^2+d}dx$$

input
```
int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

3.171.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.171.6 Sympy [N/A]

Not integrable

Time = 53.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (fx)^m\,(a+b\,\text{acsch}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

3.171.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.171.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int \sqrt{ex^2+d}(b\,\text{arcsch}\,(cx)+a)(fx)^m\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

3.171.9 Mupad [N/A]

Not integrable

Time = 5.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (fx)^m\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

input
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

output
```
int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.172 $\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

3.172.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \text{Int}\left(\frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.172.2 Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

3.172.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}} \, dx$$

$$\Big\downarrow \; 6866$$

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

3.172.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.172.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(f x \right)^{m} \left(a +b \,\operatorname{arccsch}\left(c x \right)\right)}{\sqrt{e \,x^{2}+d}}d x$$

input
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

3.172.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas"
)
```

output
```
integral((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.172.6 Sympy [N/A]

Not integrable

Time = 23.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\,(a+b\,\text{acsch}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((f*x)**m*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

3.172.7 Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.172.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)(fx)^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

3.172.9 Mupad [N/A]

Not integrable

Time = 5.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{{(f\,x)}^m\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

output
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.173 $\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

3.173.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.173.2 Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

3.173.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}dx$$

input
```
Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

3.173.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.173.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\left(f x \right)^{m} \left(a +b \,\operatorname{arccsch}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{\frac{3}{2}}}d x$$

input
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

3.173.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2
 + d^2), x)
```

3.173.6 Sympy [N/A]

Not integrable

Time = 146.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{(d + ex^2)^{3/2}}\,dx = \int \frac{(fx)^m \left(a + b\,\text{acsch}\,(cx)\right)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((f*x)**m*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

3.173.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a + b\text{csch}^{-1}(cx)\right)}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima"
)
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.173.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\,\text{arcsch}\,(cx)+a)(fx)^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

3.173.9 Mupad [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{(fx)^m \left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex^2)^{3/2}}\,dx = \int \frac{(f\,x)^m \left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

output
```
int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.174 $\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.174.1 Optimal result

Integrand size = 26, antiderivative size = 395

$$\begin{aligned}\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = &-\frac{4b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{7b(1-c^2x^2)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}\\
&-\frac{13b(1-c^2x^2)^{5/2}\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}+\frac{3b(1-c^2x^2)^{7/2}\sqrt{1+c^2x^2}}{70c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}\\
&-\frac{b(1-c^2x^2)^{9/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}\\
&+\frac{(1-c^4x^4)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}\\
&-\frac{(1-c^4x^4)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}\\
&+\frac{4b\sqrt{1+c^2x^2}\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}x}\end{aligned}$$

output
```
1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsch(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+
b*arccsch(c*x))/c^12+7/90*b*(-c^2*x^2+1)^(3/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1
+1/c^2/x^2)^(1/2)-13/150*b*(-c^2*x^2+1)^(5/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+
1/c^2/x^2)^(1/2)+3/70*b*(-c^2*x^2+1)^(7/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c
^2/x^2)^(1/2)-1/90*b*(-c^2*x^2+1)^(9/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/
x^2)^(1/2)+4/15*b*arctanh((-c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^13/x/(1+
1/c^2/x^2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c
^2/x^2)^(1/2)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

3.174.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.54

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{105a\sqrt{1-c^4x^4}(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(768-36c^2x^2+78c^4x^4-5c^6x^6+35c^8x^8)}{1+c^2x^2}+105b\sqrt{1-c^4x^4}(}{3150}$$

input
```
Integrate[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 - 36*c^2*x^2 + 78*c^4*x^4 - 5*c^6
*x^6 + 35*c^8*x^8))/(1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4
 + 3*c^8*x^8)*ArcCsch[c*x] + 840*b*Log[x + c^2*x^3] - 840*b*Log[1 + c^2*x^
2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^12
```

3.174.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.58, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {6864, 27, 7272, 1388, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$$

$$\Big\downarrow \; 6864$$

$$\frac{b\int -\frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{30c^{12}\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{30c^{13}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{7272}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^4x^4}\left(3c^8x^8+4c^4x^4+8\right)}{x\sqrt{c^2x^2+1}}dx}{30c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^2x^2}\left(3c^8x^8+4c^4x^4+8\right)}{x}dx}{30c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2331}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^2x^2}\left(3c^8x^8+4c^4x^4+8\right)}{x^2}dx^2}{60c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2123}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \left(-3c^2\left(1-c^2x^2\right)^{7/2}+9c^2\left(1-c^2x^2\right)^{5/2}-13c^2\left(1-c^2x^2\right)^{3/2}+7c^2\sqrt{1-c^2x^2}+\frac{8\sqrt{1-c^2x^2}}{x^2}\right)dx^2}{60c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\left(1-c^4x^4\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}$$

$$\Big\downarrow \text{2009}$$

3.174. $\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$

$$-\frac{(1-c^4x^4)^{5/2}(a+b\text{csch}^{-1}(cx))}{10c^{12}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{3c^{12}}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^{12}}-\frac{b\sqrt{c^2x^2+1}\left(-16\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{2}{3}(1-c^2x^2)^{9/2}-\frac{18}{7}(1-c^2x^2)^{7/2}+\frac{26}{5}(1-c^2x^2)^{5/2}-\frac{14}{3}(1-c^2x^2)^{3/2}+\right.}{60c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}$$

input
```
Int[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/c^12 + ((1 - c^4*x^4)^(3/2)*
(a + b*ArcCsch[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcCsch[c*x])
)/(10*c^12) - (b*Sqrt[1 + c^2*x^2]*(16*Sqrt[1 - c^2*x^2] - (14*(1 - c^2*x^
2)^(3/2))/3 + (26*(1 - c^2*x^2)^(5/2))/5 - (18*(1 - c^2*x^2)^(7/2))/7 + (2
*(1 - c^2*x^2)^(9/2))/3 - 16*ArcTanh[Sqrt[1 - c^2*x^2]]))/(60*c^13*Sqrt[1
+ 1/(c^2*x^2)]*x)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.174.4 Maple [F]

$$\int \frac{x^{11}(a+b\ \mathrm{arccsch}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.97

$$\int \frac{x^{11}\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx =$$

$$-\frac{105\,(3\,bc^{10}x^{10}+3\,bc^8x^8+4\,bc^6x^6+4\,bc^4x^4+8\,bc^2x^2+8\,b)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(35\,bc^9x^9$$

input
```
integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/3150*(105*(3*b*c^10*x^10 + 3*b*c^8*x^8 + 4*b*c^6*x^6 + 4*b*c^4*x^4 + 8*
b*c^2*x^2 + 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 1)/(c*x)) + (35*b*c^9*x^9 - 5*b*c^7*x^7 + 78*b*c^5*x^5 - 36*b*c^3*x^3 +
 768*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 420*(b*c^2*
x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2
)) + 1)/(c^2*x^2 + 1)) + 420*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4
 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 105*(3*a*c^1
0*x^10 + 3*a*c^8*x^8 + 4*a*c^6*x^6 + 4*a*c^4*x^4 + 8*a*c^2*x^2 + 8*a)*sqrt
(-c^4*x^4 + 1))/(c^14*x^2 + c^12)
```

3.174.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\text{Timed out}$$

input
```
integrate(x**11*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.174.7 Maxima [F]

$$\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\,\text{arcsch}\,(cx)+a)x^{11}}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima"
)
```

output
```
-1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*s
qrt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*
log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)
*c^12) - 30*integrate((x^11*log(c) + x^11*log(x))*e^(-1/2*log(c^2*x^2 + 1)
 - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 30*integrate(1/30*(3*c^10*x
^11 - 3*c^8*x^9 + 4*c^6*x^7 - 4*c^4*x^5 + 8*c^2*x^3 - 8*x)/(sqrt(c^2*x^2 +
 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^10 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^10)
, x))
```

3.174.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^{11}(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11}(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^{11}\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^11*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^11*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.175 $\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.175.1 Optimal result

Integrand size = 26, antiderivative size = 264

$$\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{b(1-c^2x^2)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{b(1-c^2x^2)^{5/2}\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8} + \frac{(1-c^4x^4)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\sqrt{1+c^2x^2}\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)}{3c^9\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

output
```
1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsch(c*x))/c^8+1/18*b*(-c^2*x^2+1)^(3/2)*(c
^2*x^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/30*b*(-c^2*x^2+1)^(5/2)*(c^2*x
^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)+1/3*b*arctanh((-c^2*x^2+1)^(1/2))*(c
^2*x^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^
2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(
1/2)/c^8
```

3.175.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.68

$$\int \frac{x^7 (a + b\text{csch}^{-1}(cx))}{\sqrt{1 - c^4x^4}} \, dx =$$
$$-\frac{15a\sqrt{1-c^4x^4}(2+c^4x^4) + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}(28-c^2x^2+3c^4x^4)}{1+c^2x^2} + 15b\sqrt{1-c^4x^4}(2+c^4x^4)\,\text{csch}^{-1}(cx) + 30b}{90c^8}$$

input
```
Integrate[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x
*Sqrt[1 - c^4*x^4]*(28 - c^2*x^2 + 3*c^4*x^4))/(1 + c^2*x^2) + 15*b*Sqrt[1
 - c^4*x^4]*(2 + c^4*x^4)*ArcCsch[c*x] + 30*b*Log[x + c^2*x^3] - 30*b*Log[
1 + c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^8
```

3.175.3 Rubi [A] (warning: unable to verify)

Time = 1.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.59, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {6864, 27, 7272, 1388, 1579, 517, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 (a + b\text{csch}^{-1}(cx))}{\sqrt{1 - c^4x^4}} \, dx$$
$$\Big\downarrow \ 6864$$
$$\frac{b\int -\frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{6c^8\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c} + \frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8} - \frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$
$$\Big\downarrow \ 27$$
$$-\frac{b\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{6c^9} + \frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8} - \frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$
$$\Big\downarrow \ 7272$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^4x^4}(c^4x^4+2)}{x\sqrt{c^2x^2+1}}dx}{6c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{ 1388}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^2x^2}(c^4x^4+2)}{x}dx}{6c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{ 1579}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^2x^2}(c^4x^4+2)}{x^2}dx^2}{12c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{ 517}$$

$$-\frac{b\sqrt{c^2x^2+1}\int -\frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{1-c^2x^2}}{6c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{b\sqrt{c^2x^2+1}\int \frac{x^4(c^4x^8-2c^4x^4+3c^4)}{1-x^4}d\sqrt{1-c^2x^2}}{6c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{b\sqrt{c^2x^2+1}\int \left(-c^4x^8+c^4x^4-2c^4+\frac{2c^4}{1-x^4}\right)d\sqrt{1-c^2x^2}}{6c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{(1-c^4x^4)^{3/2}(a+b\text{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^8}$$

$$\Big\downarrow \text{ 2009}$$

3.175. $\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

$$\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6c^8}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^8}-\frac{b\sqrt{c^2x^2+1}\left(-2c^4\operatorname{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{c^4x^{10}}{5}-\frac{c^4x^6}{3}+2c^4\sqrt{1-c^2x^2}\right)}{6c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}$$

input
```
Int[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/c^8 + ((1 - c^4*x^4)^(3/2)*(
a + b*ArcCsch[c*x]))/(6*c^8) - (b*Sqrt[1 + c^2*x^2]*(-1/3*(c^4*x^6) + (c^4
*x^10)/5 + 2*c^4*Sqrt[1 - c^2*x^2] - 2*c^4*ArcTanh[Sqrt[1 - c^2*x^2]]))/(6
*c^13*Sqrt[1 + 1/(c^2*x^2)]*x)
```

3.175.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 517
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2*(e^m/d^(m + 2*p + 1))   Subst[Int[x^(2*n + 1)*(-c + x^
2)^m*(b*c^2 + a*d^2 - 2*b*c*x^2 + b*x^4)^p, x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && ILtQ[m, 0] && IntegerQ[n + 1/2]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1579
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

3.175. $\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6864
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.175.4 Maple [F]

$$\int \frac{x^7(a+b\ \text{arccsch}\,(cx))}{\sqrt{-c^4x^4+1}}dx$$

input
```
int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.23

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{15\,(bc^6x^6+bc^4x^4+2\,bc^2x^2+2\,b)\sqrt{-c^4x^4+1}\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(3\,bc^5x^5-bc^3x^3+28\,bcx)\sqrt{-c^4x^4}}{}$$

input
```
integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/90*(15*(b*c^6*x^6 + b*c^4*x^4 + 2*b*c^2*x^2 + 2*b)*sqrt(-c^4*x^4 + 1)*l
og((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (3*b*c^5*x^5 - b*c^3*x
^3 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 15*(b*c^
2*x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c^2*x^2 + 1)) + 15*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^
4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 15*(a*c^6*x
^6 + a*c^4*x^4 + 2*a*c^2*x^2 + 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 + c^8)
```

3.175.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^7 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\text{Timed out}$$

input
```
integrate(x**7*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Timed out
```

3.175.7 Maxima [F]

$$\int \frac{x^7 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\,\mathrm{arcsch}\,(cx)+a)x^7}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*
x^8 + c^4*x^4 - 2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x
+ 1)*sqrt(-c*x + 1)*c^8) - 6*integrate((x^7*log(c) + x^7*log(x))*e^(-1/2*l
og(c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 6*integrate(
1/6*(c^6*x^7 - c^4*x^5 + 2*c^2*x^3 - 2*x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)
*sqrt(-c*x + 1)*c^6 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^6), x))
```

3.175.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \int \frac{x^7\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.176 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.176.1 Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = \frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} - \frac{bx\arctan\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1-c^2x^2}}\right)}{2c^3\sqrt{-c^2x^2}}$$

output
```
-1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(-c^2*x^2-1)^(1/2))/c^3/(-c^2*x^2)^(1/2
)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/c^4+1/2*b*x*(-c^4*x^4+1)^(1/2)
/c^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

3.176.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{a\sqrt{1-c^4x^4} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{1-c^4x^4}}{1+c^2x^2} + b\sqrt{1-c^4x^4}\text{csch}^{-1}(cx) + b\log\left(x+c^2x^3\right) - b\log\left(1+c^2x^2+c\sqrt{1+\right.}{2c^4}$$

input
```
Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(a*Sqrt[1 - c^4*x^4] + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]
)/(1 + c^2*x^2) + b*Sqrt[1 - c^4*x^4]*ArcCsch[c*x] + b*Log[x + c^2*x^3] -
b*Log[1 + c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^4
```

3.176.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6864, 27, 1896, 1388, 243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}dx$$

$$\Big\downarrow \text{6864}$$

$$\frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{c}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1+\frac{1}{c^2x^2}}x^2}dx}{2c^5}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{1896}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{c^2x^2+1}}dx}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{1388}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^2x^2}}{x}dx}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{243}$$

$$-\frac{b\sqrt{c^2x^2+1}\int \frac{\sqrt{1-c^2x^2}}{x^2}dx^2}{4c^5x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4}$$

$$\Big\downarrow \text{60}$$

$$-\frac{b\sqrt{c^2x^2+1}\left(\int \frac{1}{x^2\sqrt{1-c^2x^2}}dx^2+2\sqrt{1-c^2x^2}\right)}{4c^5x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{73}$$

$$-\frac{b\sqrt{c^2x^2+1}\left(2\sqrt{1-c^2x^2}-\frac{2\int \frac{1}{\frac{1}{c^2}-\frac{x^4}{c^2}}d\sqrt{1-c^2x^2}}{c^2}\right)}{4c^5x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^4}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\sqrt{1-c^4x^4}(a+b\text{csch}^{-1}(cx))}{2c^4}-\frac{b\sqrt{c^2x^2+1}\left(2\sqrt{1-c^2x^2}-2\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)}{4c^5x\sqrt{\frac{1}{c^2x^2}+1}}$$

input
```
Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

output
```
-1/2*(Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/c^4 - (b*Sqrt[1 + c^2*x^2]*(
2*Sqrt[1 - c^2*x^2] - 2*ArcTanh[Sqrt[1 - c^2*x^2]]))/(4*c^5*Sqrt[1 + 1/(c^
2*x^2)]*x)
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1388

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1896

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q])   Int[x^(m + mn*q)*(1 + d*(1/(x^
mn*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] &&
EqQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

rule 6864

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Simp[(a + b*ArcCsch[c*x])   v, x] + Simp[b/c   Int[SimplifyIntegr
and[v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]
] /; FreeQ[{a, b, c}, x]
```

3.176. $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

3.176.4 Maple [F]

$$\int \frac{x^3(a + b \operatorname{arccsch}(cx))}{\sqrt{-c^4x^4 + 1}}dx$$

input
```
int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

output
```
int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(110) = 220.

Time = 0.28 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.04

$$\int \frac{x^3(a + b\operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4x^4}}dx =$$
$$-\frac{2\sqrt{-c^4x^4 + 1}bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2\sqrt{-c^4x^4 + 1}(bc^2x^2 + b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (bc^2x^2 + b)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4}}{c}\right.}{4(c^6x^2 + c^4}$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*sqrt(-c
^4*x^4 + 1)*(b*c^2*x^2 + b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x)) - (b*c^2*x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + (b*c^2*x^2 + b)*log(-(c^2*x^2 - sqr
t(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 2*
sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 + a))/(c^6*x^2 + c^4)
```

3.176.6 Sympy [F]

$$\int \frac{x^3(a + b\operatorname{csch}^{-1}(cx))}{\sqrt{1 - c^4x^4}}dx = \int \frac{x^3(a + b \operatorname{acsch}(cx))}{\sqrt{-(cx - 1)(cx + 1)(c^2x^2 + 1)}}dx$$

input
```
integrate(x**3*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral(x**3*(a + b*acsch(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
), x)
```

3.176.7 Maxima [F]

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*((c^4*x^4 - 1)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*
x + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate((x^3*log(c) + x^3*log(x))*e^(-1/2
*log(c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 2*integrat
e(1/2*(c^2*x^3 - x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2 +
sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

3.176.8 Giac [F]

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{(b\operatorname{arcsch}(cx)+a)x^3}{\sqrt{-c^4x^4+1}}\,dx$$

input
```
integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\operatorname{csch}^{-1}(cx))}{\sqrt{1-c^4x^4}}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((x^3*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

output
```
int((x^3*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.177 $\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

3.177.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.177.2 Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

3.177.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$
$$\Big\downarrow \text{ 6866}$$
$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.177.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.177.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\operatorname{arccsch}\left(cx\right)}{x\sqrt{-c^{4}x^{4}+1}}dx$$

input
```
int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

3.177.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^5 - x), x)
```

3.177.6 Sympy [N/A]

Not integrable

Time = 7.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{a + b\,\text{acsch}\,(cx)}{x\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)))
, x)
```

3.177.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{a + b\text{csch}^{-1}(cx)}{x\sqrt{1 - c^4x^4}}\,dx = \int \frac{b\,\text{arcsch}\,(cx) + a}{\sqrt{-c^4x^4 + 1}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*int
egrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)
*(c*x - 1))*x), x)
```

3.177.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

3.177.9 Mupad [N/A]

Not integrable

Time = 6.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.178 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

3.178.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

output
```
Unintegrable((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.178.2 Mathematica [N/A]

Not integrable

Time = 8.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

input
```
Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

3.178.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6866}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}} \, dx$$

$$\Big\downarrow \text{ 6866}$$

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}dx$$

input
```
Int[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
```

output
```
$Aborted
```

3.178.3.1 Defintions of rubi rules used

rule 6866
```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegra
ble[u*(a + b*ArcCsch[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.178.4 Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\,\operatorname{arccsch}\left(cx\right)}{x^{5} \sqrt{-c^{4} x^{4}+1}}dx$$

input
```
int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

output
```
int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

3.178.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^9 - x^5), x)
```

3.178.6 Sympy [N/A]

Not integrable

Time = 85.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{acsch}\,(cx)}{x^5\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^2x^2+1\right)}}\,dx$$

input
```
integrate((a+b*acsch(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

output
```
Integral((a + b*acsch(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1
))), x)
```

3.178.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.31

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) +
2*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c
*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

3.178.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{b\,\text{arcsch}\,(cx)+a}{\sqrt{-c^4x^4+1}x^5}\,dx$$

input
```
integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

3.178.9 Mupad [N/A]

Not integrable

Time = 5.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

input
```
int((a + b*asinh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
```

output
```
int((a + b*asinh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                      Small rewrite of logic in main function to make it*)
(*                      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
   print ("Enter grade_antiderivative for sagemath")
   print("Enter grade_antiderivative, result=",result)
   print("Enter grade_antiderivative, optimal=",optimal)
   print("type(anti)=",type(result))
   print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation =" "
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```